Name: ____________________ Block:______________

Unit 1: Angles, Triangles, Transformations

January 2021				
Monday	Tuesday	Wednesday	Thursday	Friday
			7 Vocab/Angles	8 Parallel lines
11 Triangles	12 Review/**Quiz**	13 Delta Math	14 Transformations	15 Transformations
18 No School	19 Combinations	20 Delta Math	21 Triangle Congruence	22 Triangle Congruence
25 Proofs and CPCTC	26 **Quiz** and More Proofs	27 Delta Math	28 Review	29 Test

Dates are tentative and subject to change ☺

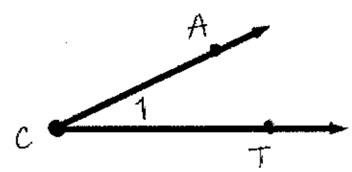
GEOMETRY VOCABULARY

Important Symbols:

∠	"m"	$\|\|$	⊥

1. Two Rays sharing the same initial point form an ____________.

- The initial point where the rays meet is called the __________ of an angle.
- We can name the angle shown right four different ways:
  - ____________
  - ____________
  - ____________
  - ____________



- Name the two rays: ___________ and _______________.
- If multiple angles share a common vertex, we must use three letters or the number label to name the angle.

2. An angle with a measure that is less than 90° is called an ___________ angle.

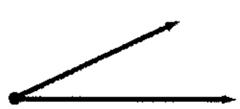
3. An angle with a measure greater than 90° is called an _____________ angle.

4. An angle with that measures exactly 180° is called a _________ angle or a ________.

5. An angle that measures exactly 90° is called a ___________ angle.

6. Label each of the following by the vocab word that describes the size of the angle.

a.



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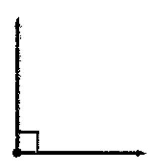
b.



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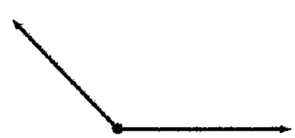
c.



____

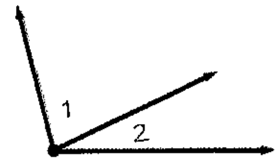
d.

____

7. Two lines that intersect to form a right angle are called ____________ lines.

- The symbol __________ indicates that one line is perpendicular to another.

8. Two angles that share a common vertex and a common ray are called __________ angles.

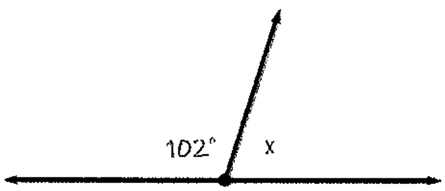


- The sum of angles 1 and 2 is 96 degrees. If angle 1 is 58 degrees, what is the measure of angle 2? ____________

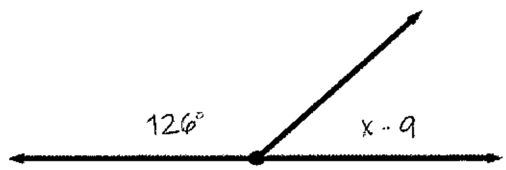
9. When the noncommon sides of two adjacent angles form a straight line, they are called a ________ ________.

- The sum of the two angles of a linear pair is __________.
- When two angles have a sum of ________ they are also called ___________.
- We can use the equation ________ + ________ = 180 to solve problems when angles are ______________.
- Solve for x:

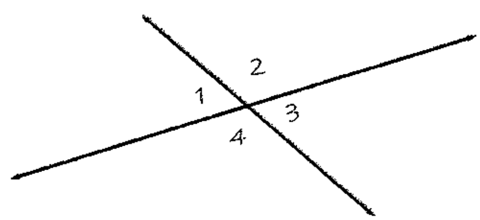
a.



b.

