Algebra 1 Name: ________________

Unit 3C – Test Review – answer key

Part One: Graphing Quadratics

Graph the following quadratic functions.

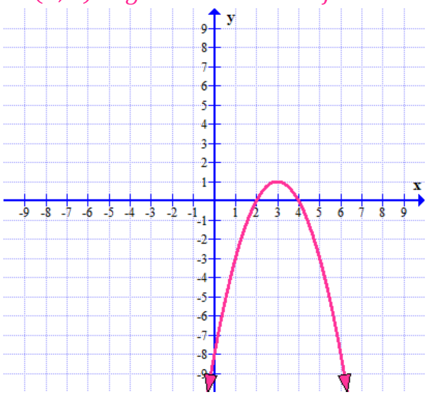
1) $f(x) = -x^2 + 6x - 8$

vertex: $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$

$y = -(3)^2 + 6(3) - 8 = 1$

vertex @ $(3, 1)$ – *goes in middle of table*

x	$f(x)$
1	-3
2	0
3	1
4	0
5	-3



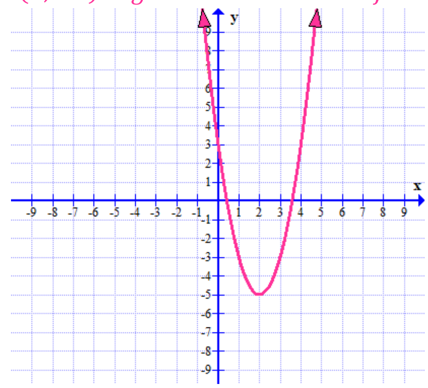
2) $f(x) = 2x^2 - 8x + 3$

vertex: $x = \frac{-b}{2a} = \frac{8}{2(2)} = \frac{8}{4} = 2$

$y = 2(2)^2 - 8(2) + 3 = -5$

vertex @ $(2, -5)$ – *goes in the middle of table*

x	$f(x)$
0	3
1	-3
2	-5
3	-3
4	3

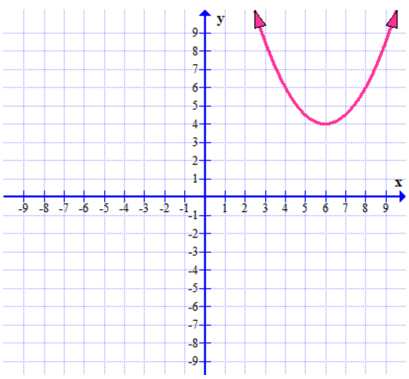


3) $f(x) = \frac{1}{2}(x - 6)^2 + 4$

$a = \frac{1}{2}$ $h = 6$ $k = 4$

vertex @ $(6, 4)$ – *goes in middle of table*

x	$f(x)$
4	6
5	4.5
6	4
7	4.5
8	6



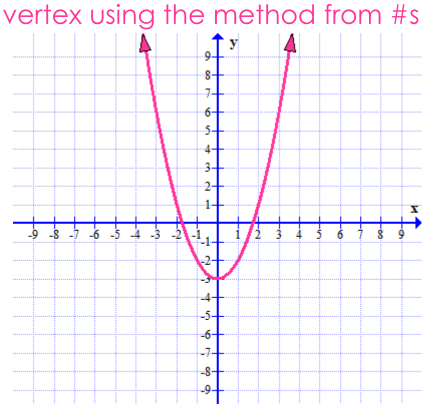
4) $f(x) = x^2 - 3$

$a = 1$ $h = 0$ $k = -3$

vertex @ $(0, -3)$ – *goes in middle of table*

