Algebra 1

Unit 2 Part 3

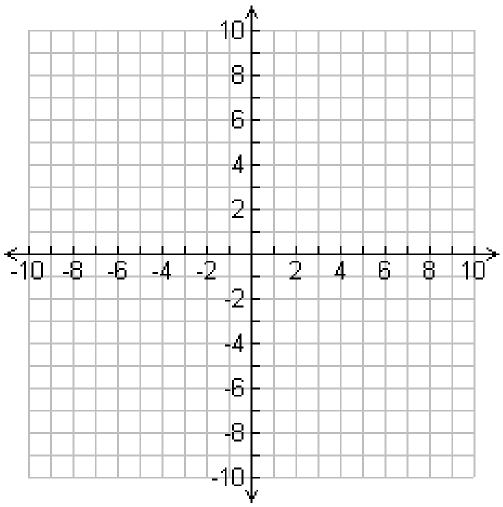
Quadratic Functions

			Thursday, March 11th	Friday, March 12th
			Transformations of Quadratic Functions	Graphing in Vertex Form Characteristics
Monday, March 15th	Tuesday, March 16th	Wednesday, March 17th	Thursday, March 18th	Friday, March 19th
Graphing in Standard Form Characteristics	Graphing Characteristics **Quiz Opens at 3:30 PM**	Converting Between Vertex Form and Standard Form **Quiz Due By Midnight**	Quadratic Word Problems	
Monday, March 22nd	Tuesday, March 23rd	Wednesday, March 24th	Thursday, March 25th	Friday, March 26th
Quadratic Word Problems	Review	**Unit 2 Part 3 Test (during class)**		

Transformations of Quadratic Functions Notes

The **parent function** of a function is the simplest form of a function. The parent function for a quadratic function is **$y = x^2$** or **$f(x) = x^2$**. Complete the table and graph the parent function below.

x	x^2
-3	
-2	
-1	
0	
1	
2	
3	



As you can see, the graph of a quadratic function looks very different from the graph of a linear function.

The U-shaped graph of a quadratic function is called a ___________________.

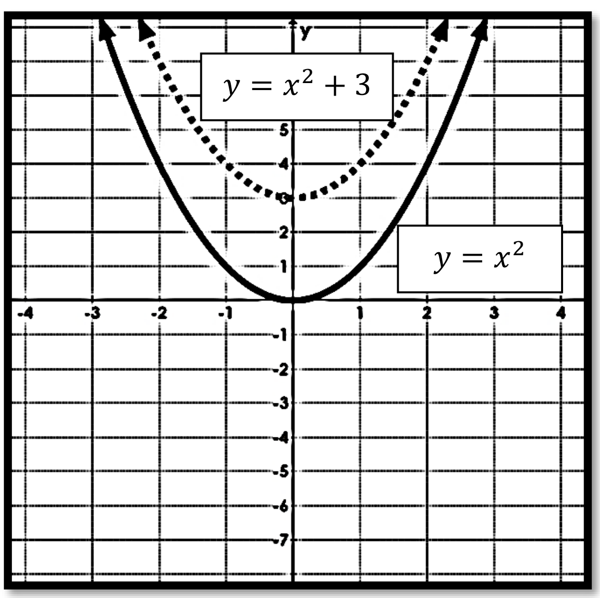
The highest/lowest point (or turning point) on a parabola is called the ___________.

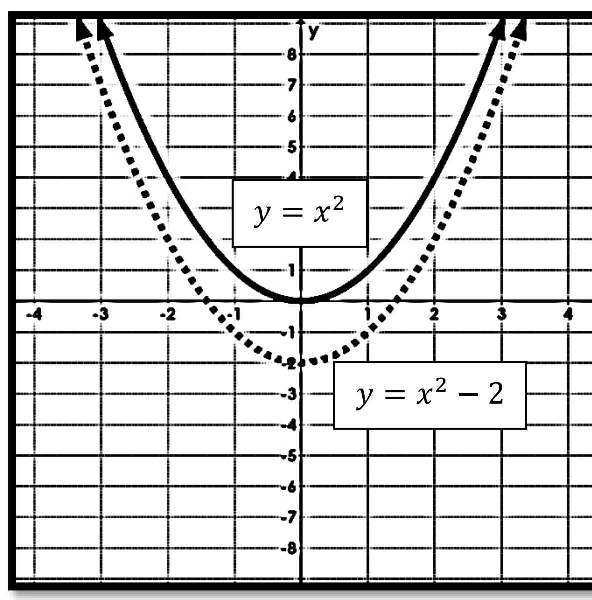
Remember, in order for a function to be a quadratic function, one term must have _____.

The graph above is our parent function – it represents a quadratic function that has not been changed in any way. We are going to talk about the transformations of quadratic functions and how those transformations are represented in the equation of a quadratic function.

Exploring the "k"

Answer the following questions about the transformation **from the parent graph (solid graph)to the new function (dotted parabola)**.





Describe the transformation:

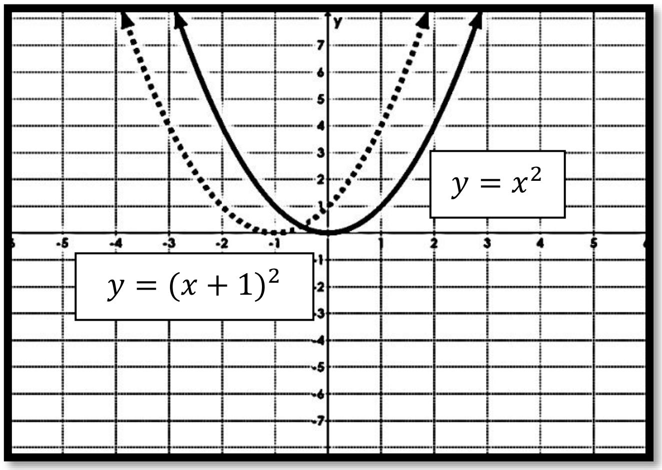
What is the vertex of the new function?

Describe the transformation:

What is the vertex of the new function?

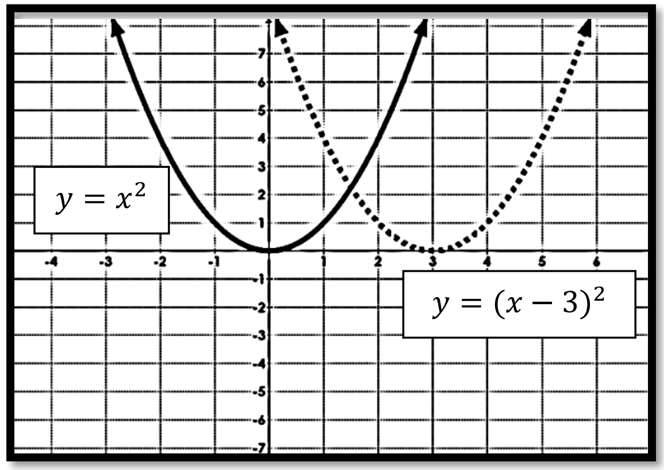
Exploring the "h" Value

Answer the following questions about the transformation from the parent graph (solid graph)to the new function (dotted parabola).



Describe the transformation:

What is the vertex of the new function?

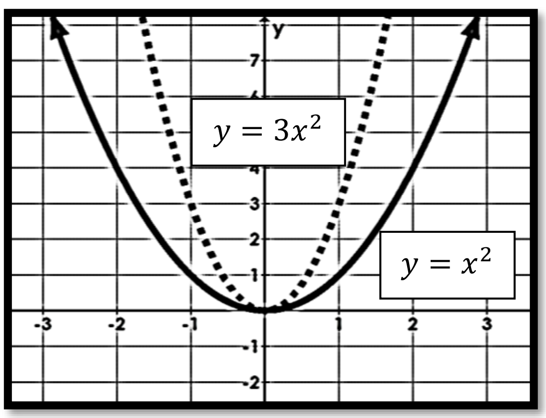


Describe the transformation:

What is the vertex of the new function?

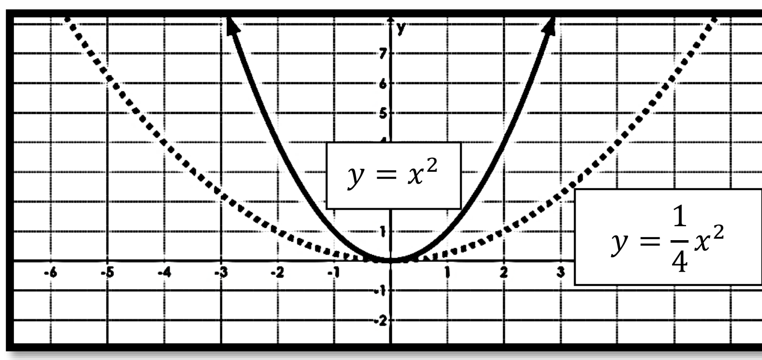
Exploring the "a" Value

Answer the following questions about the transformation from the parent graph (solid graph)to the new function (dotted parabola).



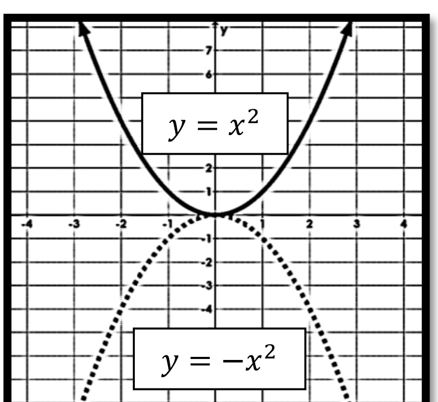
Describe the transformation:

What is the vertex of the new function?



Describe the transformation:

What is the vertex of the new function?



Describe the transformation:

What is the vertex of the new function?

Summary

Vertex Form: ____________________________

Variable	Summary of the Effects of the Transformations			
a				
h				
k				

vertex: __________

Practice

1) Given the equations below, describe the transformations from the parent function and name the vertex:

a. $y = -(x-4)^2 + 7$

b. $y = -2(x+2)^2 + 5$

c. $y = \frac{1}{2}(x-3)^2 - 8$

2) Create an equation to represent the following transformations:

a. Shifted down 4 units, right 1 unit, and reflected across the x-axis

b. Shifted up 6 units, reflected across the x-axis, and stretch by a factor of 3

c. Shifted up 2 units, left 4 units, reflected across the x-axis, and shrunk by a factor of ¾.

Identifying Transformations Practice

Equation	a, h, k values	Reflection?	Vertical Stretch or Shrink?	Horizontal Translation?	Vertical Translation?
$y = -2x^2 + 4$					
$y = \frac{3}{2}(x + 1)^2$					
$y = \frac{1}{4}(x - 2)^2 - 5$					
$y = -0.4x^2$					
$y = \frac{2}{3}(x - 3)^2 + 4$					
$y = 4x^2 - 2$					
$y = (x + 1)^2 - 5$					
$y = -3(x - 4)^2 + 1$					
$y = \frac{1}{2}x^2$					
$y = 2(x + 3)^2$					
$y = x^2 + 4$					
$y = (x + 4)^2$					
$y = 1.5x^2 - 9$					
$y = -x^2 + 2$					
$y = -0.8(x - 4)^2$					
$y = -3.2x^2 + 11$					

Writing Equations in Vertex Form Practice

Write the equation for a quadratic function which has been…

1) reflected across the x-axis and translated down 3 units.

2) vertically stretched by a factor of 2, and translated right 5 units.

3) reflected across the x-axis, vertically stretched by a factor of 1.5, and translated left 1 unit.

4) vertically shrunk by a factor of ½, translated right 2 units, and translated down 4 units.

5) translated left 3 units, reflected across x-axis, and translated up 2 units.

6) translated down 1 unit, translated right 7 units, and vertically shrunk by a factor of 0.3.

7) vertically stretched by a factor of 2.5, translated right 1.5 units, translated up 3.3 units, and reflected across the x-axis.

8) translated left 6 units, translated down 2 units, and reflected across the x-axis.

9) neither stretched nor shrunk but has a vertex at (3, 4).