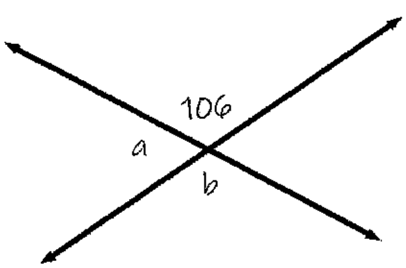


10. When two lines intersect, four angles are formed.

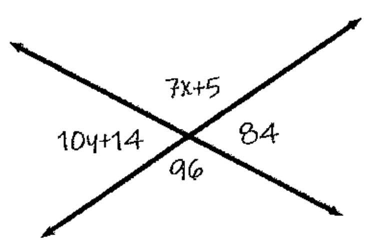


- The adjacent angles form _______ _______ and their sum is __________.
- The non-adjacent angles are called __________ angles. __________ angles are always congruent.
- For Vertical angles we can use the equation _________ = _______ to solve.
  - If the measure of angle 1 is 54 degrees, and the measure of angle 3 can be expressed as $(12x + 6)$, solve for x.

a. Determine the measure of ∠a and ∠b

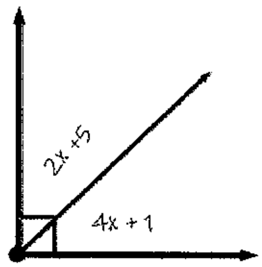


b. Solve for x and y.

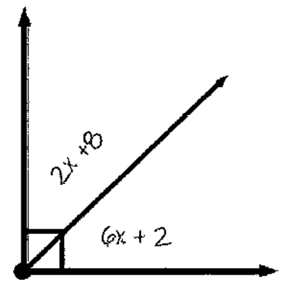


11. When the sum of two angles is 90° we say that they are ________________. We can use the equation __________ + ___________ = 90 to solve.

a.



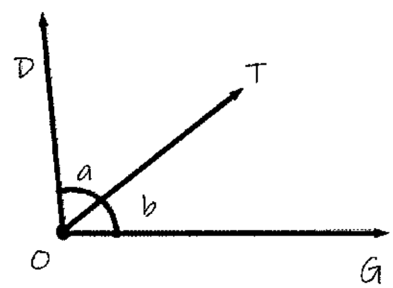
b.



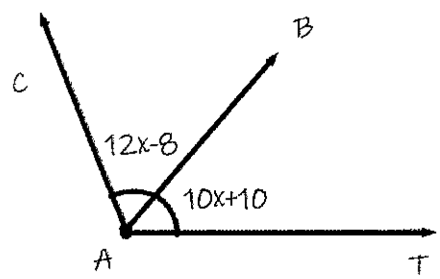
12. When a ray cuts an angle exactly in half it is called an __________ ___________.

- This creates two equal pieces.

a. ∠DOG is 98 degrees. $\overrightarrow{OT}$ is an angles bisector. Find the measure of ∠a and ∠b.



b. $\overrightarrow{AB}$ is and angle bisector. Solve for x.

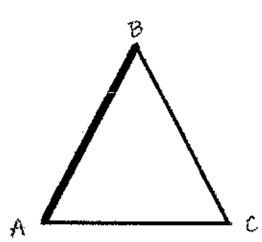


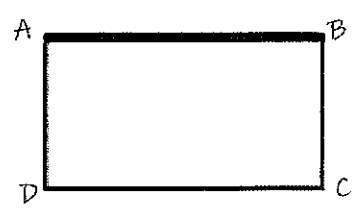
13. Naming sides of polygons

- The sides of polygons are formed by line segments. To name a side, we use two letters with a bar over them.

A. Name the Left side of the triangle.

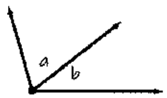
B. Name the top side of the rectangle.

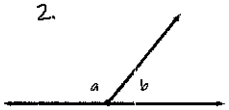




Angles Practice Page

Name the relationship between angle a and angle b.

1.


2.


