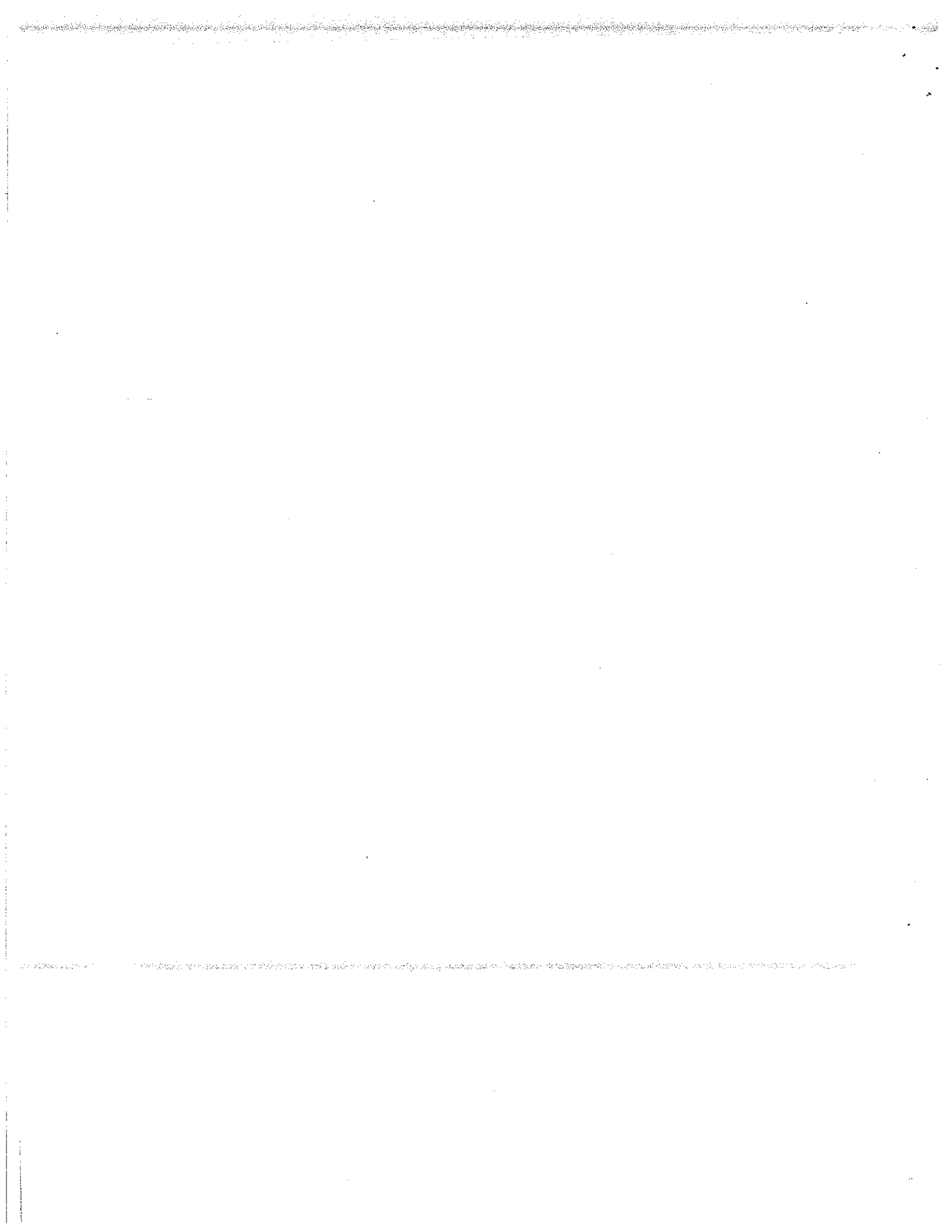


# Unit 5: Probability ●

April				
Monday	Tuesday	Wednesday	Thursday	Friday
12 Set Notation, Venn Diagrams, Experimental Probability	13 Mutually Exclusive and Overlapping Events	14 Conditional Probability  At School	15 Review/Quiz	16 Virtual Day, Delta Review Assignment
19 Independent and Dependent Events	20 Using Formulas and Working Backwards	21 <i>Test Review</i>	22 Unit 5 Test	23

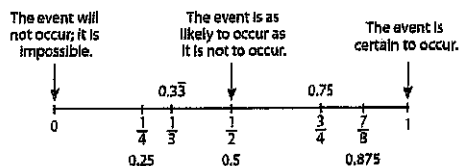


Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Vocabulary, Set Notation, & Venn Diagrams

### Probability

- A number from 0 to 1
- As a percent from \_\_\_\_\_ to \_\_\_\_\_
- Indicates how likely an \_\_\_\_\_ will occur.



### Experiment

- Any process or action that has observable results
- Example: \_\_\_\_\_

### Outcomes

- \_\_\_\_\_
- Example: \_\_\_\_\_

### Sample Space

- The set (or list) of \_\_\_\_\_
- Also known as the \_\_\_\_\_
- Example: \_\_\_\_\_

### Event

- A subset of an \_\_\_\_\_
- An outcome or \_\_\_\_\_
- Example: \_\_\_\_\_

### Set

- \_\_\_\_\_

### Subset

- List or collection of \_\_\_\_\_ all contained within another set.
- Denoted by \_\_\_\_\_ if all the elements of A are also in B.

---

**Empty Set**

- A set that has \_\_\_\_\_
- Also called a \_\_\_\_\_
- Denoted by \_\_\_\_\_

---

**Union**

- Denoted by \_\_\_\_\_
- To unite
- Everything in \_\_\_\_\_ sets

---

**Intersection**

- Denoted by \_\_\_\_\_
- Only what the sets \_\_\_\_\_ in common.

---

**Complement**

- Denoted two different ways: \_\_\_\_\_ or \_\_\_\_\_
- Everything \_\_\_\_\_ of this set

---

**Hector has entered the following names in the contact list of his new cellphone: Alicia, Brisa, Steve, Don, and Ellis.**

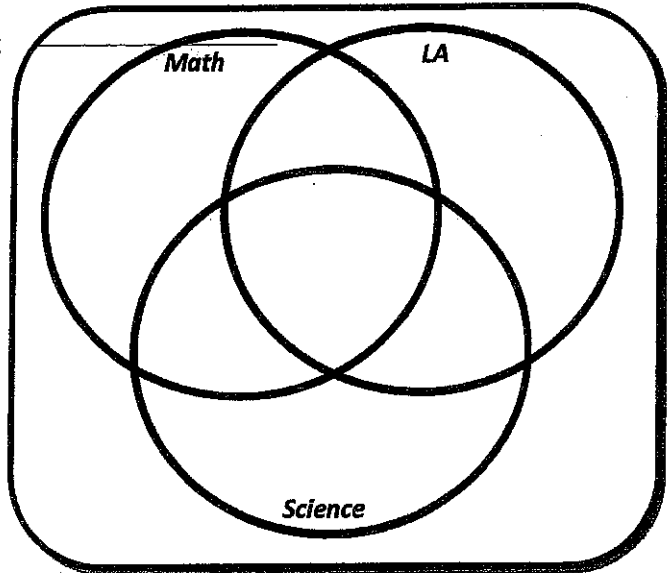
1. Draw a venn diagram to represent this.
  2. List the outcomes of B.
  3. List the outcomes of E.
  4. List the outcomes of  $B \cap E$ .
  5. List the outcomes of  $B \cup E$ .
  6. List the outcomes of  $B'$ .
  7. List the outcomes of  $(B \cup E)'$ .
-

Applications of Probability: Sets

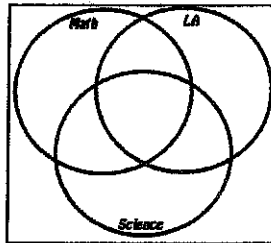
Name: \_\_\_\_\_

1. A. Make a VENN diagram of the following Chart showing what classes each student was enrolled in this semester.

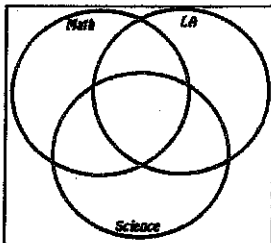
Name	Math	Language Arts	Science
Betsy		✓	✓
Devonte	✓	✓	✓
Frank	✓		✓
Heather		✓	
Jessica		✓	
Krista	✓		



