Sample Problems

Prove each of the following identities.

1. $\tan x \sin x + \cos x = \sec x$
2. $\dfrac{1}{\tan x} + \tan x = \dfrac{1}{\sin x \cos x}$
3. $\sin x - \sin x \cos^2 x = \sin^3 x$
4. $\dfrac{\cos \alpha}{1 + \sin \alpha} + \dfrac{1 + \sin \alpha}{\cos \alpha} = 2 \sec \alpha$
5. $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2 \tan x$
6. $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$
7. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$
8. $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$
9. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$
10. $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$
11. $\tan^2 \theta = \csc^2 \theta \tan^2 \theta - 1$
12. $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$
13. $\dfrac{\csc \beta}{\sin \beta} - \dfrac{\cot \beta}{\tan \beta} = 1$
14. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$
15. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$
16. $\dfrac{\sin^2 x + 4 \sin x + 3}{\cos^2 x} = \dfrac{3 + \sin x}{1 - \sin x}$
17. $\dfrac{\cos x}{1 - \sin x} - \tan x = \sec x$
18. $\tan^2 x + 1 + \tan x \sec x = \dfrac{1 + \sin x}{\cos^2 x}$

What else can we "discover"?

How about rewriting the following functions using sin x and cos x.

csc x = _____

sec x = _____

tan x = _____

cot x = _____

} Just another way to substitute …makes your trig life a bit easier at times.

Let's have some fun by substituting these new trigonometric properties to prove both sides are equal!

Problem 1 Using substitution, try to convert *sin x cot x* into *cos x*.

sin x cot x = _____

= _____

= _____

Prove it here (in exact form!): Let $u = 45°$

Problem 2 Using substitution, try to convert *csc x – sin x* into *cot x cos x*.

csc x – sin x = _____

(hint: add fractions) = _____

= _____

Prove it here (in exact form!): Let $u = 240°$

Problem 3 Using substitution, try to convert *(1 + sin x) (1 – sin x)* into $cos^2 x$

(1 + sin x) (1 – sin x) = _____

(hint: F.O.I.L) = _____

Prove it here (in exact form!): Let $u = 300°$

Reciprocal Identities: Isn't it true that a value multiplied by its reciprocal equals 1? Again, just another way to look at things.

1. (sin x) (_____) = 1 2. (cos x) (_____) = 1 3. (tan x) (_____) = 1

4. (cot x) (_____) = 1 5. (csc x) (_____) = 1 6. (sec x) (_____) = 1