Transformations of Quadratic Functions - Matching

1) ____	Up 4 and left 2	a.	$f(x) = (x-2)^2 + 4$
2) ____	Reflect across x-axis and up 3	b.	$f(x) = -3(x+5)^2$
3) ____	Vertical stretch by 3 and right 5	c.	$f(x) = \frac{1}{2}(x-2)^2 + 4$
4) ____	Vertical shrink by $\frac{1}{3}$ and right 5	d.	$f(x) = -(x+3)^2$
5) ____	Right 2 and up 4	e.	$f(x) = \frac{1}{3}x^2 - 5$
6) ____	Vertical stretch by 3 and down 5	f.	$f(x) = -x^2 - 3$
7) ____	Reflect across x-axis and down 3	g.	$f(x) = (x - \frac{1}{2})^2 + 4$
8) ____	Vertical shrink of $\frac{1}{3}$ and down 5	h.	$f(x) = \frac{1}{2}x^2 + 4$
9) ____	Up 4 and right $\frac{1}{2}$	i.	$f(x) = 3(x-5)^2$
10) ____	Reflect across x-axis and left 3	j.	$f(x) = (x+2)^2 + 4$
11) ____	Vertical stretch of 2, right 4 and up 3	k.	$f(x) = -x^2 + 3$
12) ____	Reflect across x-axis, vertical stretch of 3 and left 5	l.	$f(x) = 3x^2 - 5$
13) ____	Vertical shrink by $\frac{1}{2}$, right 2 and up 4	m.	$f(x) = 2(x+3)^2 + 4$
14) ____	Vertical shrink by $\frac{1}{2}$ and up 4	n.	$f(x) = 2(x-4)^2 + 3$
15) ____	Vertical stretch of 2, left 3 and up 4	o.	$f(x) = \frac{1}{3}(x-5)^2$

Graphing and Characteristics of Quadratic Functions

[vertex form]

To graph a quadratic function that is in vertex form, follow these steps:

① Create an x-y table with 5 rows

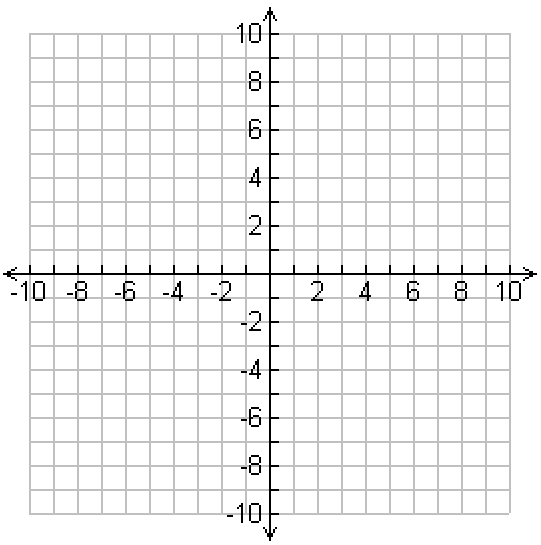
② Find the vertex – this goes in the middle row

③ Fill out the two x-values before and after the vertex

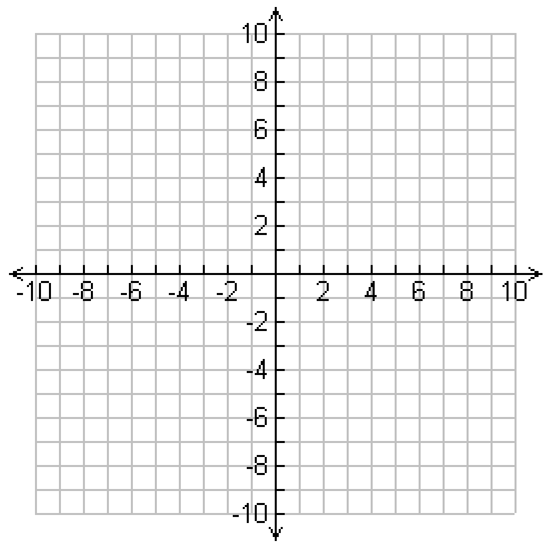
④ Use your calculator to find the y-values and graph

Graph the following quadratic functions.

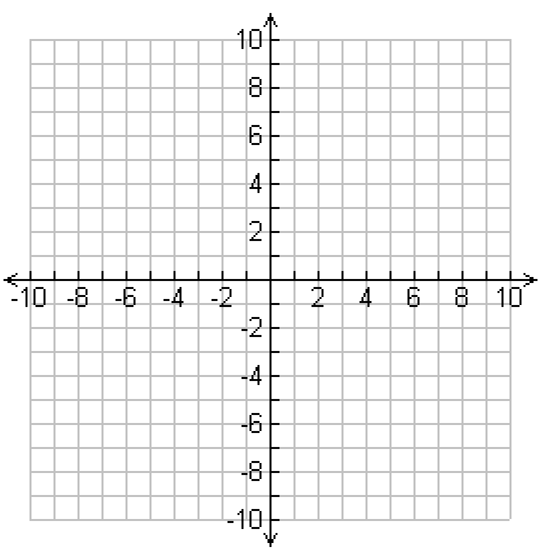
1) $y = (x - 3)^2 + 2$



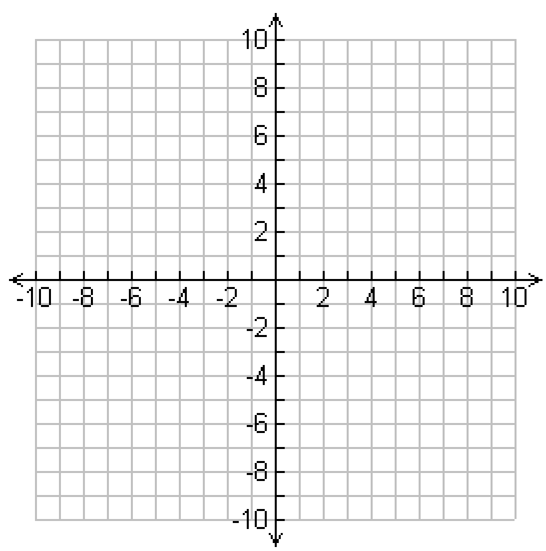
2) $f(x) = -(x + 1)^2 - 4$



3) $f(x) = -2x^2 + 7$

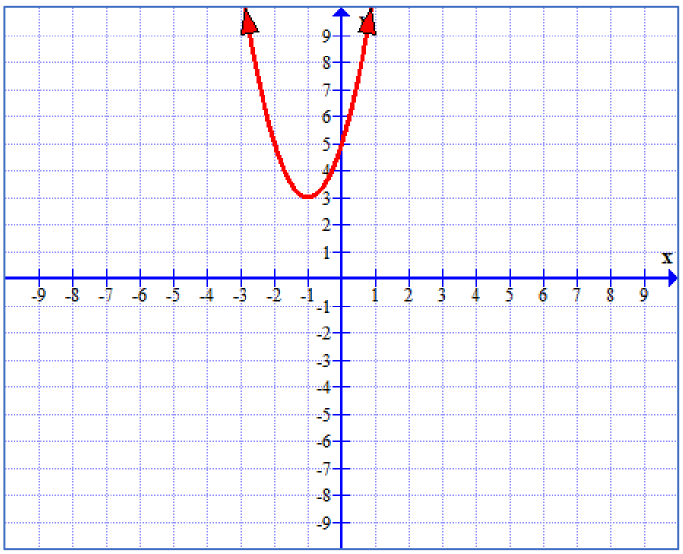


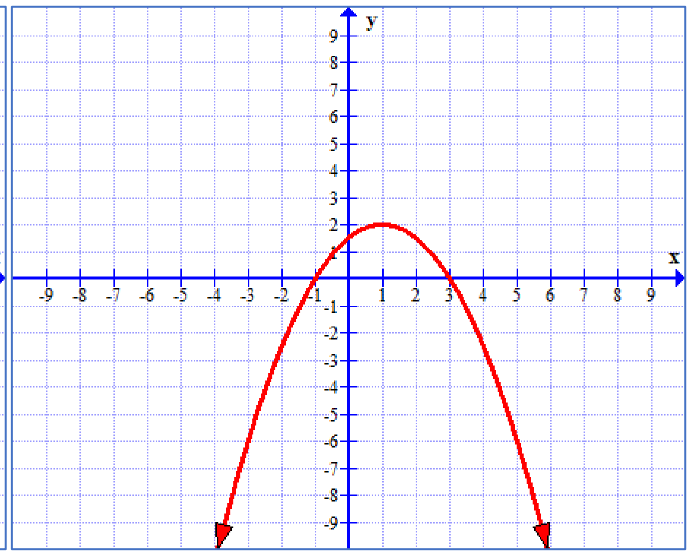
4) $y = \frac{3}{2}(x + 4)^2 - 1$



Characteristics
– Domain and Range –

Domain		
Define: All possible values of x	**Think:** How far left to right does the graph go?	**Write:** [#, #]
Range		
Define: All possible values of y	**Think:** How far down to how far up does the graph go?	**Write:** [#, #]



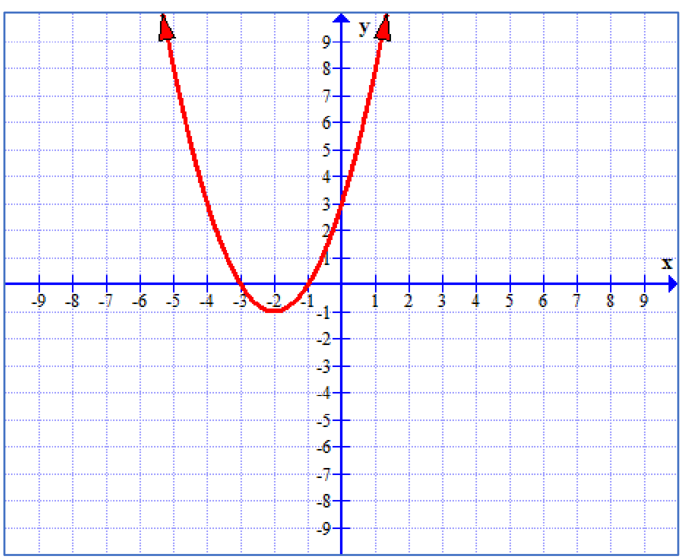


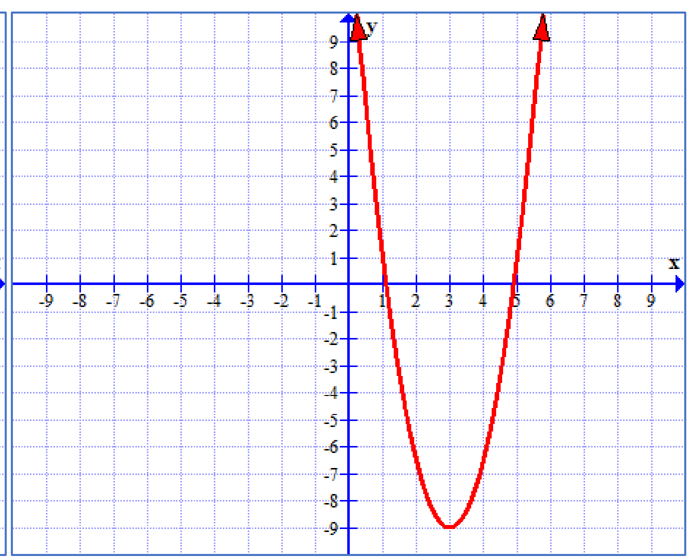
Domain:

Range:

Domain:

Range:





Domain:

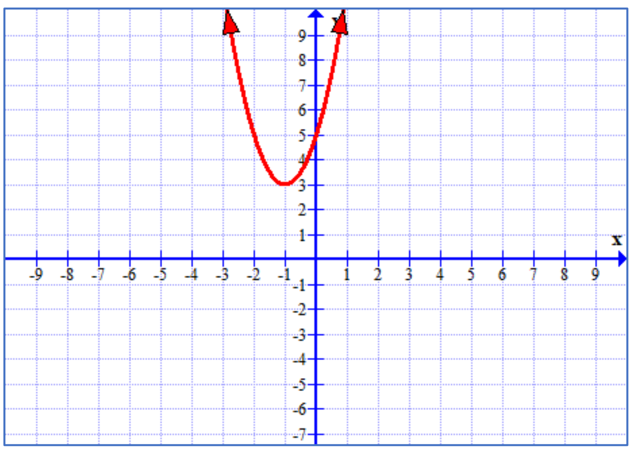
Range:

Domain:

Range:

– zeros and intercepts –

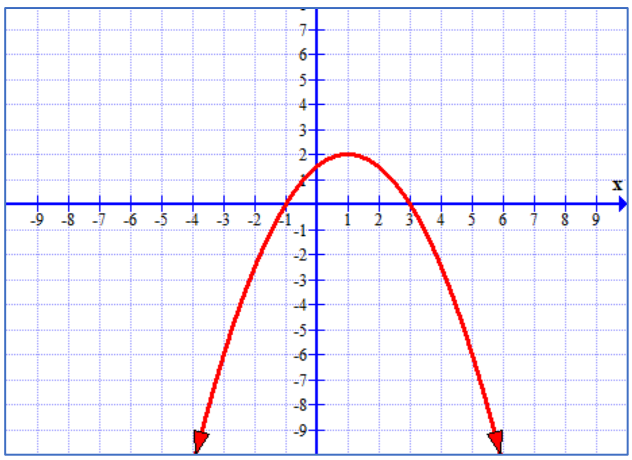
Y-Intercept		
Define: Point where the graph crosses the y-axis	**Think:** At what coordinate point does the graph cross the y-axis?	**Write:** (0, b)
X-Intercept		
Define: Point where the graph crosses the x-axis	**Think:** At what coordinate point does the graph cross the x-axis?	**Write:** (a, 0)
Zero		
Define: Where the function (y-value) equals 0	**Think:** At what x-value does the graph cross the x-axis?	**Write:** x = ____



Y-Intercept:

X-Intercept(s):

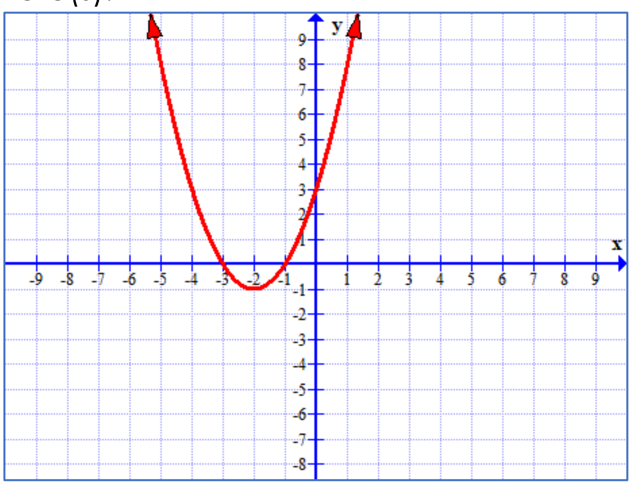
Zero(s):



Y-Intercept:

X-Intercept(s):

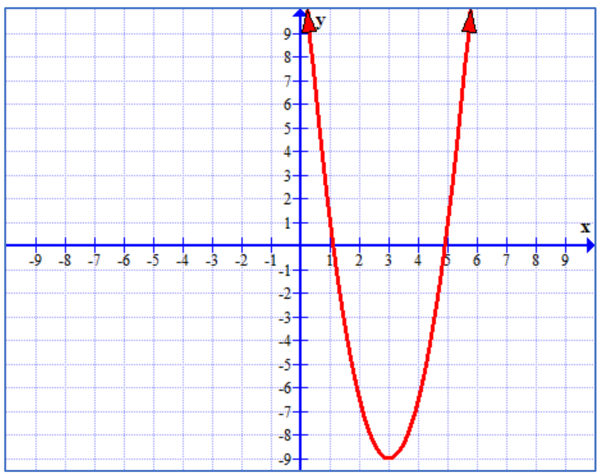
Zero(s):



Y-Intercept:

X-Intercept(s):

Zero(s):



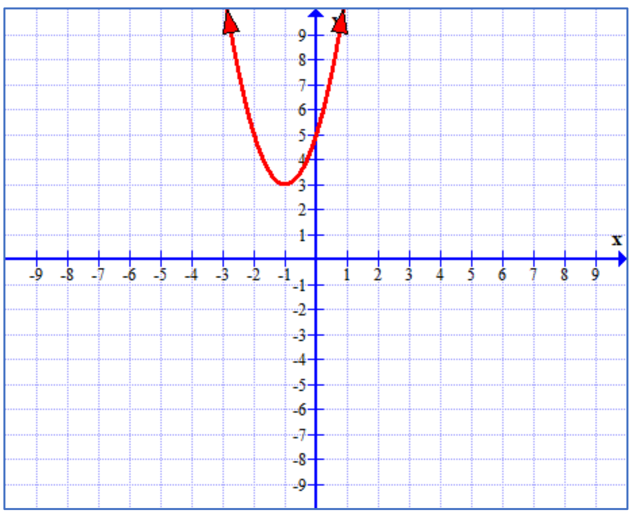
Y-Intercept:

X-Intercept(s):

Zero(s):

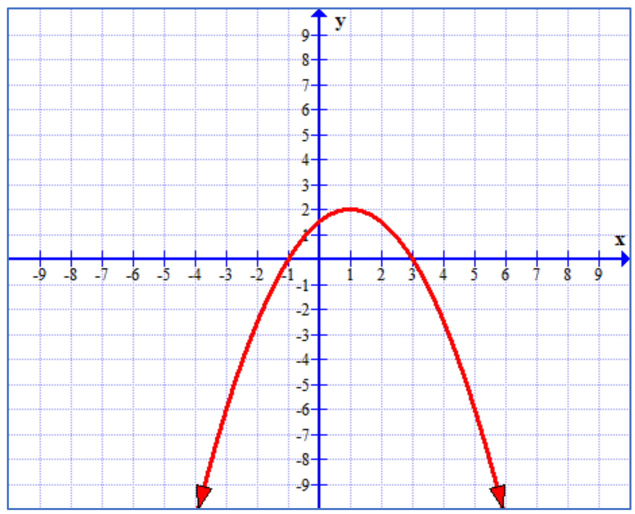
– vertex and axis of symmetry –

Vertex		
Define: Highest or lowest point or peak of a parabola	**Think:** What is my highest or lowest point on my graph?	**Write:** Name the point (h, k)
Axis of Symmetry		
Define: The vertical line that divides the parabola into mirror images and runs through the vertex	**Think:** What imaginary, vertical line would make the parabola symmetrical?	**Write:** x = h (x value of the vertex)



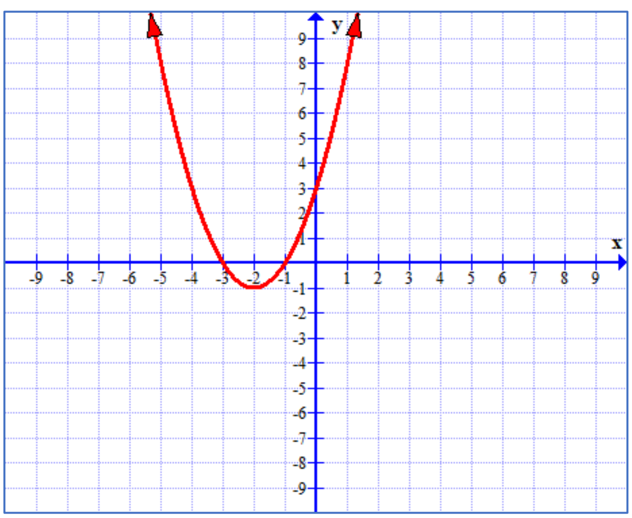
Vertex:

Axis of Symmetry:



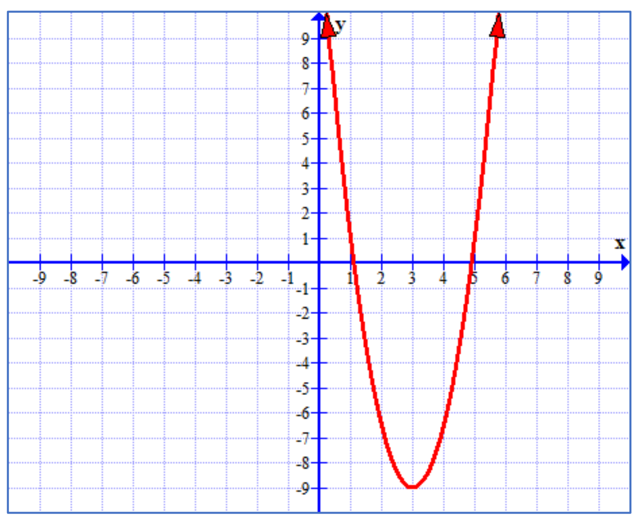
Vertex:

Axis of Symmetry:



Vertex:

Axis of Symmetry:

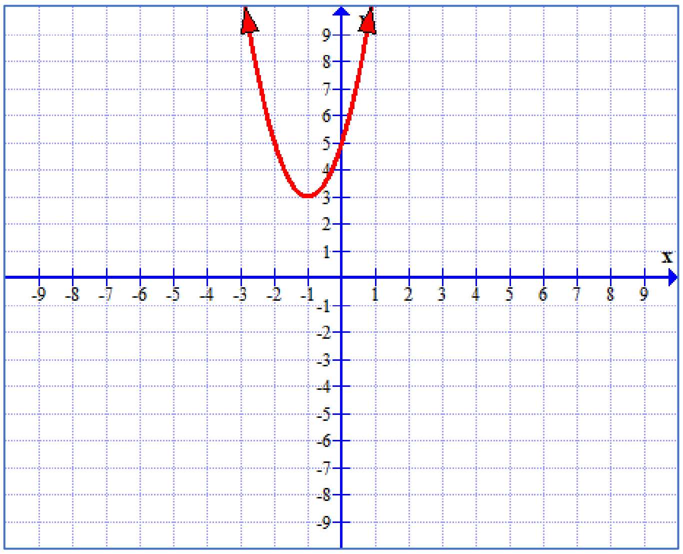


Vertex:

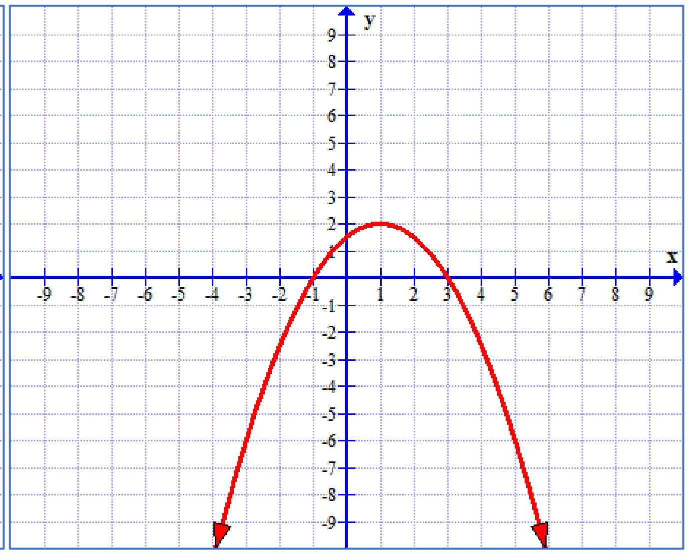
Axis of Symmetry:

– extrema –

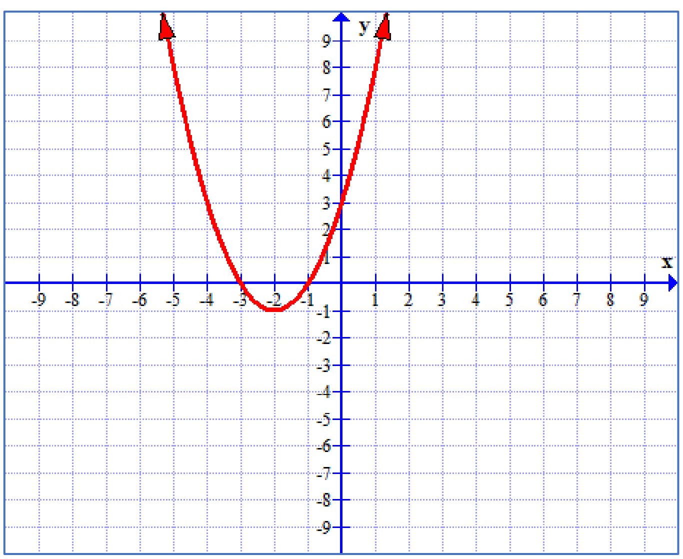
Maximum		
Define: Highest point or peak of a function.	**Think:** What is my highest point on my graph?	**Write:** y = k (y-value of the vertex)
Minimum		
Define: Lowest point or valley of a function.	**Think:** What is the lowest point on my graph?	**Write:** y = k (y-value of the vertex)