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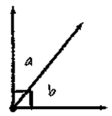

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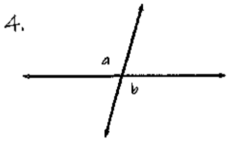
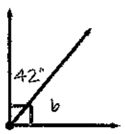
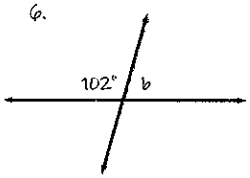
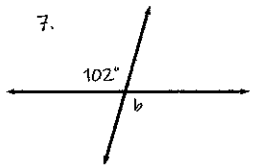


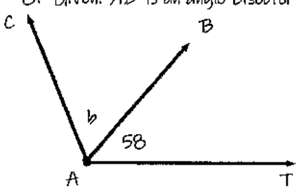
______ ______ ______ ______

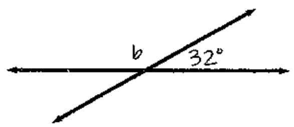
Find the measure of angle b in each diagram.

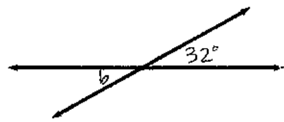
5.


6.


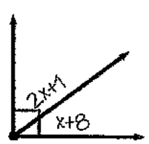
7.


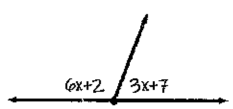
8. Given: $\overrightarrow{AB}$ is an angle Bisector


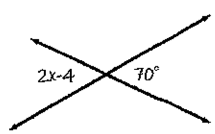
9.


10.


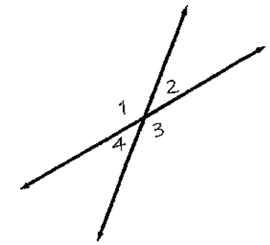
9. Solve for x.

a.


b.


c.


10. Answer the questions using the diagram to the right.



a. Name the two pairs of vertical angles: ______

b. Name 4 linear pairs: ______

c. If the measure of angle 1 is 107, what is the measure of angle 2? ______

d. If the measure of angle 1 is 107, which other angle must also be 107?

Word Problems:

11. Two angles are complementary. The first angle is 5 times the measure of the second angle. Find both angles.

12. Two angles are supplementary. The first angle is 9 less than 2 times the second angle. Find both angles.

13. Two angles are supplementary. One angle is 2/3 the measure of the other one. Find both angles.

14. Two angles are complementary. One angle is 9 more than twice the other angle. Find both angles.

15. CHALLENGE SECTION: Find the measure of all the numbered angles below.

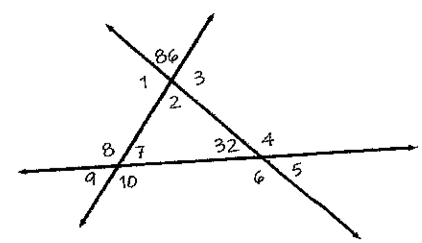
∠1 = ______ ∠2 = ______

∠3 = ______ ∠4 = ______

∠5 = ______ ∠6 = ______

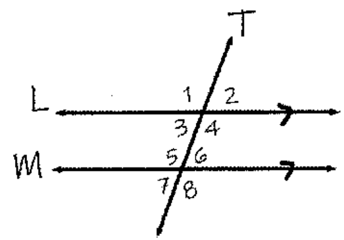
∠7 = ______ ∠8 = ______

∠9 = ______ ∠10 = ______



Parallel Lines Cut by a Transversal

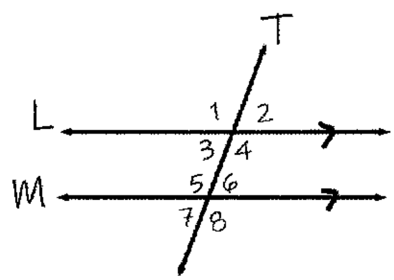
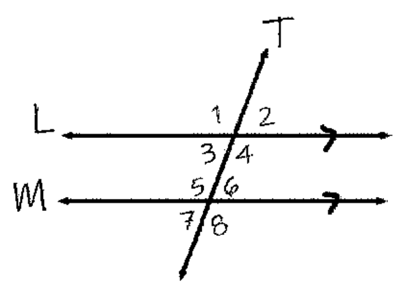
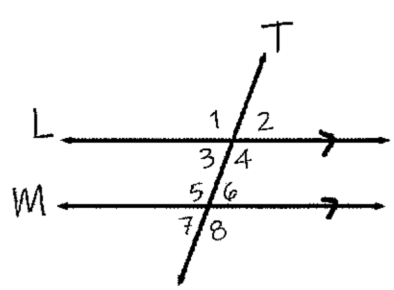
What does it mean to be parallel? Lines in the same plane (2D space), that never intersect.

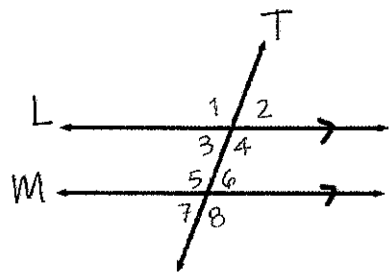
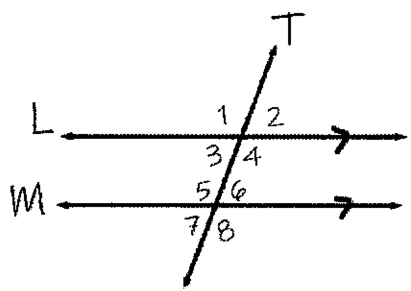


Notation: In the diagram to the right, we use the notation line "L || M, " to say that line "L" is parallel to line "M." The extra arrows on L and M also denote that the lines are parallel.

What is a transversal? A transversal is a line that "transverses" or cuts through other lines. Line "T" is the transversal because is cuts through L and M. Where the **transversal** intersects the parallel lines, angles are created.

Angle Relationships:

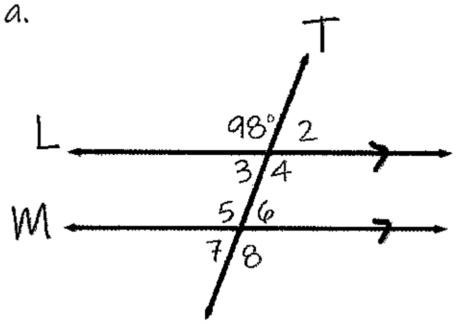
Corresponding Angles In the ______ ______ on different parallel lines. They are always __________. Equation: ______ = ______		Examples: ∠1 & ∠ ______ ∠2 & ∠ ______ ∠3 & ∠ ______ ∠4 & ∠ ______
Alternate Interior ________ sides of the transversal, ________ lines L and M. They are always __________. Equation: ______ = ______		Examples: ∠3 & ∠ ______ ∠4 & ∠ ______
Alternate Exterior ________ sides of the transversal, ________ lines L and M. They are always __________. Equation: ______ = ______		Examples: ∠1 & ∠ ______ ∠2 & ∠ ______

<u>Same Side Interior</u> __________ side of the transversal, _________ lines L and M. They are always ___________. Equation: _____ +_____ = 180		Examples: ∠3 & ∠ ______ ∠4 & ∠ ______
<u>Same Side Exterior</u> __________ side of the transversal, _________ lines L and M. They are always ___________. Equation: _____ +_____ = 180		Examples: ∠1 & ∠ ______ ∠2 & ∠ ______

<u>You Try!</u>

1. Given the measure of one angle, find all the other angles:

a.