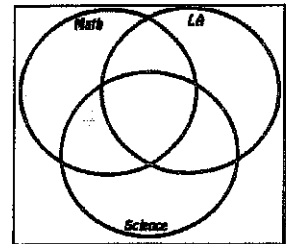
B. (LA):



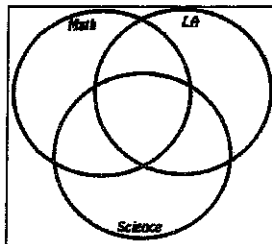
C. (Math ∩ Science):



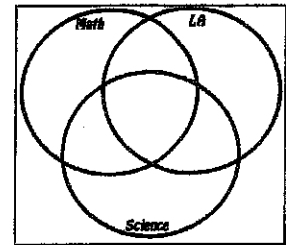
D. (Math)':



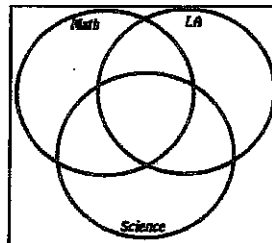
E. (Math ∪ LA):



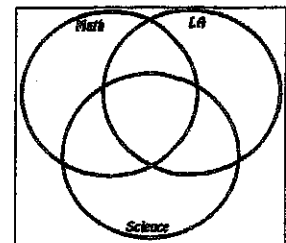
F. (Math ∪ LA)':



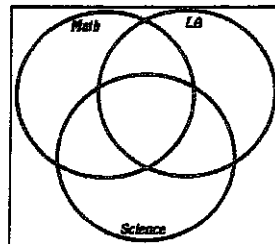
G. (Math ∩ LA)':



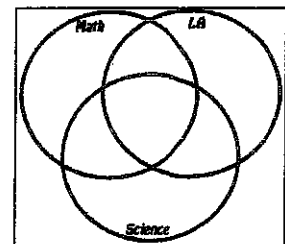
H. (Math ∩ LA ∩ Science):



I. (Math ∪ LA) ∩ (Science):



J. (Math ∩ LA) ∪ (Science):



2. Given  $A = \{1, 2, 3, 6, 7, 9\}$ ,  $B = \{2, 4, 6, 7, 8\}$ , and  $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  answer the following.

A.  $(A \cap B)$ :

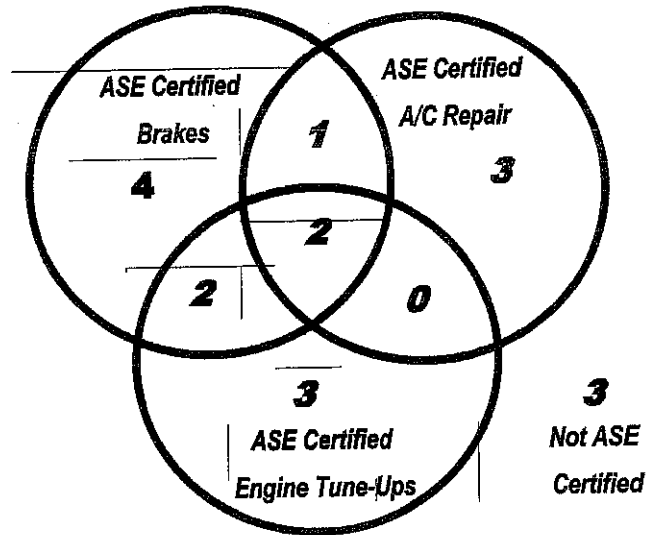
B.  $(A \cup B)$ :

C.  $(A)'$ :

D.  $(A \cap B)'$ :

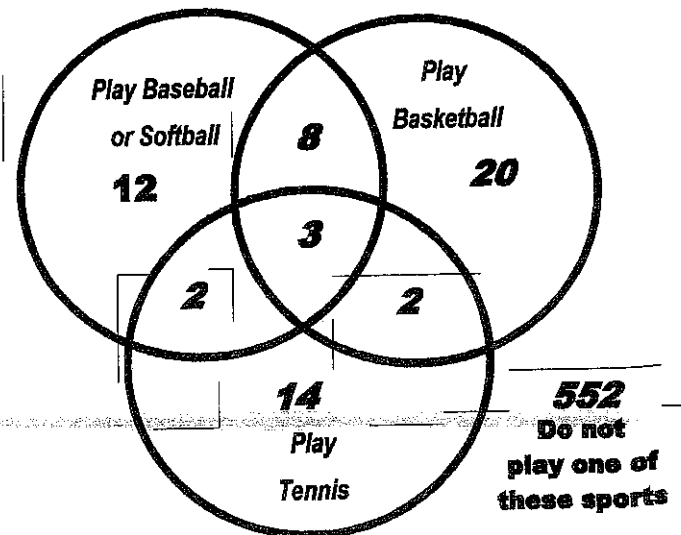
3. A manager that owns 3 local area Car Maintenance Garages was researching certifications of mechanics that worked for her company. Consider the following Venn diagram.

- How many mechanics worked for her company?
- How many of the mechanics are certified by ASE to do work on Brakes?
- How many of the mechanics are certified by ASE to do work on Brakes and Tune-Ups ( $\text{Brakes} \cap \text{Tune-Ups}$ )?
- How many of the mechanics are certified by ASE to do work on either A/C or Tune-Ups ( $\text{A/C} \cup \text{Tune-Ups}$ )??
- How many of the mechanics have their certification in Brakes or A/C but not in Tune-Ups???  
( $\text{Brakes} \cup \text{A/C} \cap (\text{Tune Ups})'$ )



4. The following Venn diagram shows a breakdown of a small high schools sports program.

- How many students play only Tennis?
- How many students play basketball and tennis?
- How many students play basketball or softball/baseball?  
 $\text{Basketball} \cup \text{Baseball/Softball}$
- How many students play baseball/softball or tennis but not basketball?  
 $(\text{Baseball/Softball} \cup \text{Tennis}) \cap (\text{Basketball})'$
- How many students that play a sport do not play basketball?
- How many students attend this school?
- How many students do not play tennis in total?



5. In the state of Oregon, all of the area codes start with a number greater than 4 and end in an odd number (e.g. 503-232-1235, 971-923-5648). Let A represent the set of all area codes that start with an even number. Let B represent the set of all area codes that could be used in Oregon by the requirements stated earlier.

Which might be an area code that belongs to the set  $(A \cap B)$ ?

- A. 403                      B. 792                      C. 892                      D. 631

Which might be an area code that belongs to the set  $(A \cap B')$ ?

- A. 402                      B. 792                      C. 892                      D. 631

Which might be an area code that belongs to the set  $(A' \cap B')$ ?

- A. 403                      B. 392                      C. 892                      D. 631

6. In a particular state, the first character on a license plate is always a letter. The last character is always a digit from 0 to 9. Let V represent the set of all license plates beginning with a vowel, and O represents the set of all license plates that end with an odd number,

Which might be a license plate that belongs to the set  $(V \cap O)$ ?

- A.       B.       C.       D. 

Which might be a license plate that belongs to the set  $(V \cap O')$ ?

- A.       B.       C.       D. 

Which might be a license plate that belongs to the set  $(V' \cap O')$ ?

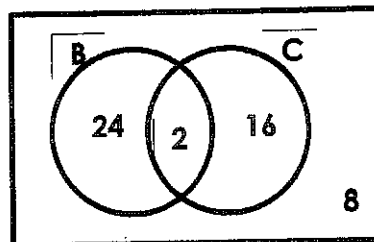
- A.       B.       C.       D. 