can also find vertex using the method from #s 1-2

x	$f(x)$
-2	1
-1	-2
0	-3
1	-2
2	1

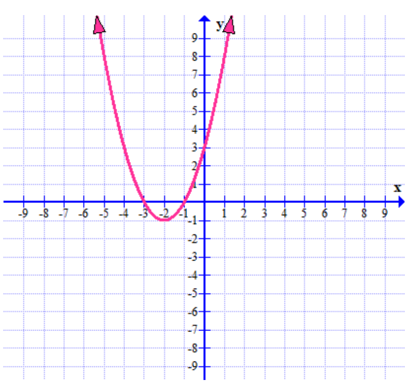


5) $f(x) = (x + 2)^2 - 1$

$a = 1$ $h = -2$ $k = -1$

vertex @ $(-2, -1)$ – *goes in middle of table*

x	$f(x)$
-4	3
-3	0
-2	-1
-1	0
0	3



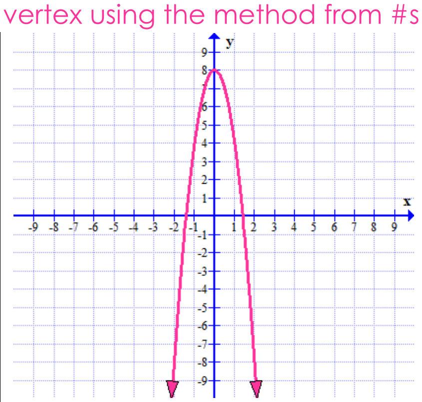
6) $f(x) = -4x^2 + 8$

$a = -4$ $h = 0$ $k = 8$

vertex @ $(0, 8)$ – *goes in middle of table*

can also find vertex using the method from #s 1-2

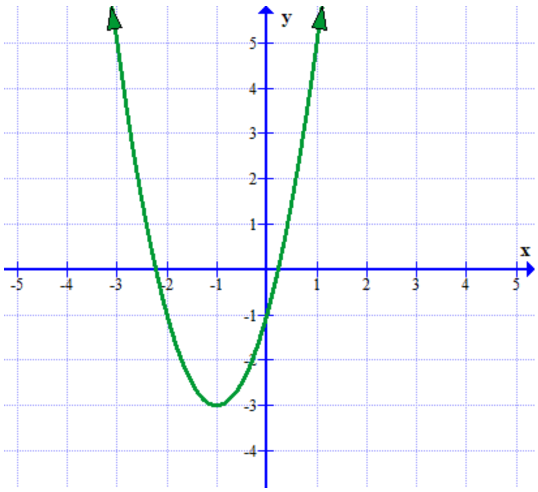
x	$f(x)$
-2	-8
-1	4
0	8
1	4
2	-8



Part Two: Characteristics of Graphs

Identify the listed characteristics for each graph.

7)



Domain: $(-\infty, \infty)$ or all real numbers

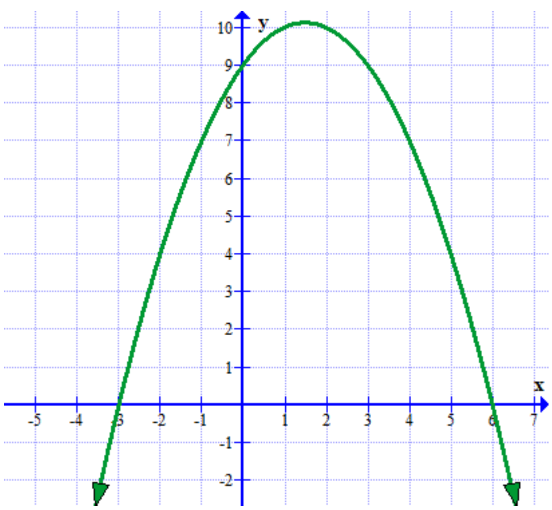
Range: $[-3, \infty)$

Vertex: $(-1, -3)$

Extrema/extrema value: minimum at $y = -3$

Axis of Symmetry: $x = -1$

8)



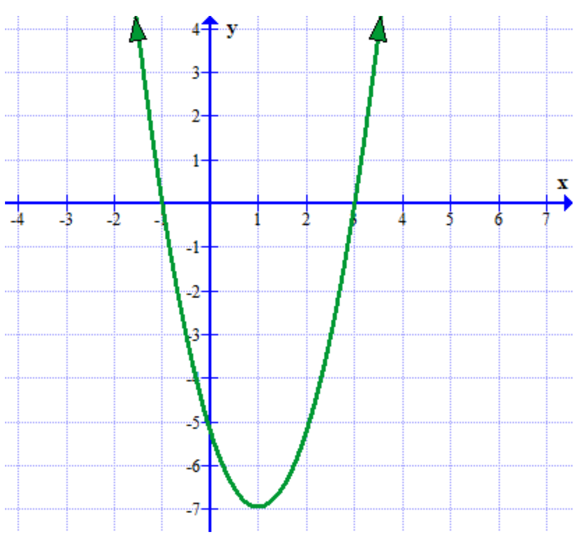
Y-Intercept: $(0, 9)$

X-Intercept(s): $(-3, 0)$ *and* $(6, 0)$

Solution(s): $x = -3$ *and* $x = 6$

Extrema type: maximum

9)