∠2 = ____________

∠3 = ____________

∠4= ____________

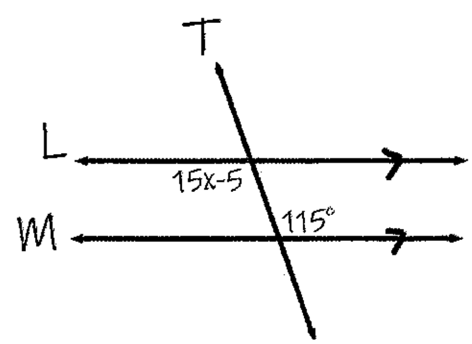
∠5= ____________

∠6= ____________

∠7= ____________

∠8= ____________

b. Solve for X.



X = ______________

Parallel Lines Practice

1. Find the measures of all the numbered angles:

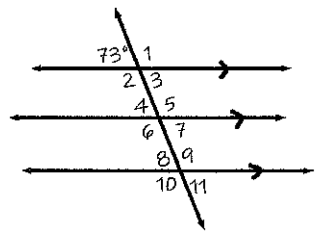
∠ 1 = ____________ ∠ 2 = ____________

∠ 3 = ____________ ∠ 4= ____________

∠ 5= ____________ ∠ 6= ____________

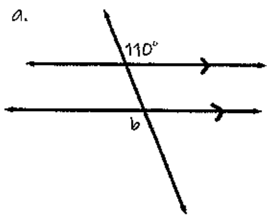
∠ 7= ____________ ∠ 8= ____________

∠ 9 = ____________ ∠ 10 = ____________ ∠ 11 = ____________

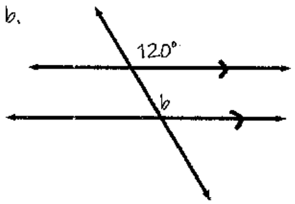


2. Name the angle relationship, and find the measure of angle b.

a.



b.



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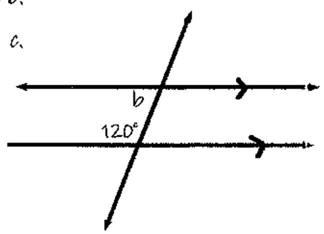

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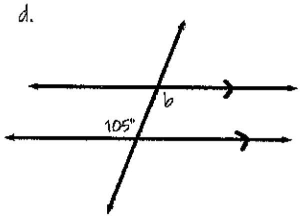
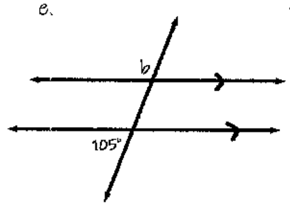
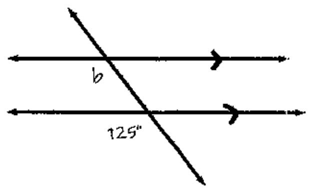


e.

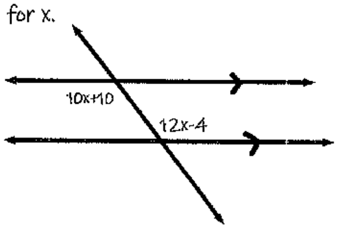


f.

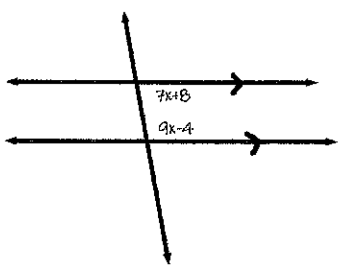


3. Solve for x.

a.

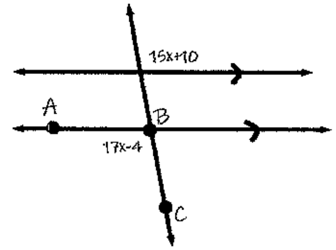


b.



4. Find the measure of ∠ABC in each diagram.

a.



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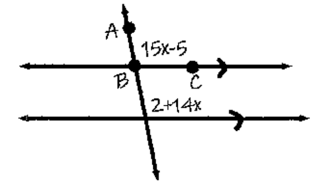
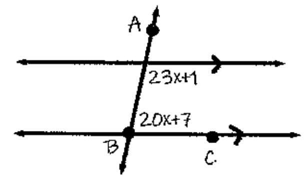
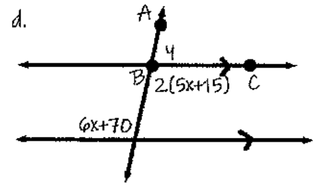


c.