Name: \_\_\_\_\_ Date: \_\_\_\_\_

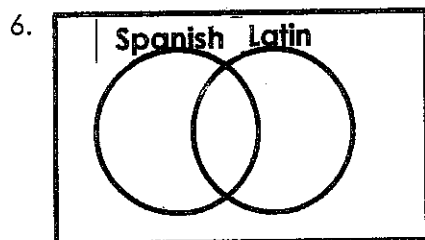
### Using Venn Diagrams

If the Venn Diagram below shows the number of people in a fine arts club who are in band (B) and choir (C), make the following determinates:

- \_\_\_\_\_ 1. How many people are in the club?
- \_\_\_\_\_ 2. Find  $P(B)$
- \_\_\_\_\_ 3. Find  $P(B \cap C)$
- \_\_\_\_\_ 4. Find  $P(B \cup C)$
- \_\_\_\_\_ 5. Find  $P(B)'$



A guidance counselor is planning schedules for 30 students. 16 want to take Spanish and 11 want to take Latin. 5 say they want to take both. Display this information on the Venn Diagram below.



- \_\_\_\_\_ 7. Find  $P(S \cap L)$
- \_\_\_\_\_ 8. Find  $P(L)$
- \_\_\_\_\_ 9. What is the probability that a student studies at least one subject?  $P(S \cup L)$
- \_\_\_\_\_ 10. What is the probability that a student studies exactly one subject?
- \_\_\_\_\_ 11. What is the probability that a student studies neither subject?  $P(S \cup L)'$
- \_\_\_\_\_ 12. What is the probability that a student studied Spanish if it is known that the student studies Latin?

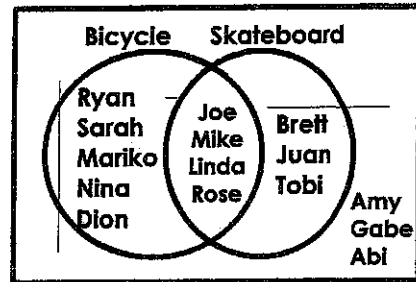


**Mr. Leary's Class:** Use the Venn Diagram showing the number of kids owning bicycles (A) and skateboards (B) to find the following probabilities.

\_\_\_\_\_ 13. Find  $P(A \cap B)$  and describe what this probability represents?

\_\_\_\_\_ 14. Find  $P(A \cup B)$  and describe what this probability represents?

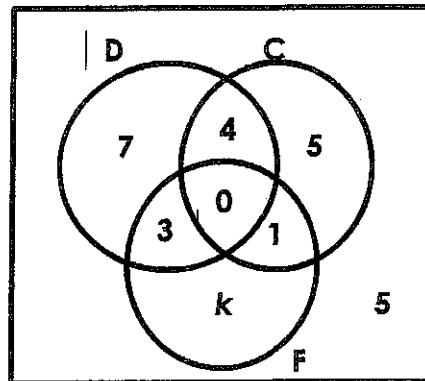
\_\_\_\_\_ 15. Find  $P(A \cup B)'$  and describe what this probability represents?



**The Venn Diagram below shows the results of a survey done by a veterinarian about the types of pets owned by 26 clients. The survey was only related to dogs (D), cats (C), and fish (F).**

\_\_\_\_\_ 16. What is the value of  $k$ ?

\_\_\_\_\_ 17. How did you determine the value?



**If a randomly selected member is asked their preference, what is the probability that the member has:**

\_\_\_\_\_ 18. Only dogs?

\_\_\_\_\_ 19. Dogs and cats?

\_\_\_\_\_ 20. None of these animals?

\_\_\_\_\_ 21. At least one of these pets?

\_\_\_\_\_ 22. All of the pets?

\_\_\_\_\_ 23. Fish and dogs, but not cats?

\_\_\_\_\_ 24. Fish or dogs?

### Mutually Exclusive vs. Overlapping

\*If two or more events cannot occur at the same time they are termed **mutually exclusive**.

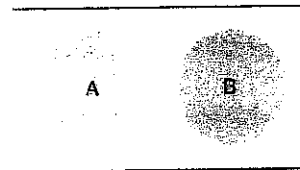
\*They have **no** common outcomes.

\* **Overlapping** events have **at least one common outcome**.

1

### Mutually Exclusive Formula

$$P(A \text{ or } B) = P(A) + P(B)$$



2

**OR  
Means  
you ADD**

3

### Sum of Rolling 2 Dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

4

#### Example 2:

\*When rolling two dice find

**P(sum 4 or sum 5)**

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

5

#### Example 1:

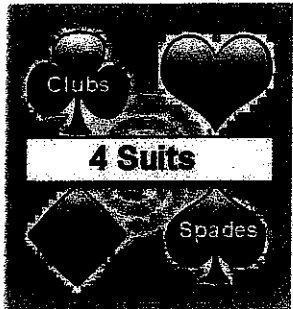
\* Find the probability that a girl's favorite department store is **Macy's or Nordstrom**.

\* Find the probability that a girl's favorite store is **not** JC Penny's.

Macy's	0.25
Saks	0.20
Nordstrom	0.20
JC Penny's	0.10
Bloomingdale's	0.25

6

### Deck of Cards



- 52 total cards
- 4 Suits
- 13 cards in each suit
- 3 Face cards in each suit

7

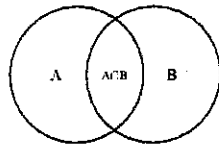
### Example 3:

\*In a deck of cards, find  
**P(Queen or Ace)**

8

### Overlapping Events Formula

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$



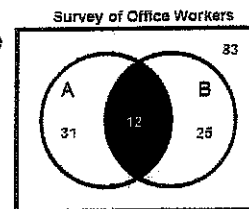
9

### Example 4:

\*Find the probability that a person will drink **both**.

\*A = drink coffee

\*B = drink soda



10

### Example 5:

- \* Find the  $P(A \cup B)$
- \* A = band members
- \* B = club members
- \* A = 195 students
- \* B = 565 club members
- \* 35 students do both band and a club.
- \* 1200 total students at the High School

11

### Example 6:

\*In a deck of cards find  
**P(King or Club)**

12

**Example 7:**

\*Find the P(picking a **female** or a person from **Florida**).

	Female	Male
FL	8	4
AL	6	3
GA	7	3

13

**Example 8:**

\*When rolling 2 dice, find P(an **even sum** or a number **greater than 10**).

1	2	3	4	5	6	
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

14

**Example 9: Complementary Events**

Find  $P(\overline{A \cup B}) =$

- \* A = band members
- \* B = club members
- \* A = 195 students
- \* B = 565 club members
- \* 35 students do both band and a club.
- \* 1200 total students at the High School

15

**Example 10: Complementary Events**

A = plays volleyball  
(26 students)

B = plays softball (37  
Students)

There are 454 total  
athletes

What is the probability  
that someone does  
**not** play volleyball?

$$P(\overline{A}) =$$

16

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Mutually Exclusive Practice

Determine if the following events are mutually exclusive or overlapping.

- \_\_\_\_\_ 1. The experiment is rolling a die.  
 The 1st event: the number is greater than 3  
 The 2nd event: the number is even.
- \_\_\_\_\_ 2. The experiment is year in school.  
 The 1st event: the person is a senior.  
 The 2nd event: the person is a junior.
- \_\_\_\_\_ 3. The experiment is answering multiple choice questions.  
 The 1st event: the correct answer is chosen  
 The 2nd event: the answer A is chosen.
- \_\_\_\_\_ 4. The experiment is selecting a chocolate bar.  
 The 1st event: the bar has nuts  
 The 2nd event: the bar has caramel.

- \_\_\_\_\_ 5. One card is randomly drawn from a deck of 52 cards. The card is face down on the table. What is the probability of getting a Jack or a Spade?

Use the general addition rule to compute the probability that if you roll two six-sided dice.

- \_\_\_\_\_ 6. you get doubles or a sum of 4

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- \_\_\_\_\_ 7. you get doubles or a sum of 7

- \_\_\_\_\_ 8. you get a 5 on the first die or you get a 5 on the second die.

Use the Venn Diagram to answer the following questions.