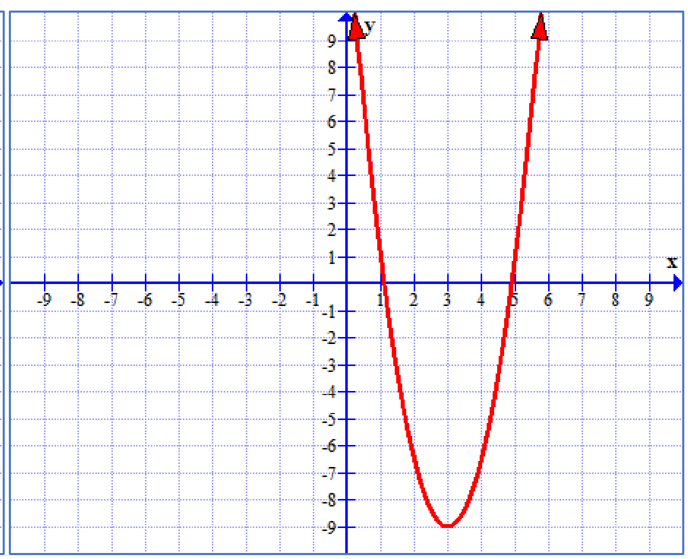
Extrema:



Extrema:



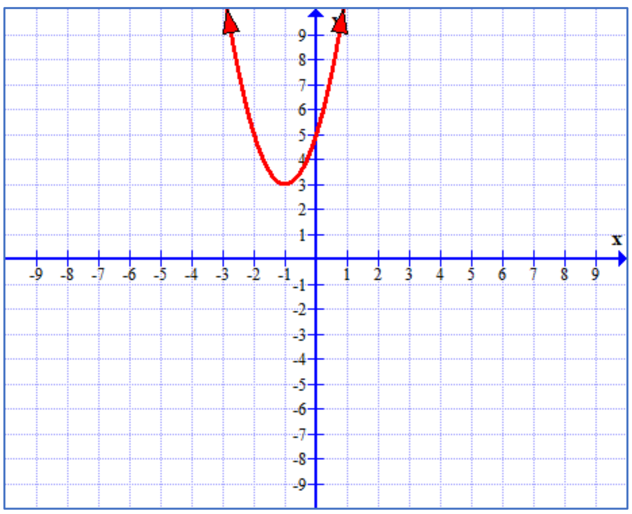
Extrema:



Extrema:

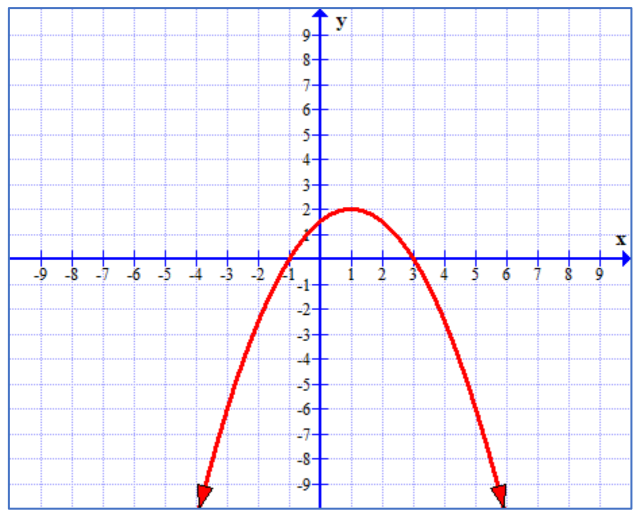
– end behavior –

Define: Behavior of the ends of the function (what happens to the y-values or f(x)) as x approaches positive or negative infinity. The arrows indicate the function goes on forever so we want to know where those ends go.	
Think: As x goes to the left (negative infinity), what direction does the left arrow go?	**Write:** As x → -∞, f(x) → _____
Think: As x goes to the right (positive infinity), what direction does the right arrow go?	**Write:** As x → ∞, f(x) → _____



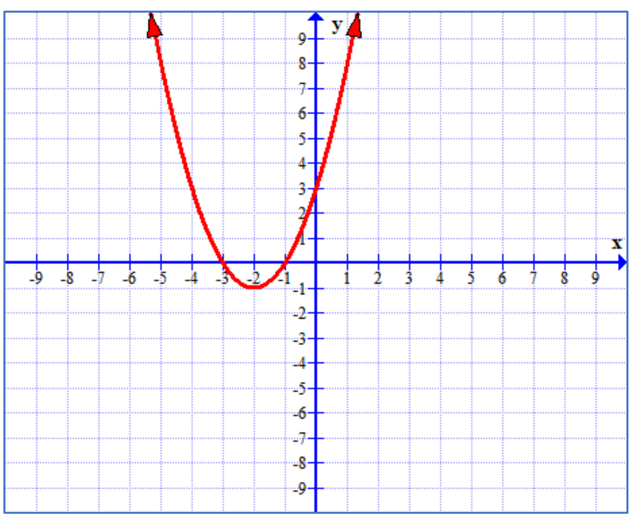
As x → -∞, f(x) → _____

As x → ∞, f(x) → _____



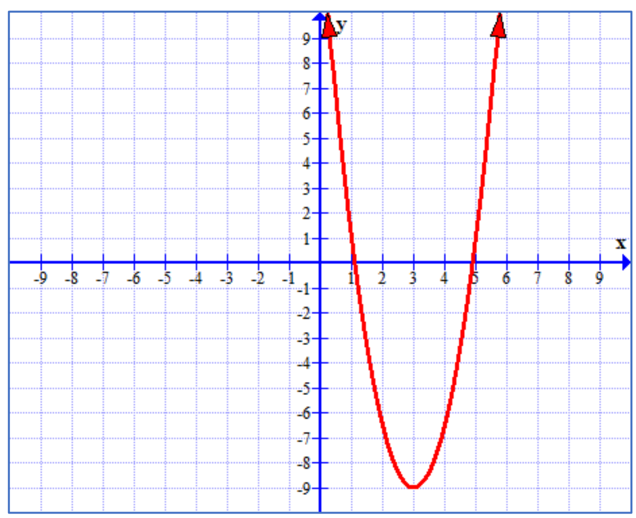
As x → ∞, f(x) → _____

As x → -∞, f(x) → _____



As x → -∞, f(x) → _____

As x → ∞, f(x) → _____

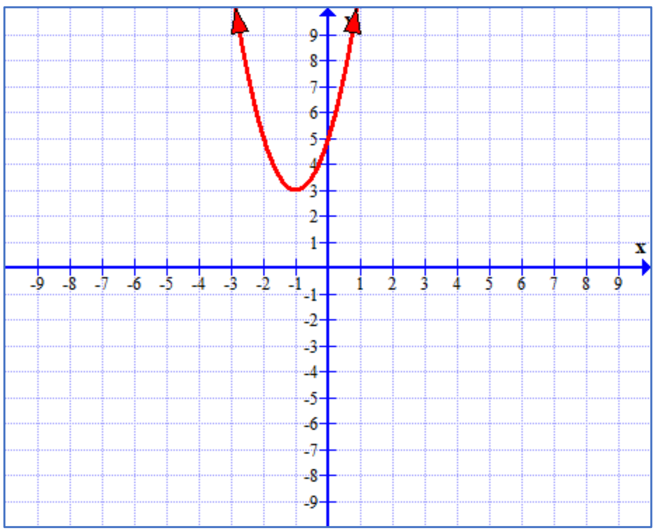


As x → -∞, f(x) → _____

As x → ∞, f(x) → _____

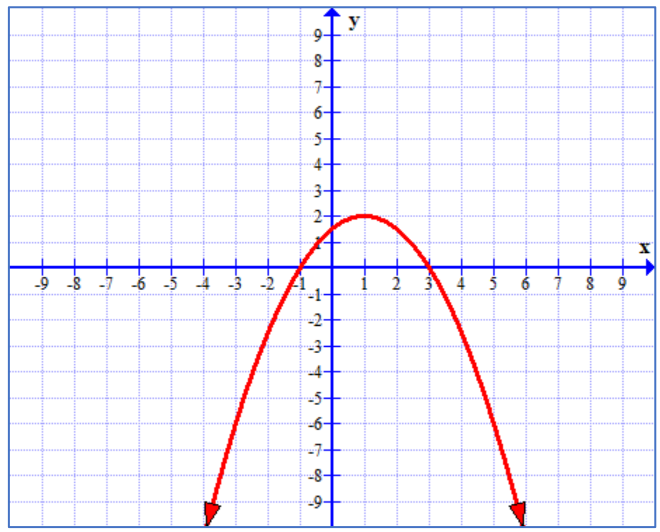
– interval of increase and decrease –

Interval of Increase		
Define: The part of the graph that is rising as you read left to right.	**Think:** From left to right, is my graph going up?	**Write:** [left, right] of portion going up
Interval of Decrease		
Define: The part of the graph that is falling as you read from left to right.	**Think:** From left to right, is my graph going down?	**Write:** [left, right] of portion going down



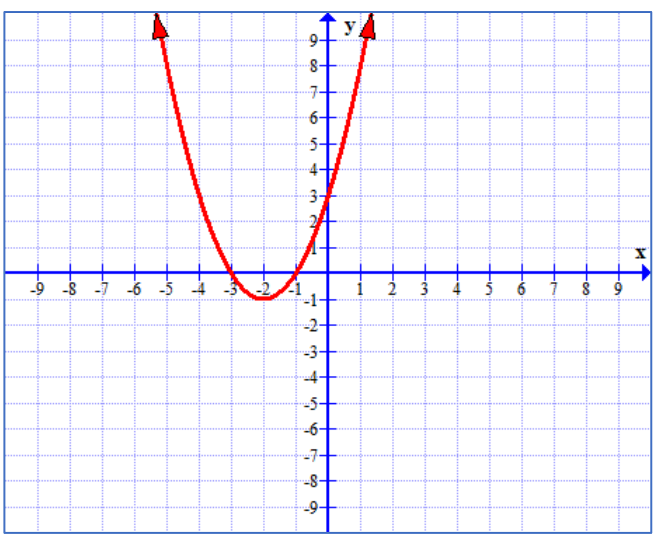
Interval of Increase:

Interval of Decrease:



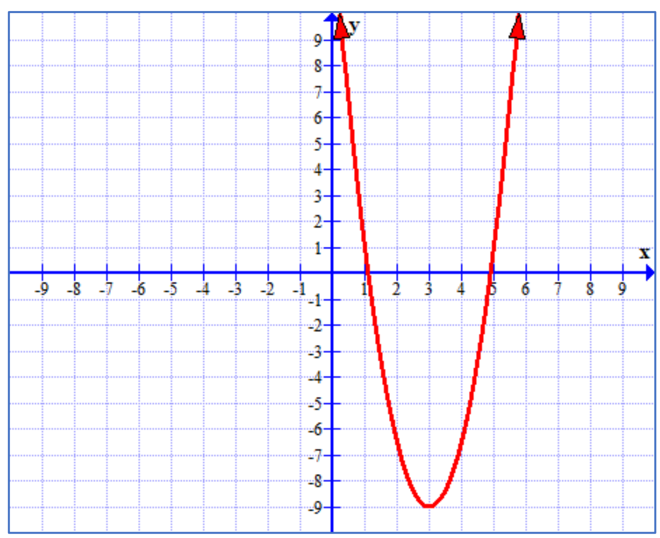
Interval of Increase:

Interval of Decrease:



Interval of Decrease:

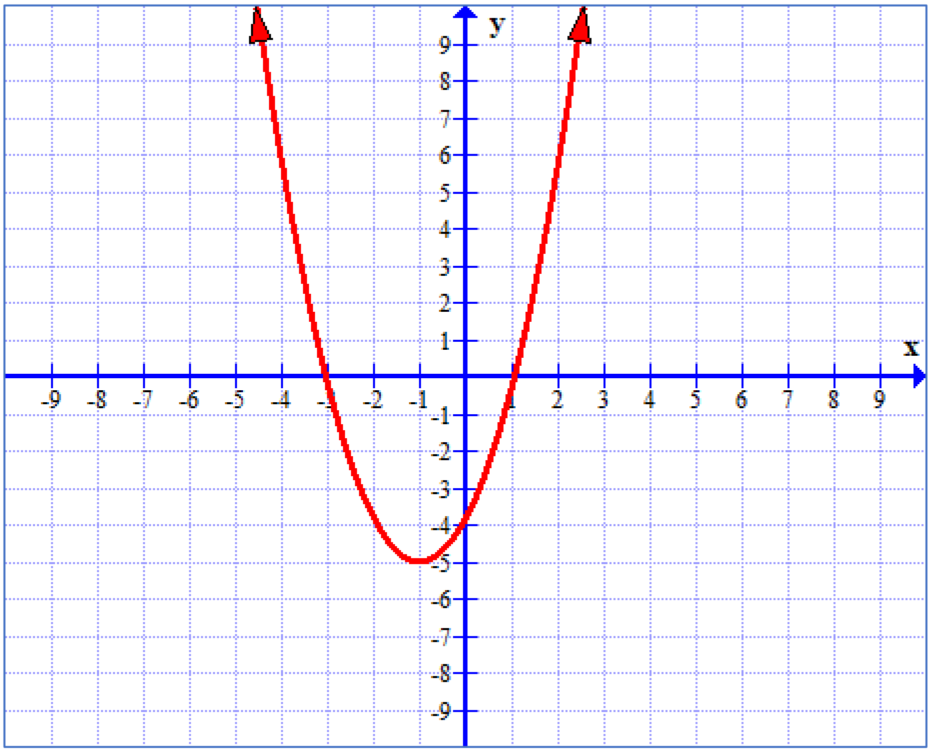
Interval of Increase:



Interval of Decrease:

Interval of Increase:

Identify the listed characteristics for the following graph.



Domain:

Range:

Vertex:

Max or Min:

Extrema Value:

Axis of Symmetry:

Zero(s):

Y-Intercept:

X-Intercept(s):

Interval of Increase:

Interval of Decrease:

As x $\rightarrow \infty$, f(x) $\rightarrow$ _____

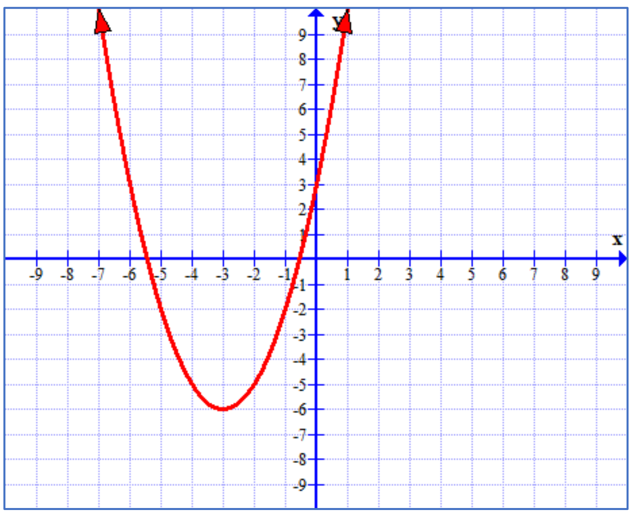
As x $\rightarrow -\infty$, f(x) $\rightarrow$ _____

Average Rate of Change Notes

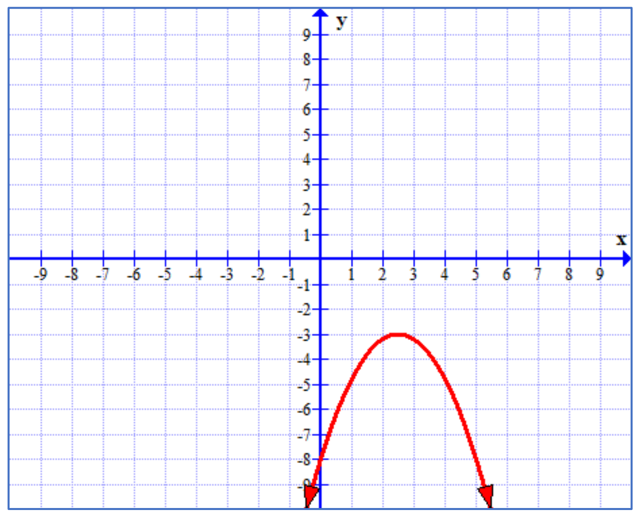
Average Rate of Change (AROC): The change in the value of a quantity divided by the elapsed time. For a function, this is the change in the y-value divided by the change in the x-value for two distinct points on the graph.

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding AROC from a graph.
Using the problem, find the two points for which you are trying to find the average rate of change between. Then, use the formula to find the AROC.



Find the AROC of the interval $[-4, -1]$.



Find the AROC between $x = 1$ and $x = 5$.

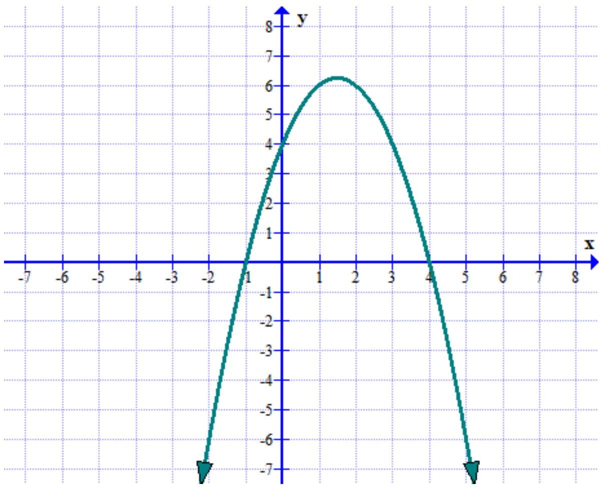
Finding AROC from a graph.
Using the problem, plug in the two x-values (one at a time) to find the two points for which you are trying to find the average rate of change between. Then, use the formula to find the AROC.

Given $y = (x-2)^2 + 6$, find the average rate of change between $x = -3$ and $x = 2$.

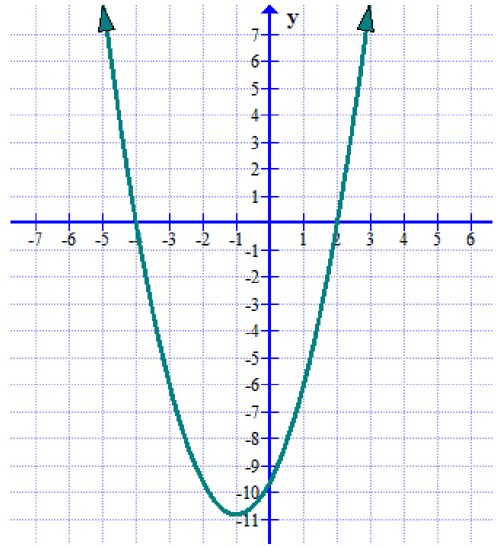
Given $y = -4x^2 + 6x + 11$, find the AROC of the interval $[0,5]$.

Average Rate of Change Practice

1) Find the average rate of change over the interval [-1, 3].



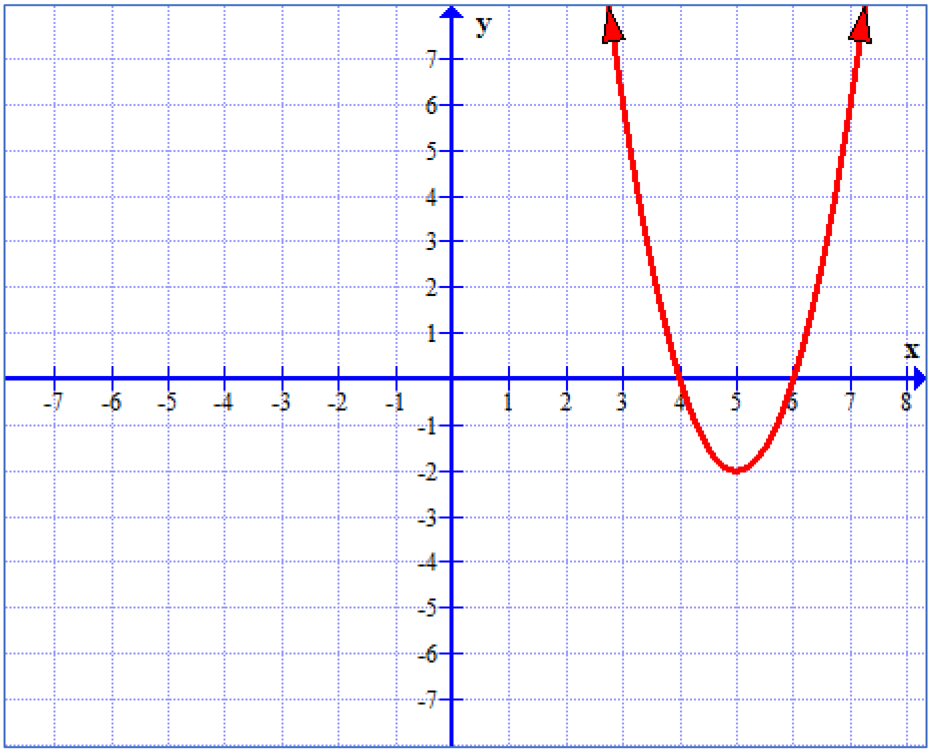
2) Find the average rate of change over the interval $-3 \leq x \leq 2$.



3) Using the equation $y = -4(x+2)^2 - 6$, find the average rate of change from x = -2 to x = 1.

4) Using the equation $y = -x^2 - 6x + 2$, find the average rate of change for the interval [-6, -2].

Characteristics Practice



Domain:

Range:

Int. of Increase:

Int. of Decrease:

Max/Min:

Extrema Value:

Zeros:

Y-Int:

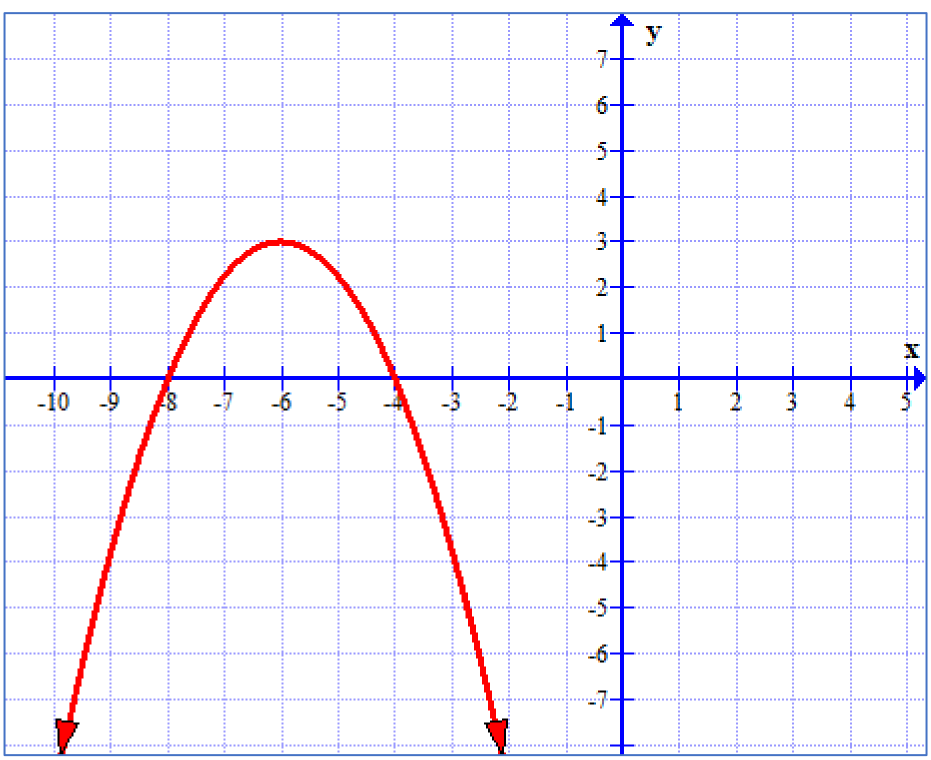
X-Int:

As $x \to \infty, f(x) \to$ ____

As $x \to -\infty, f(x) \to$ ____

Vertex:

Axis of Symmetry:



Vertex:

X-Int:

Int. of Decrease:

Zeros:

Range:

As $x \to -\infty, f(x) \to$ ____

Max/min:

Axis of Sym:

Domain:

Y-Int:

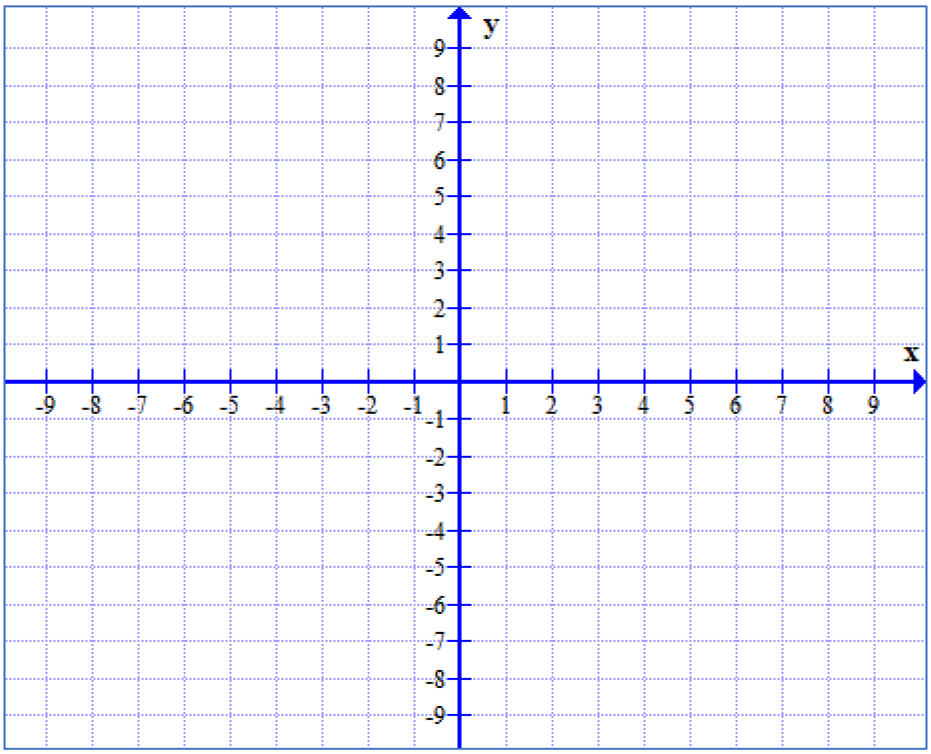
As $x \to \infty, f(x) \to$ ____

Int. of Increase:

Int. of Constant:

Draw a graph that has the following characteristics:

- ▫ Vertex at (3, 4)
- ▫ End behavior of as $x \to -\infty, f(x) \to -\infty$
- ▫ Two zeros
- ▫ A y-intercept of (0, -2)
- ▫ A domain of (- ∞, ∞)



Then, identify the following:

Axis of Symmetry:

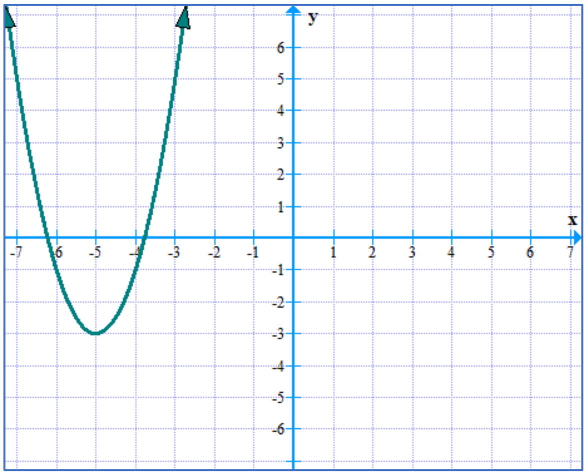
Range:

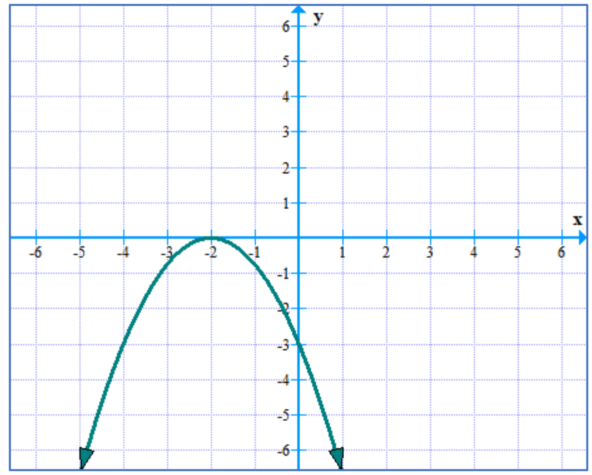
Interval of Increase:

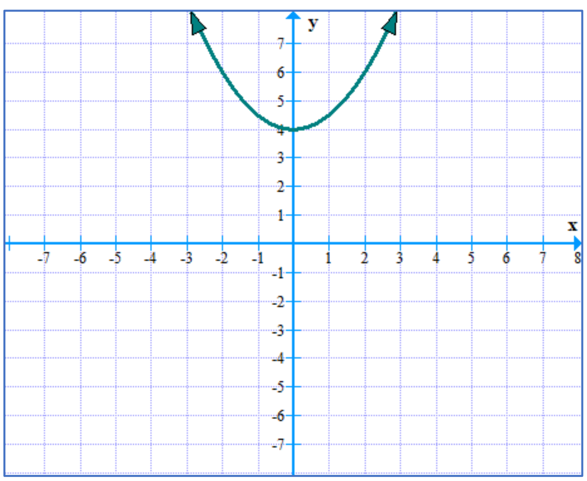
Interval of Decrease:

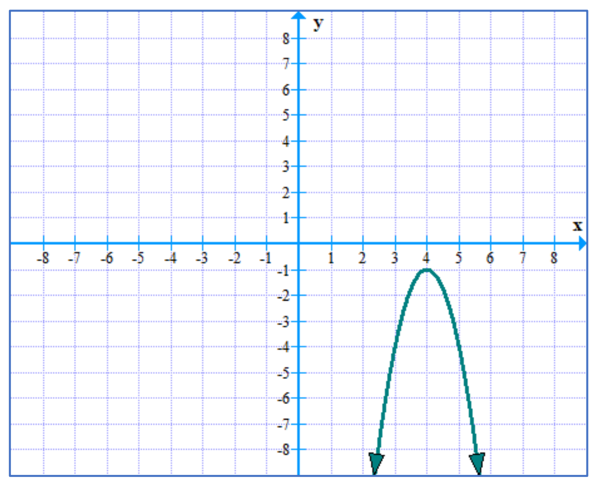
Writing Equations in Vertex Form When Given a Graph

Steps: ① Find the vertex ② Find stretch/shrink/reflection (AROC from vertex to one point to the right) ③ Plug values into equation

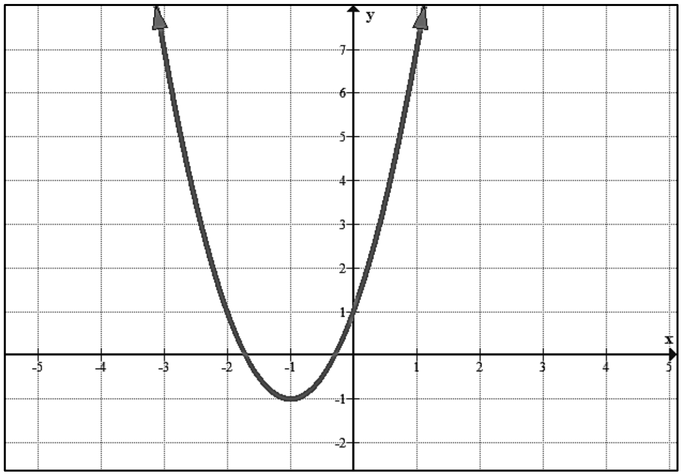








Graphing in Vertex Form – Practice



1) Determine the equation for the function graphed on the left.

a) Domain:

b) Range:

c) Extrema:

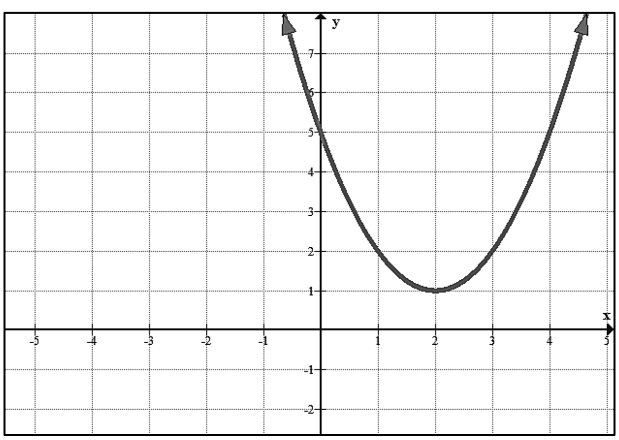
d) Axis of Symmetry:

e) Increasing:

f) Decreasing:

g) As $x \to -\infty, f(x) \to$ ____

h) AROC $-3 \leq x \leq -1$.



1) Determine the equation for the function graphed on the left.

a) Domain:

b) Range:

c) Extrema:

d) Axis of Symmetry:

e) Increasing:

f) Decreasing:

g) As $x \to -\infty, f(x) \to$ ____

h) As $x \to \infty, f(x) \to$ ____

i) AROC between $x = 1$ and $x = 4$.

Graphing and Characteristics of Quadratic Functions

[standard form]

To graph a quadratic function that is in standard form, follow these steps:

① Create an x-y table with 5 rows

② Find the vertex – this goes in the middle row

To find the x-value of the vertex: $x = \frac{-b}{2a}$

Then plug the x-value into the equation to get the y-value

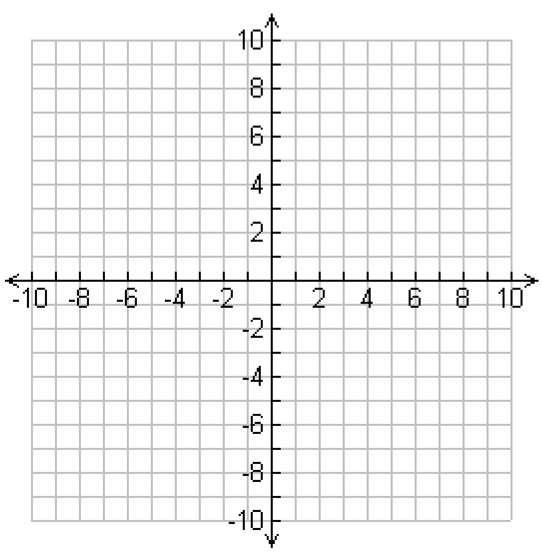
③ Fill out the two x-values before and after the vertex

④ Use your calculator to find the y-values and graph

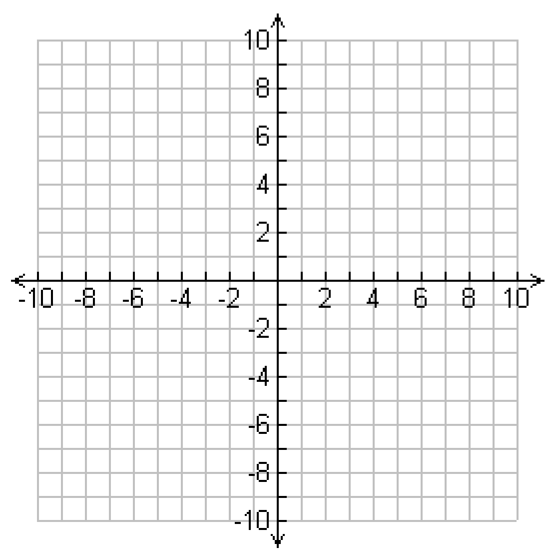
Note: the y-intercept of a quadratic function in standard form is ___________

For the following problems, find the vertex and graph the function.