Domain: $(-\infty, \infty)$ or all real numbers

Range: $[-7, \infty)$

Vertex: $(1, -7)$

Axis of Symmetry: $x = 1$

Y-Intercept: $(0, -5)$

X-Intercept(s): $(-1, 0)$ *and* $(3, 0)$

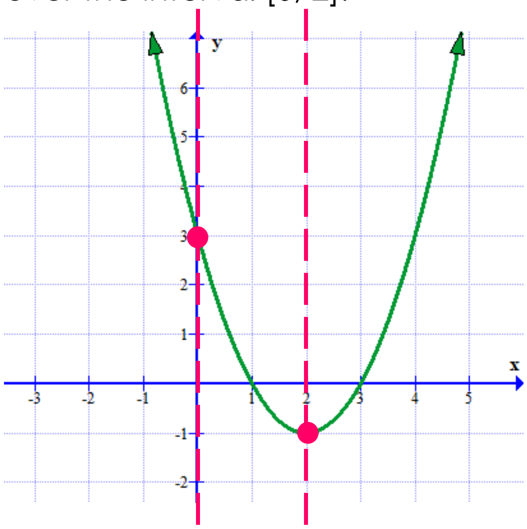
Extrema/extrema value: minimum at $y = -7$

Solution(s): $x = -1$ *and* $x = 3$

Part Three: Average Rate of Change

Find the average rate of change indicated for each function below.

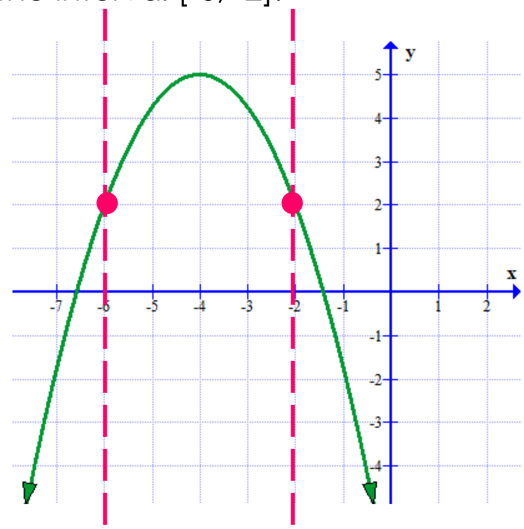
10) Find the average rate of change over the interval [0, 2].



$x = 0 \rightarrow (0, 3)$ (x_1 y_1) $x = 2 \rightarrow (2, -1)$ (x_2 y_2)

$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{2-0} = \frac{-4}{2} = \mathbf{-2}$

11) Find the average rate of change over the interval [-6, -2].



$x = -6 \rightarrow (-6, 2)$ (x_1 y_1) $x = -2 \rightarrow (-2, 2)$ (x_2 y_2)

$AROC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{-2--6} = \frac{2-2}{-2+6} = \frac{0}{4} = \mathbf{0}$

Part Four: Transformations of Quadratic Functions

Identify the transformations for each function below from the parent function $f(x) = x^2$.

12) $f(x) = -x^2 + 5$

$a = -1$ → reflection over x-axis

$h = 0$

$k = 5$ → translation up 5 units

13) $f(x) = 2(x + 4)^2$

$a = 2$ → vertical stretch of 2

$h = -4$ → translation left 4 units

$k = 0$

14) $f(x) = -3(x - 6)^2 - 2$

$a = -3$ → reflection over x-axis and vertical stretch of 3

$h = 6$ → translation right 6 units

$k = -2$ → translation down 2 units

15) $f(x) = (x + 1)^2$

$a = 1$

$h = -1$ → translation left 1 units

$k = 0$

16) $f(x) = 4(x + 3)^2 + 1$

$a = 4$ → vertical stretch of 4

$h = -3$ → translation left 3 units

$k = 1$ → translation up 1 units

17) $f(x) = -\frac{1}{2}(x - 4)^2 - 3$

$a = -\frac{1}{2}$ → reflection over x-axis vertical shrink of ½

$h = 4$ → translation right 4 units

$k = -3$ → translation down 3 units

Part Five: Vertex, Axis of Symmetry, and Extrema

For the following functions, identify the vertex, axis of symmetry and extrema.

18) $f(x) = x^2 - 6x + 1$

vertex: $x = \frac{-b}{2a} = \frac{6}{2(1)} = \frac{6}{2} = 3$

$y = (3)^2 - 6(3) + 1 = -8$

vertex: $(3, -8)$
axis of symmetry: $x = 3$
extrema: minimum (since a is +)

19) $f(x) = -2x^2 + 12x$

vertex: $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$

$y = -2(3)^2 + 12(3) = 18$

vertex: $(3, 18)$
axis of symmetry: $x = 3$
extrema: maximum (since a is -)

20) $f(x) = 3(x + 4)^2 - 1$

$a = 3 \quad h = -4 \quad k = -1$

vertex: $(-4, -1)$
axis of symmetry: $x = -4$
extrema: minimum (since a is +)

21) $f(x) = -2(x - 3)^2 + 5$

$a = -2 \quad h = 3 \quad k = 5$

vertex: $(3, 5)$
axis of symmetry: $x = 3$
extrema: maximum (since a is -)

Part Six: Converting Between Different Forms of Quadratics

Convert the following quadratic functions from vertex form to standard form.