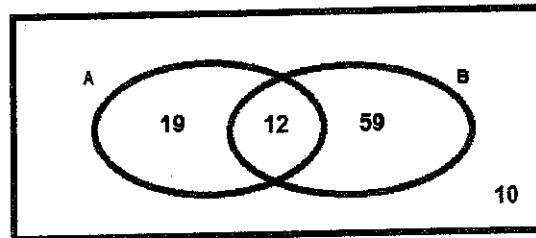
\_\_\_\_\_ 9.  $P(A)$

\_\_\_\_\_ 10.  $P(B)$

\_\_\_\_\_ 11.  $P(B)'$

\_\_\_\_\_ 12.  $P(A \cup B)$

\_\_\_\_\_ 13.  $P(A \cap B)$



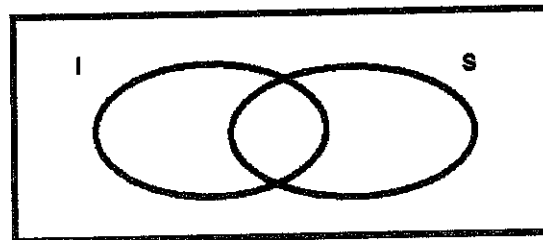
When you arrive home today, you find 27 cupcakes in a large circular plate. There are 13 that have icing 11 have sprinkles, and 4 have both.

\_\_\_\_\_ 14.  $P(I)$

\_\_\_\_\_ 15.  $P(S)$

\_\_\_\_\_ 16.  $P(I \cup S)$

\_\_\_\_\_ 17.  $P(I \cap S)$



Use the data below to find each of the following probabilities.

**Coollest Deals Sold at Ike's**

Topping choice	Ice cream choice			
	Vanilla	Chocolate	Cookie dough	Mint chip
Sprinkles	9	12	16	14
Hot fudge	11	4	16	15
Caramel	10	12	18	15

\_\_\_\_\_ 18.  $P(\text{Chocolate})$

\_\_\_\_\_ 19.  $P(\text{Chocolate})'$

\_\_\_\_\_ 20.  $P(\text{Sprinkles} \cap \text{Cookie Dough})$

\_\_\_\_\_ 21.  $P(\text{Caramel} \cup \text{Vanilla})$

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Suppose that you select a person at random from your school. Are these pairs of events mutually exclusive?

\_\_\_\_\_ 1. has ridden a roller coaster; has ridden a Ferris wheel

\_\_\_\_\_ 2. has brown hair; has brown eyes

\_\_\_\_\_ 3. is left-handed; is right-handed

\_\_\_\_\_ 4. owns a classical music CD; owns a jazz music CD

\_\_\_\_\_ 5. is a senior; is a junior

\_\_\_\_\_ 6. has shoulder-length hair; is male

7. A group of senior citizens have won free vacation packages. The vacation to Bermuda is chosen by 25% of them, 60% choose Alaska, and 15% choose Costa Rica. What is the probability that one randomly chosen senior citizen chooses to vacation in Bermuda or Costa Rica?

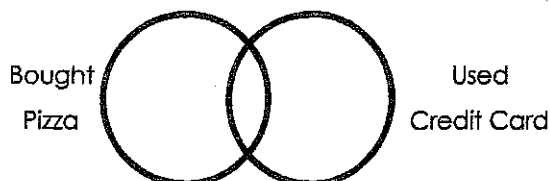
Use the general addition rule to compute the probability that if you roll two six-sided dice,

\_\_\_\_\_ 8. you get odd sum or a sum greater than 10.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

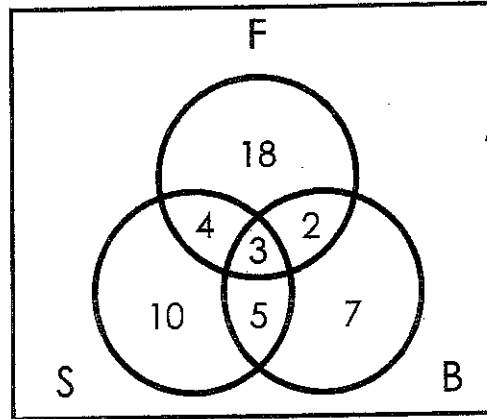
\_\_\_\_\_ 9. you get even sum or a sum of 11.

10. Of the 220 people who came into the Italian deli on Friday, 104 bought pizza and 82 used a credit card. Half of the people who bought pizza used a credit card. What is the probability that a customer bought pizza or used a credit card?



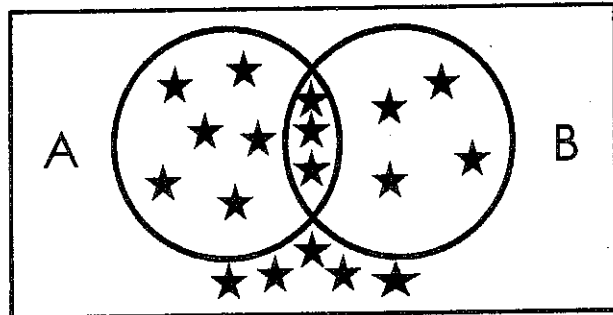
A group of 60 students were asked if they played field hockey (F), basketball (B) or soccer (S). The diagram below displays the results. Use the information given to find the following probabilities.

- \_\_\_\_\_ 11.  $P(B \cap S)$
- \_\_\_\_\_ 12.  $P(F \cup B)$
- \_\_\_\_\_ 13.  $P(F)'$
- \_\_\_\_\_ 14.  $P(F \cup B \cup S)$
- \_\_\_\_\_ 15.  $P(F \cup B \cup S)'$



Given the Venn Diagram below with set A and set B determine the following:

- \_\_\_\_\_ 16.  $P(A \cap B)$
- \_\_\_\_\_ 17.  $P(A \cup B)$
- \_\_\_\_\_ 18.  $P(\bar{A} \cup B)$
- \_\_\_\_\_ 19.  $P(A \cap \bar{B})$



20. Suppose 80% of people can swim. Suppose 70% of people can whistle. Suppose 55% of people can do both. What percentage of people can swim or whistle?

21. At Harrison, 60% of the students carry a backpack or a wallet, 40% carry only a backpack, and 30% carry only a wallet. If a student is selected at random, find the probability that the student carries both a backpack and a wallet.



# Conditional Probability

## Definition:

The probability that A occurs given that B occurs is called the conditional probability of "A given B" and is written  $P(A | B)$

## Examples of Conditional Scenarios

- What is the probability that someone is in the Harrison band if you know the person is a freshman?
- What is the probability of drawing an ace in a game of go fish if you know that you already have two of them?
- What is the probability of drawing a Queen if you know that it is a face card?

## The "Lawyers and Liars" Scenario

- There are 100 people at a party.
- Forty are liars.
- Twenty-five are lawyers.
- 15 of the lawyers are liars.

A non-conditional Scenario: What is the probability that you randomly pick a lawyer?

## The "Lawyers and Liars" Scenario

- There are 100 people at a party.
- Forty are liars.
- Twenty-five are lawyers.
- 15 of the lawyers are liars.

A Conditional Scenario: What is the probability that you pick a liar if you know that the person you are talking to is a lawyer?

## A Formula and How to Use it

- There are 100 people at a party.
- Forty are liars.
- Twenty-five are lawyers.
- 15 of the lawyers are liars.

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

A Conditional Scenario: What is the probability that you pick a liar if you know that the person you are talking to is a lawyer?

Using the formula

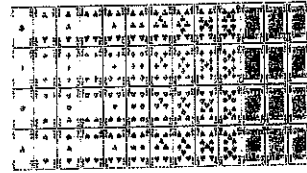
Determine  $P(B|A)$  using the following data:

$$P(A) = .54$$

$$P(B) = .3$$

$$P(A \cap B) = .216$$

What is the probability of drawing a Queen if you know that it is a face card?



In a class of students, the following data table summarizes how many students passed a test and complete the homework due the day of the test. What is the probability that a student completed the homework given that they passed the test?

	Passed Test	Failed Test
Completed HW	15	3
Did not Complete HW	4	6

More Conditional From a table

The following table shows data collected about new born babies. What is the probability of picking a baby with brown eyes if you know the baby has a slow heart rate?

	Slow heart rate (<100 bpm)	Fast heart rate (>100 bpm)
Brown eyes	6	2
Blue eyes	4	8

More Conditional From a table

The following table shows data collected about new born babies. What is the probability of picking a baby with a fast heart rate if you know the baby has blue eyes?

	Slow heart rate (<100 bpm)	Fast heart rate (>100 bpm)
Brown eyes	6	2
Blue eyes	4	8

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### The Conditional Probability from Tables

The frequencies of the marbles in a bag are shown in the table.