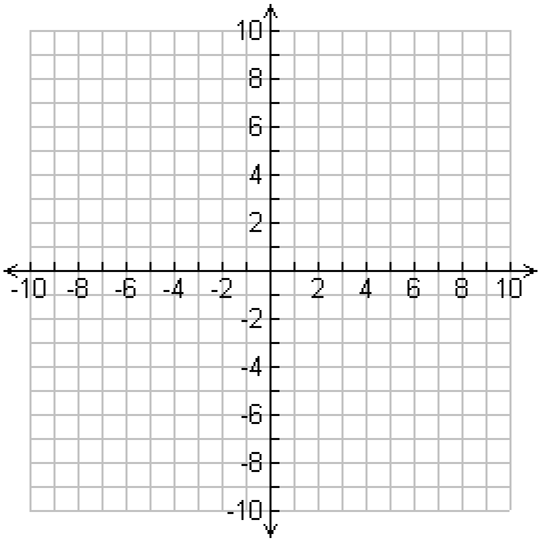
1) $y = x^2 - 2x - 1$



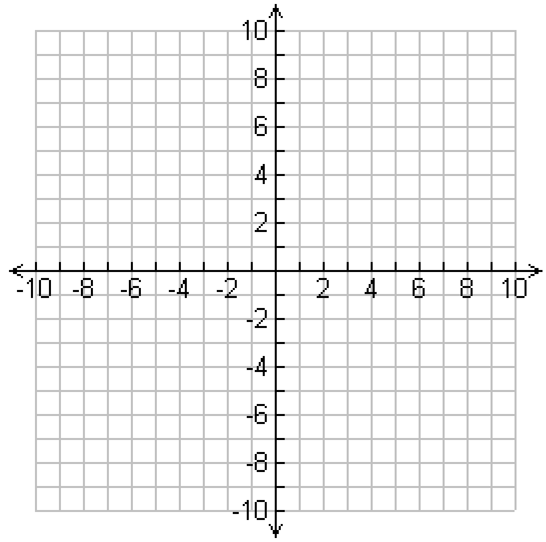
2) $y = 3x^2 - 6x$



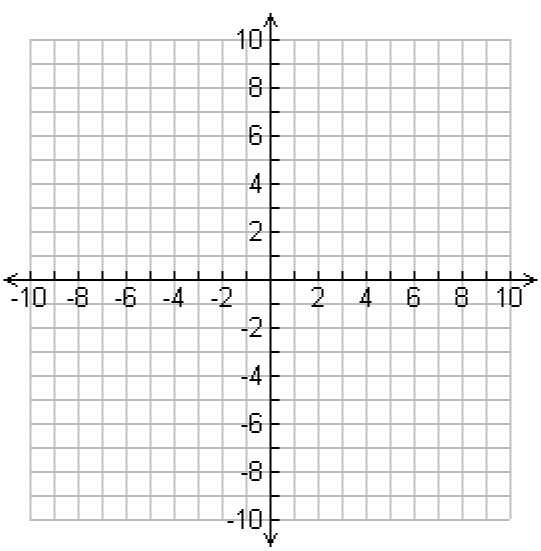
3) $f(x) = -x^2 + 6x - 9$



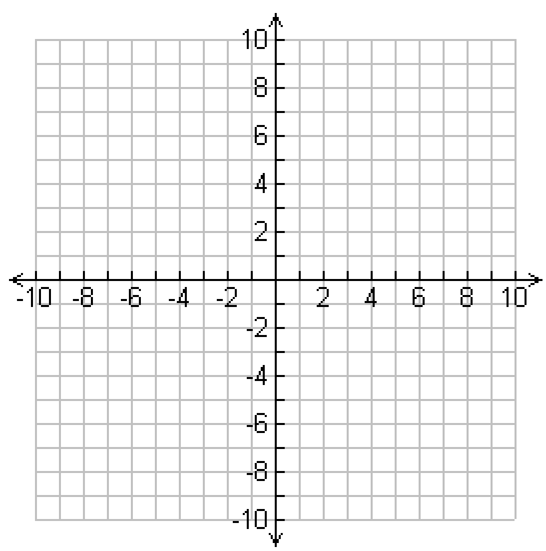
4) $y = \frac{1}{2}x^2 + 2x - 6$



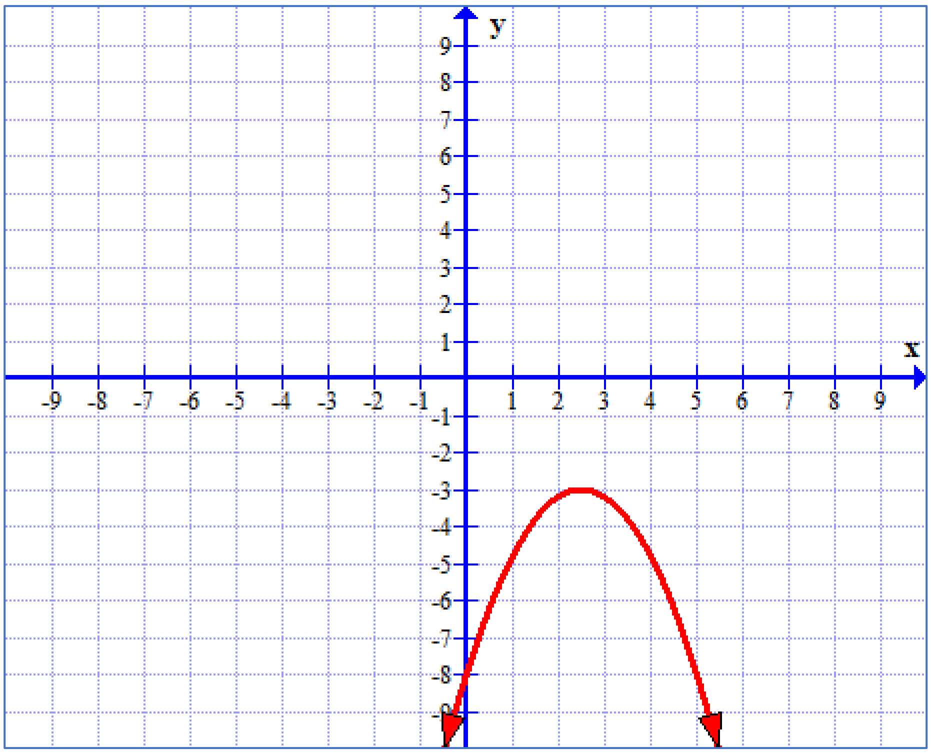
5) $f(x) = -1.2x^2 + 8$



6) $y = 2x^2 - 10x + 3$



For the graphs below, find the characteristics listed.



Domain:

Range:

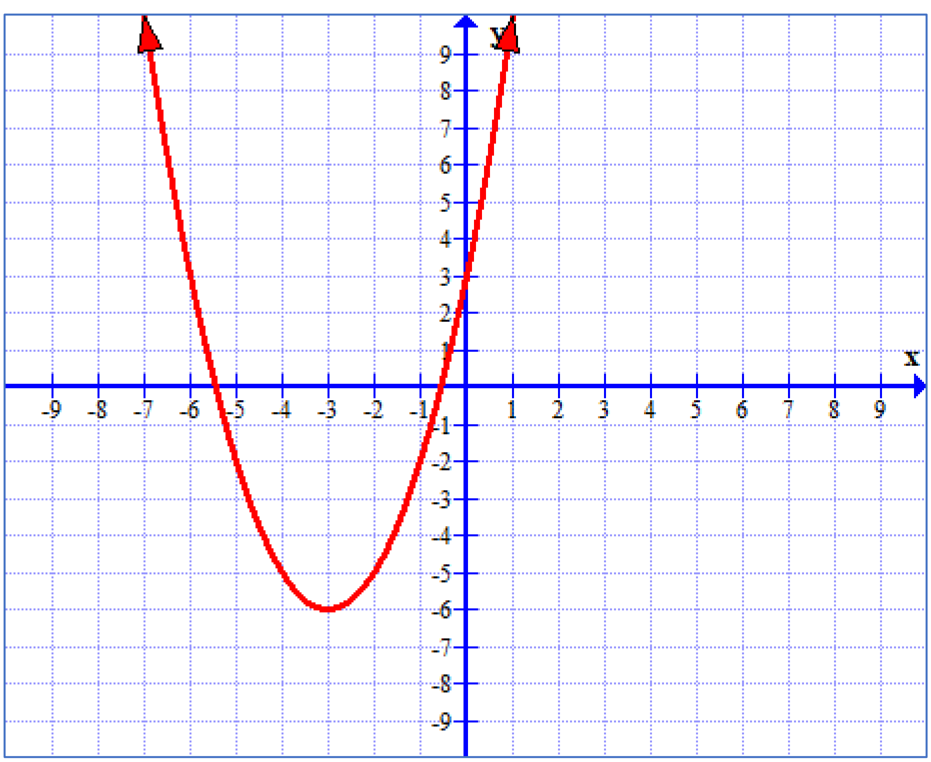
Zeros:

Y-Intercepts:

Interval of Increase:

As x → ∞, f(x) → _____

Extrema:



Range:

X-Intercepts:

Max or Min:

As x → ∞, f(x) → _____

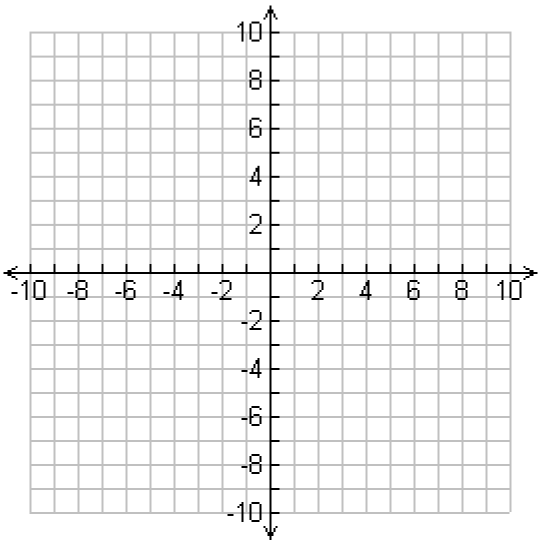
Interval of Increase:

Interval of Decrease

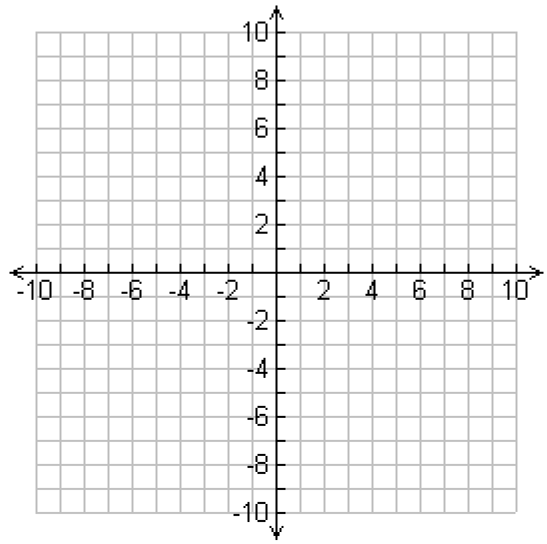
Graphing in Standard Form – Practice

Graph the following.