22) $f(x) = -0.5(x + 4)^2 - 2$

$f(x) = -0.5(x + 4)(x + 4) - 2$

$f(x) = -0.5(x^2 + 4x + 4x + 16) - 2$

$f(x) = -0.5(x^2 + 8x + 16) - 2$

$f(x) = -0.5x^2 - 4x - 8 - 2$

$\boldsymbol{f(x) = -0.5x^2 - 4x - 10}$

23) $f(x) = 3(x - 1)^2 + 4$

$f(x) = 3(x - 1)(x - 1) + 4$

$f(x) = 3(x^2 - 1x - 1x + 1) + 4$

$f(x) = 3(x^2 - 2x + 1) + 4$

$f(x) = 3x^2 - 6x + 3 + 4$

$\boldsymbol{f(x) = 3x^2 - 6x + 7}$

Convert the following quadratic functions from standard form to vertex form.

24) $f(x) = 2x^2 + 8x - 6$

$a = 2 \quad b = 8 \quad c = -6$

vertex: $x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2$ → h

$y = 2(-2)^2 + 8(-2) - 6 = -14$ → k

$f(x) = a(x - h)^2 + k$

$f(x) = 2(x - -2)^2 - 14$

$\boldsymbol{f(x) = 2(x + 2)^2 - 14}$

25) $f(x) = -x^2 + 6x + 3$

$a = -1 \quad b = 6 \quad c = 3$

vertex: $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$ → h

$y = -(3)^2 + 6(3) + 3 = 12$ →k

$f(x) = a(x - h)^2 + k$

$\boldsymbol{f(x) = -1(x - 3)^2 + 12}$

or $\boldsymbol{f(x) = -(x - 3)^2 + 12}$

Part Seven: Applications of Quadratic Functions
Solve the following word problems.

26) A person standing at the edge of a building throws a baseball vertically upward. The quadratic function $f(x) = -16x^2 + 64x + 32$ models the baseball's height above the ground, f(x) in meters, x seconds after it was thrown.

a) From what height was the baseball thrown?

starting value → y-intercept (plug in 0 for x)

$f(0) = -16(0)^2 + 64(0) + 32 = \mathbf{32\ meters}$

b) When did the baseball hit it's maximum height?

x-value of vertex

$x = \frac{-b}{2a} = \frac{-64}{2(-16)} = \frac{-64}{-32} = \mathbf{2\ seconds}$

c) What was the baseball's maximum height?

y-value of the vertex → plus in the x-value of the vertex (2) to find the y-value

$f(2) = -16(2)^2 + 64(2) + 32 = \mathbf{96\ meters}$

d) A bird is flying 100 feet above the ground – is the bird in danger of being hit?

No – the baseball reaches a maximum height of only 96 meters so it will not hit the bird

e) When did the baseball land?

x-intercept → quadratic formula

$x = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(32)}}{2(-16)} = \frac{-64 \pm \sqrt{6144}}{-32}$

→ $\frac{-64+\sqrt{6144}}{-32} = ~~-0.45\ seconds~~$ → *can't have negative time*

→ $\frac{-64-\sqrt{6144}}{-32} = \mathbf{4.45\ seconds}$

Jennifer hit a golf ball from the ground and it followed the projectile $h(t) = -16t^2 + 100t$, where t is the time in seconds, and h is the height of the ball.

a) When did the ball hit it's maximum height?

x-value of vertex

$x = \frac{-b}{2a} = \frac{-100}{2(-16)} = \frac{-100}{-32} = \mathbf{3.125\ seconds}$

b) What was the maximum height?

y-value of the vertex → plus in the x-value of the vertex (3.125) to find the y-value

$f(2) = -16(3.125)^2 + 100(3.125) = \mathbf{156.25\ meters}$

c) When did the golfball land?

x-intercept → quadratic formula

$x = \frac{-100 \pm \sqrt{(100)^2 - 4(-16)(0)}}{2(-16)} = \frac{-100 \pm \sqrt{10000}}{-32}$

→ $\frac{-100+\sqrt{10000}}{-32} = 0\ seconds$

→ $\frac{-100-\sqrt{10000}}{-32} = \mathbf{6.25\ seconds}$