\_\_\_\_\_ 1. Find  $P(\text{small})$

\_\_\_\_\_ 2. Find  $P(\text{green} | \text{large})$

	GREEN	BLUE
LARGE	2	4
SMALL	8	12

A town planning committee is considering a new system for public transit. Residents of the town were randomly selected to answer two questions: "Do you work less than 5 miles from home?" and "Would you use the new system to get to work, if it were available?" The results are shown in the table below.

		Work less than 5 miles from home?	
		YES	NO
Use new system?	YES	24	32
	NO	44	20

\_\_\_\_\_ 3. If residents work less than 5 miles from home, what is the probability that they would use the new system?

\_\_\_\_\_ 4. If residents are willing to use the new system, what is the probability that they don't work less than 5 miles from home?

The table shows the results of a poll of randomly selected high school students who were asked if they prefer to hear all school announcements in the morning or afternoon.

	Underclassmen	Upperclassmen
Morning	8	14
Afternoon	18	10

\_\_\_\_\_ 5. Find  $P(\text{Morning} | \text{Underclassmen})$

\_\_\_\_\_ 6. Find  $P(\text{Afternoon} | \text{Upperclassmen})$

The table shows the results of a customer satisfaction survey for a cellular service provider, by location of the customer. In the survey, customers were asked whether they would recommend a plan with the provider to a friend.

\_\_\_\_\_ 7. Find  $P(\text{Yes})$

\_\_\_\_\_ 8. Find  $P(\text{Yes} | \text{Arlington})$

\_\_\_\_\_ 9. Are the 2 probabilities the same?

	Arlington	Towson	Parkville
Yes	40	35	41
No	18	10	6

Roberto is the owner of a car dealership. He is assessing the success rates of his top three sales people in order to offer one of them a promotion. Over two months, for each attempted sale, he records whether the sales person made a successful sale or not. The results are shown in the cart below.

\_\_\_\_\_ 10. Find  $P(\text{Successful} | \text{Becky})$

\_\_\_\_\_ 11. Find  $P(\text{Unsuccessful} | \text{Darrell})$

	Successful	Unsuccessful
Becky	6	6
Earl	4	5
Darrell	6	9

Mrs. Koehler surveyed 430 men and 200 women about their vehicles. Of those surveyed, 160 men and 85 women said they own a blue vehicle.

\_\_\_\_\_ 12. If a randomly chosen person is a man, what is the probability of that person having a blue car?

\_\_\_\_\_ 13.  $P(\text{Blue})'$

\_\_\_\_\_ 14.  $P(\text{Women} | \text{Not Blue})$

\_\_\_\_\_ 15.  $P(\text{Men} \cap \text{Not Blue})$

	Blue	Not Blue
Men		
Women		

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Conditional Probability

A random survey was taken to gather information about grade level and car ownership status of students at a school. This table shows the results of the survey.

**Car Ownership by Grade**

	Owens a Car	Does Not Own a Car	TOTAL
Junior	6	10	16
Senior	12	8	20
TOTAL	18	18	36

- \_\_\_\_\_ 1. Find the probability that a randomly selected student will be a junior, given that the student owns a car.
- \_\_\_\_\_ 2. Find the probability that a randomly selected student will own a car, given that the student is a senior.

The table below shows numbers of registered voters by age in the United States in 2004 based on the census. Find each probability in decimal form.

Age	Registered Voters (in thousands)	Not Registered to Vote (in thousands)
18-24	14,334	13,474
25-44	49,371	32,763
45-64	51,659	19,355
65 and over	26,706	8,033

- \_\_\_\_\_ 3. Find the probability that a randomly selected person is registered to vote, given that the person is between the ages of 18 and 24.
- \_\_\_\_\_ 4. Find the probability that a randomly selected person is not registered to vote, given that they are 65 and over.
- \_\_\_\_\_ 5. Find the probability that a randomly selected person is between the ages of 45 and 64 and is not registered to vote.

A faculty advisor at Ridge High School surveyed 100 students about their preference for a social event. Of the 100 students surveyed, 50 were tenth graders and 50 were eleventh graders. Of the tenth graders, 30 chose a bowling party and 20 chose a dance. Of the eleventh graders, 20 chose a bowling party and 30 chose a dance.

6. Make a two way frequency table to represent the data.

	Bowling (B)	Dance (D)	
10 <sup>th</sup> graders (T)			
11 <sup>th</sup> graders (E)			

\_\_\_\_\_ 7. Find  $P(B)$ .

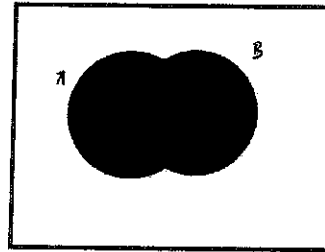
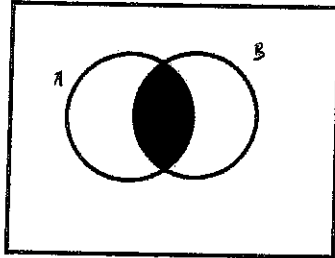
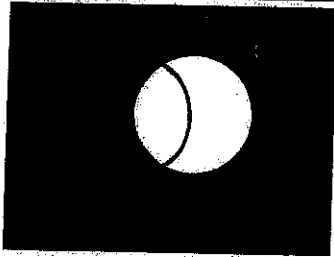
\_\_\_\_\_ 8. Find  $P(B | T)$ .

\_\_\_\_\_ 9. Do you think that the probability of liking bowling is dependent on whether a student is in the 10<sup>th</sup> or 11<sup>th</sup> grade?

**Probability Practice Quiz**

Set Notation and Venn Diagrams: Match the set notation to its Venn diagram. Each option will be used only once.

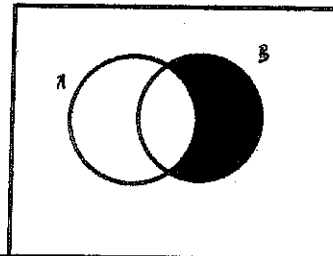
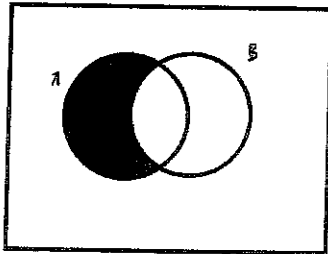
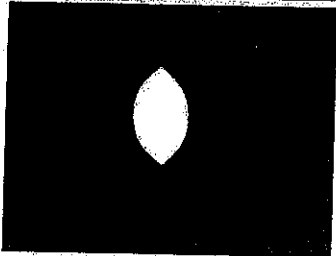
- a.  $A \cap B$     b.  $(A \cap B)'$     c.  $A \cup B$     d.  $B'$     e.  $B \cap A'$     f.  $A \cap B'$



1.

2.

3.



4.

5.

6.

Set Notation: Answer the following questions based on the universal set and subsets given below.

**(Universal)**  $\Omega = \{A, B, C, D, E, F, G, 1, 2, 3, 4, 5, 6, 8, 9, 11\}$

$\epsilon = \{A, D, E, F, 1, 2, 4, 6, \}$

$\xi = \{A, B, C, F, 1, 2, 3, 5\}$

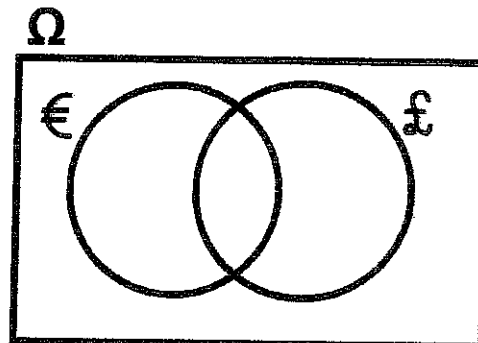
7. List the elements of the set  $(\epsilon \cap \xi)$

8. List the elements of the set  $(\epsilon \cup \xi)$

9. List the elements of the set  $(\epsilon \cup \xi)'$

10. List the elements of the set  $(\epsilon \cap \xi)'$

11. Draw a Venn Diagram to Represent the Sets:



## Probability

Find the experimental probability of the following events. Your answer should be a fully reduced fraction.