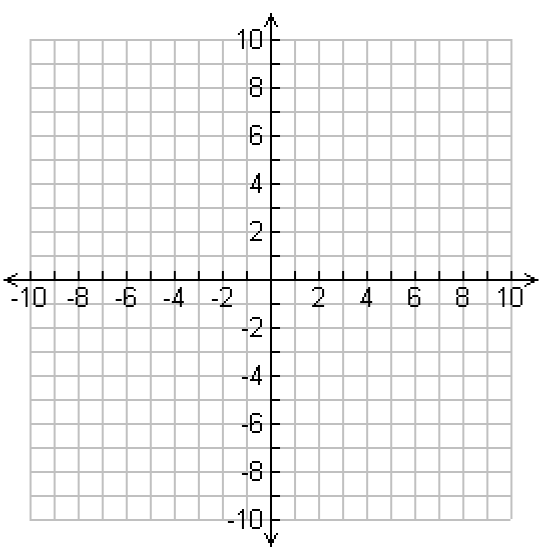
1) $y = 2x^2 + 6x + 3$



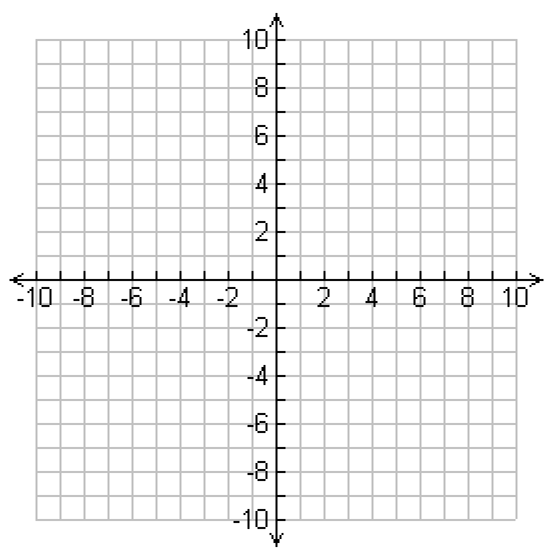
2) $y = x^2 - 2x - 1$



3) $y = -\frac{1}{2}x^2 + 4x - 3$



4) $y = 2x^2 - 8x + 6$



Converting Between Vertex and Standard Form

Converting From Standard Form to Vertex Form

1) Identify a, b, and c from the equation

2) Find the x-value of the vertex by using $x = \frac{-b}{2a}$

3) Find the y-value of the vertex by plugging in the x-value from step #2

4) Plug a (from original equation), h (the x-value of vertex), and k (the y-value of the vertex) into vertex form

1) $y = x^2 + 12x + 32$

2) $f(x) = x^2 - 8x - 9$

3) $f(x) = x^2 + 10x - 3$

4) $y = x^2 - 6x + 15$

Converting from Vertex Form to Standard Form

1) Re-write the binomial squared as the product of a binomial multiplied by itself
2) Use the distributive property to multiply
3) Distribute the coefficient, if there is one
4) Combine like terms

1) $f(x) = 2(x - 5)^2 + 8$

2) $y = -3(x + 1)^2 + 4$

3) $y = \frac{3}{2}(x - 6)^2 - 2$

4) $f(x) = -0.75(x + 16)^2 - 12$

Converting Between Forms Practice

Part One: Convert from standard form to vertex form.

1) $y = x^2 - 8x + 15$

2) $y = x^2 - 4x$

3) $y = 2x^2 + 12x + 7$

4) $y = 2x^2 - 8x + 17$

Part Two: Convert from vertex form to standard form.

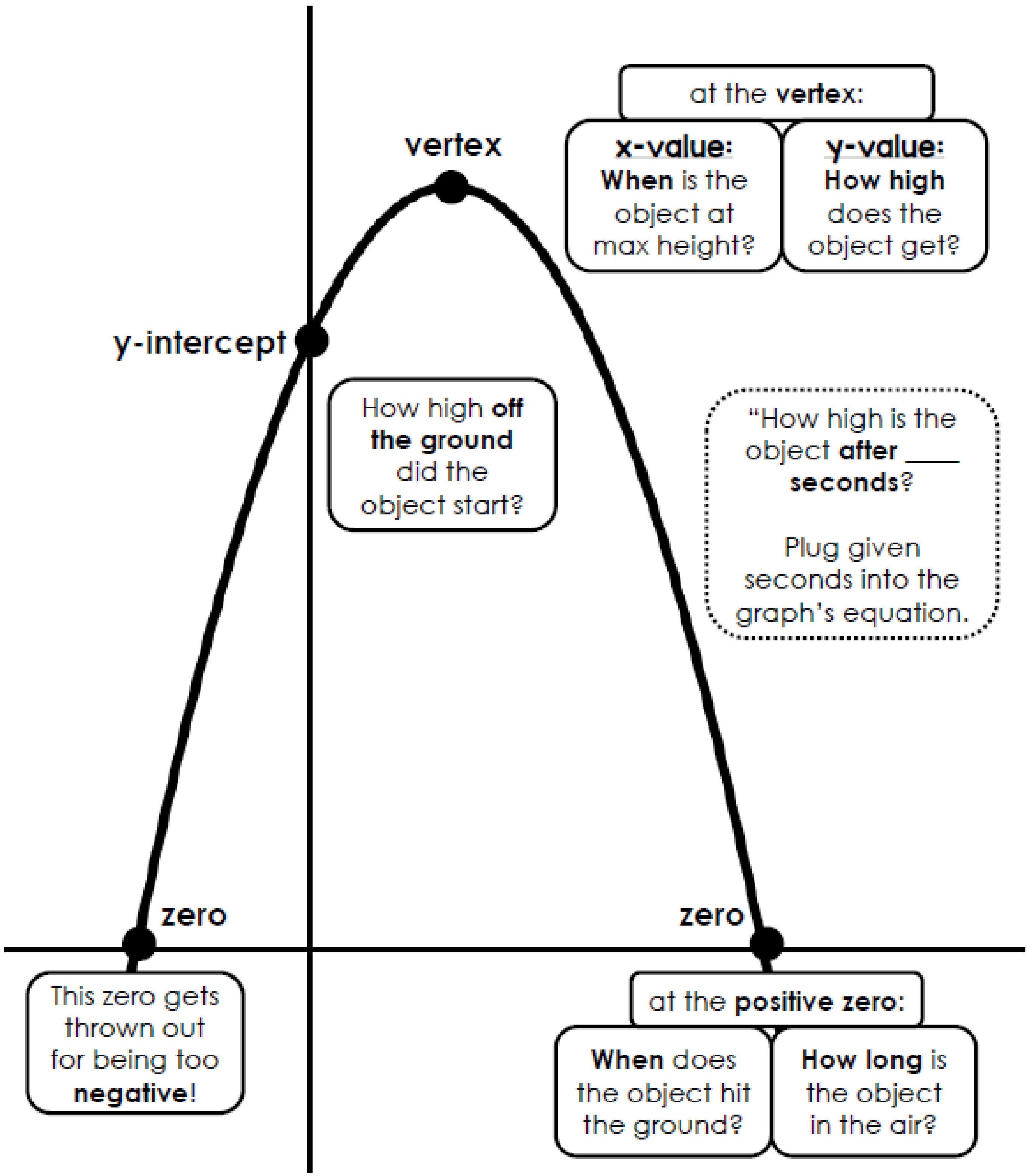
5) $y = (x + 4)^2 + 5$

6) $y = -(x + 3)^2 - 2$

7) $y = 2(x - 2)^2 - 3$

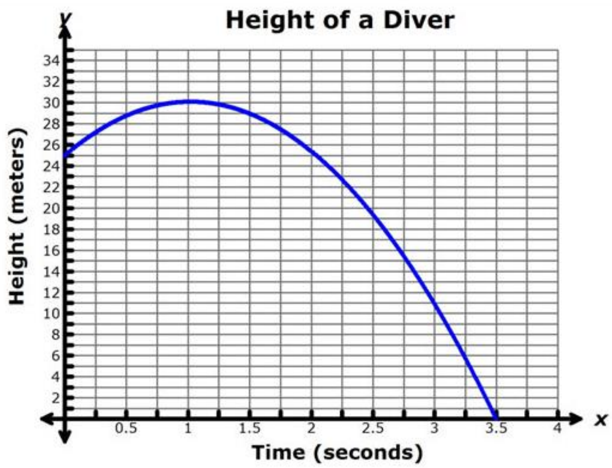
8) $y = \frac{1}{2}(x + 8)^2 + 6$

Quadratic Keywords
from scaffoldedmath.com



Applications of Quadratic Functions

1) This graph represents the height of a diver above the water vs. the time after the diver jumps from a springboard. Answer the following questions based on the information.



a) How long did it take the diver to hit the water?

b) How tall was the diving board?

c) What was the maximum height reached by the diver?

But what do we do if the graph isn't given to us? If we are not given a graph, we will be given the equation that represents the scenario.

You will need to determine whether the problem is asking you to find the vertex, the x-intercept(s), or the y-intercept.

► Vertex: maximum, minimum, highest, lowest

Vertex form: $y = a(x-h)^2 + k \rightarrow$ vertex at (h, k)

Standard form: $y = ax^2 + bx + c \rightarrow$ x-value of vertex found using $x = \frac{-b}{2a}$ and then plug that in to find y-value

► X-Intercept: ending, landing, ground level, sea level

Solve by: factoring, compl. the square, taking square roots, quadratic formula

► Y-Intercept: starting value

Plug in 0 for x in the given equation

2) An object is launched from a platform. Its height (in meters), x seconds after the launch, is modeled by: $h(x) = -5x^2 + 20x + 60$. What is the height of the object at the time of launch?

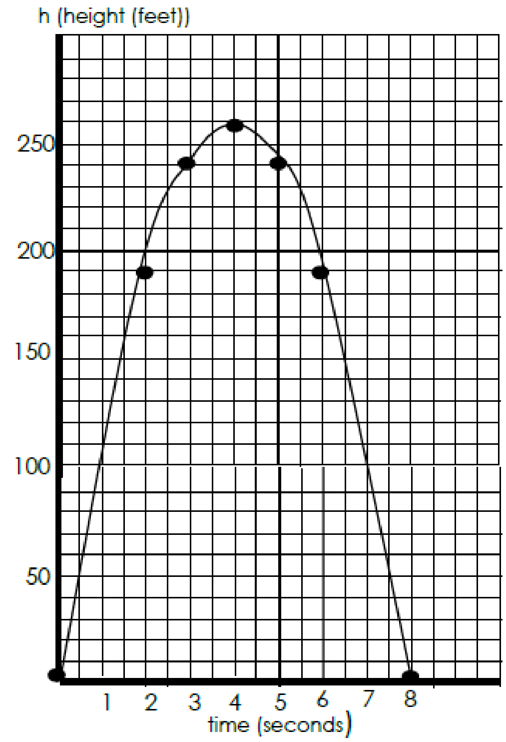
3) The height, h, in feet of an object above the ground is given by $h = -16t^2 + 64t + 190$, $t \geq 0$, where t is the time in seconds.

a) Find the time it takes the object to strike the ground.

b) Find the maximum height of the object.

Applications of Quadratic Functions Practice

1) Using the graph at the right, it shows the height, h, if feet of a small rocket t seconds after it is launched. The path of the rocket is given by the equation $h = -16t^2 + 128t$.



a) How long is the rocket in the air?

b) What is the greatest height that the rocket reaches?

c) About how high is the rocket after 1 second?

d) After 2 seconds, how high is the rocket? Is it going up or going down?

e) After 6 seconds, how high is the rocket? Is it going up or going down?

f) What is the average speed between $t = 0$ seconds and $t = 2$ seconds.

g) Using the equation, find the exact height of the rocket at 6.5 seconds.

h) What is the domain of the graph?

i) What is the range of the graph?

j) Identify the vertex.

k) Identify the axis of symmetry.

Quadratic Formula Word Problems

1) Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach his maximum height?

b. What was the highest point that Jason reached?

c. Jason hit the water after how many seconds?

2) If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height h after t seconds is given by the equation $h(t) = -16t^2 + 218t$ (if air resistance is neglected).

a. How long will it take for the rocket to return to the ground?

b. After how many seconds will the rocket be 112 feet above the ground?

c. How long will it take the rocket to hit its maximum height?

d. What is the maximum height?

3) A rocket is launched from atop a 101 – foot cliff with an initial velocity of 116 ft/s. It's height is represented by the quadratic equation $y = -16x^2 + 116x + 101$, where x represents the amount of time since launch in seconds and y represents the height of the rocket in feet.

Use the quadratic formula to find out how long the rocket will take to hit the ground after it is launched. Round to the nearest tenth of a second.

4) A blue jay swoops down from the top of a 10m tree to chase away some house sparrows. The blue jay's path follows a parabolic path given by the function $h(t) = 2t^2 - 8t + 10$ where t is time in seconds and $h(t)$ is height in meters.

a) What is the blue jay's lowest height? When did the blue jay reach the lowest height?

b) What is the blue jay's starting height?

c) Does the blue jay ever touch the ground? If so, at what time?