12. Bridget categorized all the people who entered the theatre for a show by their hair color. The data is shown in the table below

black	13
gray	35
brown	22

a. Determine  $P(\text{black})$  : \_\_\_\_\_

b. Determine  $P(\text{grey})$  : \_\_\_\_\_

c. Determine  $P(\text{black or brown})$  : \_\_\_\_\_

13. An ice cream store recorded its sales for the week in the summer. Their data is shown below.

mint	307
rocky road	219
mocha	244

What is the experimental probability that the next cone sold is mocha? Your answer should be a fully reduced fraction.

\_\_\_\_\_

14. Determine the following probabilities from the table below:

	Organic	Non-organic
Apples	13	8
Peaches	6	3
Blueberries	13	12

a.  $P(\text{Apple and Non-Organic}) =$  \_\_\_\_\_

b.  $P(\text{Apple or Non- Organic}) =$  \_\_\_\_\_

c.  $P(\text{Blueberries or Peaches}) =$  \_\_\_\_\_

d.  $P(\text{Blueberry and Organic}) =$  \_\_\_\_\_

e.  $P(\text{Blueberries or Organic}) =$  \_\_\_\_\_

15. Determine the Following probabilities from the data:

$P(A) = .8$   $P(B) = .5$   $P(A \cap B) = .4$  Determine:  $P(A \cup B) =$  \_\_\_\_\_

16.  $P(C) = .6$   $P(B) = .5$   $P(C \cup B) = .8$  Determine:  $P(C \cap B) =$  \_\_\_\_\_



**Determine if the following are mutually exclusive or overlapping, then find the probability of each.**

17. In a deck of cards find the probability of drawing a Jack or a heart.

Mutually Exclusive or Overlapping? \_\_\_\_\_

P (Jack or Heart) = \_\_\_\_\_

18. In a deck of cards, find the probability of drawing a Queen or Even Numbered card.

Mutually Exclusive or Overlapping? \_\_\_\_\_

P(Queen or Even Numbered Card) = \_\_\_\_\_

19. You're rolling two dice and looking at their sum. What is the probability of rolling an even sum or a sum greater than 8?

Mutually Exclusive or Overlapping? \_\_\_\_\_

P(Even sum or Sum greater than 8) = \_\_\_\_\_

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

20. Answer the questions about probabilities from the sum chart.

a. P (Even sum and sum greater than 8) = \_\_\_\_\_

b. P(odd sum or sum less than 6) = \_\_\_\_\_

c. P(sum of 12 or sum of 3) = \_\_\_\_\_

# Probability

## Independent vs. Dependent events

1

## Independent Events

- Two events A and B, are independent if the fact that A occurs does not affect the probability of B occurring.
- Examples- EX 1. Landing on heads from two different coins; EX 2. rolling a 4 on a die, then rolling a 3 on a second roll of the die.
- Probability of A and B occurring:  
 $P(A \text{ and } B) = P(A) \cdot P(B)$

2

## Experiment 1

- A coin is tossed and a 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.



3

## Experiment 2

- A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and an eight?



4

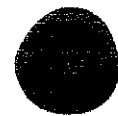
## Experiment 3

- A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and a yellow marble?

5

## Experiment 4

- A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with replacement, what is the probability that all three students like pizza?



6

### Dependent Events

- Two events A and B, are dependent if the fact that A occurs affects the probability of B occurring.
- Examples- Picking a blue marble and then picking another blue marble if I don't replace the first one.
- Probability of A and B occurring:  
 $P(A \text{ and } B) = P(A) \cdot P(B | A)$

7

### Experiment 1

- A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. A second marble is chosen without replacing the first one. What is the probability of choosing a green and a yellow marble?

8

### Experiment 2

- An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly select a second fish. What is the probability that both fish are male?

9

### Experiment 3

- A random sample of parts coming off a machine is done by an inspector. He found that 5 out of 100 parts are bad on average. If he were to do a new sample, what is the probability that he picks a bad part and then, picks another bad part if he doesn't replace the first?

10

## Independent vs. Dependent

Determining if 2 events  
are independent

11

### Independent Events

- Two events are independent if the following are true:  
 $P(A | B) = P(A)$   
 $P(B | A) = P(B)$   
 $P(A \text{ AND } B) = P(A) \cdot P(B)$
- To show 2 events are independent, you must prove one of the above conditions.

12

### Experiment 1

- Let event  $G$  = taking a math class. Let event  $H$  = taking a science class. Then,  $G$  AND  $H$  = taking a math class and a science class.
- Suppose  $P(G) = 0.6$ ,  $P(H) = 0.5$ , and  $P(G \text{ AND } H) = 0.3$ .
- Are  $G$  and  $H$  independent?

13

### Experiment 2

- In a particular college class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair.
- Let  $F$  be the event that the student is female. Let  $L$  be the event that the student has long hair.
- One student is picked randomly. Are the events of being female and having long hair independent?

14

### Approach #2

- If they are independent,  $P(L|F)$  should equal  $P(L)$ .
- $0.75 \neq 0.5$

15

Independent Probability

Name: \_\_\_\_\_

Two events are said to be **Independent** if the occurrence of the first event does **NOT** affect the probability of the second event and events are independent if  $P(A) \cdot P(B) = P(A \text{ and } B)$

**INDEPENDENT PROBABILITY**

1. Determine the following probabilities if each of the following are **independent**.

<b>GIVEN:</b>	$P(A) = 0.8$	$P(B) = 0.25$	$P(C) = 0.6$
---------------	--------------	---------------	--------------

a.  $P(A \text{ and } C) =$

Decimal:

b.  $P(A \text{ and } B \text{ and } C) =$

Decimal:

c.  $P(\text{Rolling a 4 on a standard die and } B) =$

Decimal:

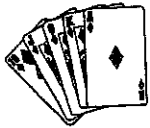
d. Find  $P(D)$  if  $D$  is an independent event and  $P(C \text{ and } D) = 0.10$

Decimal:



e.  $P(\text{Rolling a 2 on a standard die and picking a card with a "7" on it from a standard deck of cards}) =$

Decimal:



f. If your chances of losing the shell game if you randomly pick is 2 in 3. What are the chances that you would lose 5 games in a row?

Decimal:



g. If the Atlanta Hawks free throw percentage is 82%, what is the probability that a player for the Hawks will make 2 free shots in a row?

Percentage:

h. The chance of rain on a random day in May in Gwinnett is about 30%. Using this empirical probability, what would you estimate the probability of having NO rain for an entire week (7 days)?

Percentage:



<b>GIVEN:</b>	$P(M) = 0.8$	$P(N) = 0.25$	$P(R) = 0.6$
---------------	--------------	---------------	--------------

a. If the probability of  $P(M \text{ and } N) = 0.2$ , are  $M$  and  $N$  independent?

b. If the probability of  $P(N \text{ and } R) = 0.3$ , are  $N$  and  $R$  independent?

**DEPENDENT PROBABILITIES**

3. Consider that 3 consecutive cards are drawn **without replacement** from a shuffled deck of cards



A. What is the probability that the first two cards drawn are face cards?

Decimal:

B. What is the probability that the all three cards are hearts?

Decimal:

C. What is the probability that all three cards are a King?

Decimal:

D. What is the probability that all three cards are numbered?

Decimal:

4. A bag contains 4 blue marbles, 4 red marbles, and 4 green marbles:

A. What is the probability of drawing 2 green marbles **without replacement**?

Decimal:

B. What is the probability of drawing 3 marbles without replacement in a row of the same color **without replacement**?

Decimal:

5. James has 3 dimes, 4 pennies, and 2 quarters in his pocket. If each coin is equally likely to be pulled out of his pocket in order **without replacement**, what is the probability that he will pull out the 2 quarters in a row first?

Reduced Fraction:

6. In a cookie jar there are 10 chocolate chip cookies and 8 peanut butter cookies left. The cookies are randomly mixed together in the jar. What is the probability of pulling two of the same types of cookies out of the cookie jar in a row **without replacement**?



DECIMAL:

7. In a classroom there are 7 male students and 11 female students that are taking a test. If each student is equally likely to turn in their test at any given time at the end of class, what is the probability that the first 3 students to turn in their test are female students?



DECIMAL:

# Probability Part 2 Practice Quiz

## Part 1: Conditional Probability

1. Calculate the probability of  $P(A|B)$  given the data:

$$P(A) = \frac{3}{4}$$

$$P(B) = \frac{7}{8}$$

$$P(A \cap B) = \frac{5}{8}$$

$$P(A|B) = \underline{\hspace{2cm}}$$

2. Calculate the probability given the table:

The table shows the relationship between cups of coffee that Jamie drank, and number of hours of sleep she got.

	6hrs Sleep	7 hours sleep
0 cups	5	5
1 cup	4	4

a. Determine the probability Jamie slept 6 hours given that she had 0 cups of coffee.

$$P(6hr | 0 coffee) = \underline{\hspace{2cm}}$$

b. Determine the probability that Jamie drank 1 cup of coffee given that she slept 7 hours.

$$P(1 coffee | 7hr) = \underline{\hspace{2cm}}$$

3. Calculate the probabilities given the table:

The data shows the ages and types of shoes that people were wearing at a movie theatre one weekend.

	Tennis shoes	Dress Shoes
13 to 30	15	6
31 to 50	9	13
51 to 60	11	4

a. Determine the probability that a person was wearing tennis shoes given they are in the 31 to 50 age group.

$$P(\text{Tennis} | 31 \text{ to } 50) = \underline{\hspace{2cm}}$$

b. If a person had on dress shoes, what is the probability that they were in the 13 to 30 category?

$$P(13 \text{ to } 30 | \text{dress}) = \underline{\hspace{2cm}}$$

4. Determine the probability from the data:  $P(A) = .4$   $P(B) = .3$   $P(A \cap B) = .26$

What is the probability of  $P(B|A)$ ?  $\underline{\hspace{2cm}}$

## Part 2: Independent and Dependent Events

Determine if the scenario represents independent or dependent events, then calculate the probability.

5. Flipping a coin that lands on heads, and rolling a die that lands on 2.
6. A bag has 8 green marbles, 3 blue marbles, and 9 red marbles. What is the probability of drawing 3 blue marbles in a row, **without replacement**?
7. A bag has 8 green marbles, 3 blue marbles, and 9 red marbles. What is the probability of drawing 3 blue marbles in a row, **with replacement**?
8. Flipping a coin three times in a row, and getting tails all three times.
9. Drawing two jacks in a row out of a deck of cards, **without replacement**.
10. Drawing two diamonds in a row out of a deck of cards, **with replacement**.

## Part 3: Determining Independence

Show calculations to justify your answer

11. Given the following probabilities, determine if events A and B are independent.

$$P(A) = \frac{4}{9} \quad P(B) = \frac{3}{4} \quad P(A \cap B) = \frac{2}{9}$$

Independent? \_\_\_\_\_

12. Use the data in the table to determine if the events are independent.  
Are the events "waiting 3 to 7 minutes" and "ordering from Chicken Club" independent?

	Waiting 3-7 min	Chicken Club
Waiting 3-7 min	10	12
Waiting 8-12 min	2	7
Waiting 13-15 min	12	3

Independent? \_\_\_\_\_



## Using Probability Formulas and Working Backwards: Notes

Overlapping/Inclusive:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1. Given,  $P(A) = 0.4$ ,  $P(B) = 0.3$ ,  $P(A \cap B) = 0.1$ , determine the probability of  $P(A \cup B)$ .
2. Given,  $P(A) = 0.5$ ,  $P(B) = 0.3$ ,  $P(A \cup B) = 0.6$ , determine the probability of  $P(A \cap B)$ ?
3. On a soccer team there are 20 players. The probability that a player likes Gatorade or Powerade is  $\frac{18}{20}$ , the probability that they like Gatorade is  $\frac{16}{20}$ , the probability that they like Powerade is  $\frac{8}{20}$ . Find the probability that a player likes both.

Conditional/ Dependent Conditional:  $P(A | B) = \frac{P(A \cap B)}{P(B)}$  Dependent:  $P(A \cap B) = P(A) \cdot P(B | A)$

4. Given that the probability of A is .68, the probability of B is .5, and the probability of  $(A | B) = .91$ , find the probability of  $(A \cap B)$ .
5.  $P(A) = .3$   $P(B) = .44$  and the probability of  $P(B | A) = .65$  find  $P(A \cap B)$ .
6. In an experiment, the probability that a person likes to run is  $\frac{8}{9}$ . The probability that someone likes dogs given they like to run is  $\frac{1}{2}$ . Find the probability that a person likes dog and likes to run.

Independent  $P(A \cap B) = P(A) \cdot P(B)$  and  $P(A | B) = P(A)$  and  $P(B | A) = P(B)$

7. Events A and B are independent. If the probability of A is .56 and the probability of  $(A \cap B)$  is .168, determine the probability of B.
  
  
  
  
  
  
  
  
  
  
8. Events A and B are independent. If the probability of A is .26 and the probability of B is .6, determine the probability of  $(B | A)$ .
  
  
  
  
  
  
  
  
  
  
9. The probability of Kyle showing up to an event is .8. The probability of Kyle and Rob showing up is .68. Given the events are independent, what is the probability that Rob shows up.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### Using Probability Formulas and Working Backwards

**Mutually Exclusive:**  $P(A \cup B) = P(A) + P(B)$

**Overlapping:**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Independent:**  $P(A \cap B) = P(A) \cdot P(B)$

**Dependent:**  $P(A \cap B) = P(A) \cdot P(B | A)$

1. For two events A and B, it is known that  $P(A) = 0.20$ ,  $P(B) = 0.40$  and  $P(A \cup B) = 0.50$ . Find  $P(A \cap B)$ .

2. For two events X and Y, it is known that  $P(X) = 2/5$  and  $P(X \cap Y) = 1/5$ . Find  $P(Y | X)$ .

3. For two events B and C, it is known that  $P(C | B) = 0.61$  and  $P(C \cap B) = 0.48$ . Find  $P(B)$ .

4. Suppose that the probability of Eirik coming to a party is 80% and the probability of Emma coming to a party is 95%. Assuming that these events are independent, what is the probability that they both will come to a party?

5. The probability of playing basketball is 12%, and the probability of playing both basketball and football is 5%. What is the probability of a person playing football, given they play basketball?

6. Joel and Rico play basketball. The probability that Joel makes a 3 pointer is 64%. The probability that Rico makes a 3 pointer is 87.5%. The probability of at least one of them making a 3 pointer is 95.5%. What is the probability, as a percent, that both Joel and Rico will make a 3 pointer?

# PROBABILITY REVIEW

1. A bag contains 16 marbles. Four are red, ten are blue, and 2 are green. Find the following probabilities:
- You draw one marble out of the bag, find:  $P(\text{red or green})$
  - Probability of drawing red on the first draw, blue on the second draw, and green on the third draw **with replacement**.
  - Probability of drawing three blue marbles in a row, **without replacement**.
  - Probability of drawing a red marble, given someone took a green marble out earlier and never put it back.

2. Julie spins a spinner with 8 equal sections numbered 1-8. Find the following probabilities.
- Probability of spinning an even number, or a number greater than 5.
  - Probability of spinning an even number given that you got a number greater than 5.
  - Probability of spinning an even number and a number greater than five.

3. Two dice are rolled. The chart below shows the possible sums. Find the following probabilities.

- $P(\text{sum of six or sum of 10})$
- $P(\text{Sum of six and sum of 10})$
- $P(\text{even sum or sum greater than 9})$
- $P(\text{even sum} \mid \text{sum greater than 9})$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

# PROBABILITY REVIEW

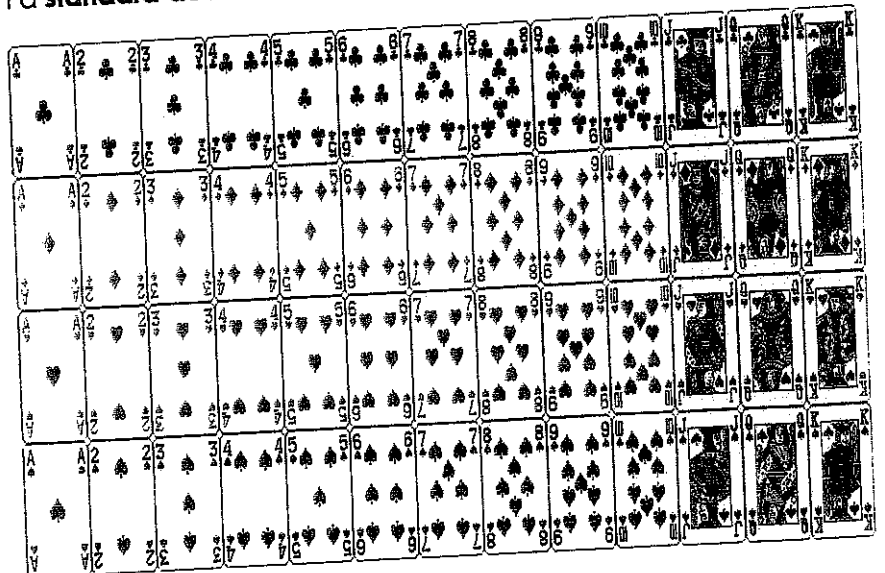
4. Find the following Probabilities according to the table.

	20	50	20
	65	100	45

- $P(\text{Domestic})$
- $P(\text{Compact and Foreign})$
- $P(\text{Domestic or SUV})$
- $P(\text{Midsize} \mid \text{Domestic})$
- $P(\text{Domestic})'$

5. The following questions are based on a **standard deck of cards**.

- Find the probability of drawing a 4 and then a king, without replacement.
- Probability of a Club and then Diamond without replacement.
- Two sevens in a row without replacement.

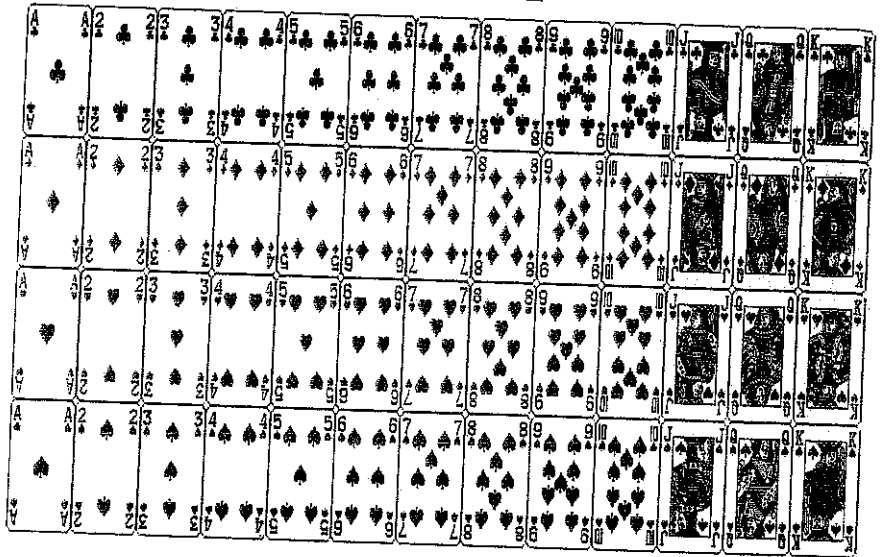


# PROBABILITY REVIEW

d. A heart and then a seven with replacement.

e. Probability of both are hearts with replacement.

f. Probability of Heart or a club.



g. Probability of Heart and Club. (Think: only one draw is happening)

h. Probability of drawing a card with a letter on it or an even numbered card.

i. Probability of drawing a heart or a card with an even number on it.

6. Answer the questions given the following Probabilities

**Events C and D are independent and  $P(C) = .2$  and  $P(D) = .7$**

Find: a.  $P(C \text{ and } D) =$  \_\_\_\_\_

b.  $P(C \text{ or } D) =$  \_\_\_\_\_

c.  $P(C | D) =$  \_\_\_\_\_

**7. Given Events G and H with  $P(G) = .3$   $P(H) = .2$  and  $P(G \text{ and } H) = .1$**

Find: a.  $P(G \text{ or } H) =$  \_\_\_\_\_

b.  $P(G | H) =$  \_\_\_\_\_

c. Are G and H Mutually exclusive? Explain. \_\_\_\_\_

d. Are G and H Independent? Explain. \_\_\_\_\_

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Probability Review**

In a bowl of marbles, there are 10 red ones, 6 green ones, and 8 blue ones.

- \_\_\_\_\_ 1. If a marble is chosen at random from the bowl, find  $P(\text{red one or a blue one})$ ?
- \_\_\_\_\_ 2. If two marbles are chosen at random with replacement, find  $P(\text{red and a blue})$ ?
- \_\_\_\_\_ 3. If two marbles are chosen at random without replacement, find  $P(\text{they are both red})$ ?

A person rolls two dice, one after the other.

- \_\_\_\_\_ 4.  $P(\text{even sum})$  or  $P(\text{sum of } 9)$
- \_\_\_\_\_ 5.  $P(\text{odd sum})$  or  $P(\text{sum less than } 5)$
- \_\_\_\_\_ 6. What is the probability that the sum of two rolls is an even number **given** at least one of the rolls is a 4?

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

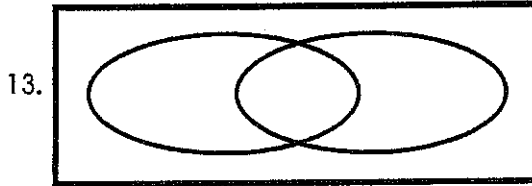
A card is chosen from a standard deck of cards. The drawer is looking for clubs and face cards.

	Club	Not a Club
Face card	3	9
Not a face card	10	30

- \_\_\_\_\_ 7. Find  $P(\text{Club})$
- \_\_\_\_\_ 8. Find  $P(\text{Club} \mid \text{Not a Face Card})$
- \_\_\_\_\_ 9. Find  $P(\text{Club} \cap \text{Face Card})$
- \_\_\_\_\_ 10. Find  $P(\text{Not a Club} \cup \text{Not a Face Card})$
- \_\_\_\_\_ 11. Are the events Club and Not a Face Card Independent of each other?

12. In an Algebra class, 22 students were male and 10 students were female. Out of those students, 11 of the guys and 4 of the girls passed the EOC. If a person is chosen at random from the class, what is the probability of choosing a girl or a person that did NOT pass the EOC?

	Pass	Not Pass
Male		
Female		



Of 500 athletes surveyed, 300 were male and 20 were left-handed. Only 8 of the left-handed athletes were female.

14. What is the probability that an athlete was male or was left-handed?

In a survey of 450 people, 200 of whom are female, it was found that 225 prefer chocolate ice cream including 99 males. Use this information to complete the table below.

	Males	Females	
Vanilla			
Chocolate			
			450

15. The person likes chocolate.

16. The person like vanilla, given they are male.

17. The person likes vanilla or is a female.

18. Are being a male and liking chocolate independent events?

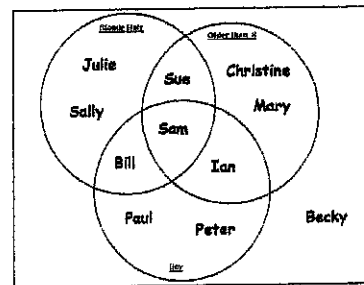
Use the Venn diagram to find the following probabilities.

19.  $P(\text{blonde hair})$

20.  $P(\text{blonde hair} \cap \text{Boy})$

21.  $P(\text{Older than 8} \cup \text{Boy})$

22.  $P(\text{Older than 8} \cup \text{Boy})'$



23. The probability of a randomly chosen boy playing basketball is 0.30. The chance that a boy plays both basketball and football is 0.05. The chance that a boy plays football is 0.25. What is the probability that a randomly chosen boy plays basketball or football?

24. Assume that the following events are dependent:

- The probability that a high school student eats breakfast is 0.8.
- The probability that a high school senior will eat breakfast and get over 6 hours of sleep is 0.2.

What is the probability that a high school senior will get over 6 hours of sleep, given that the person ate breakfast?