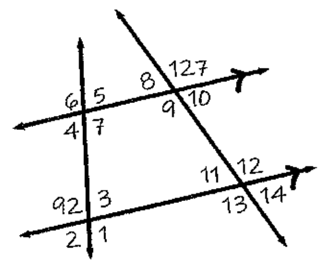


d.



CHALLENGE SECTION:

5.



∠ 1 = ____________ ∠ 2 = ____________

∠ 3 = ____________ ∠ 4= ____________

∠ 5= ____________ ∠ 6= ____________

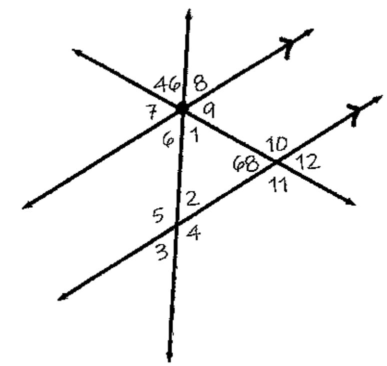
∠ 7= ____________ ∠ 8= ____________

∠ 9 = ____________ ∠ 10 = ____________

∠ 11 = ____________ ∠ 12 = ____________

∠ 13 = ____________ ∠ 14= ____________

6.



∠ 1 = ____________ ∠ 2 = ____________

∠ 3 = ____________ ∠ 4= ____________

∠ 5= ____________ ∠ 6= ____________

∠ 7= ____________ ∠ 8= ____________

∠ 9 = ____________ ∠ 10 = ____________

∠ 11 = ____________ ∠ 12 = ____________

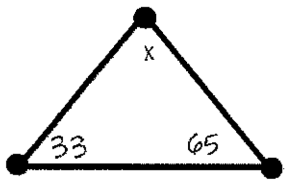
All About Triangles

1. Triangle Sum Theorem:

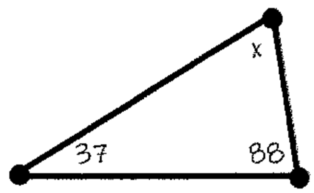
The sum of the three **interior** angles of a triangle is ________.

Examples: Find the missing angle in each triangle below.

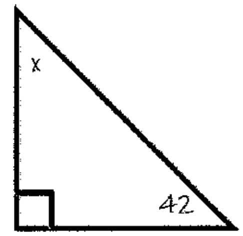
a.



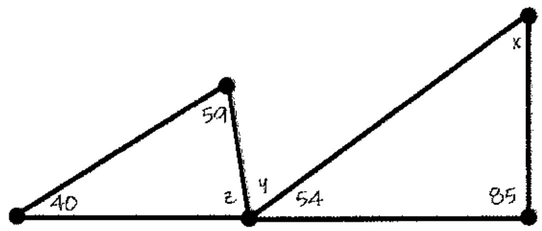
b.



c.



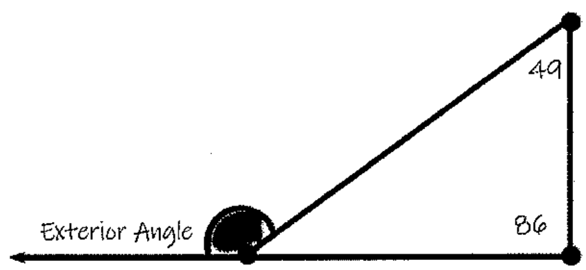
2. **Challenge:** Use the properties we have learned about angle relationships to find the missing angles in the diagrams.



X = ______________

Y= ______________ Z= ______________

3. **The Exterior Angle Theorem:** If you extend the side lengths of a triangle beyond its vertices, exterior angles are created.



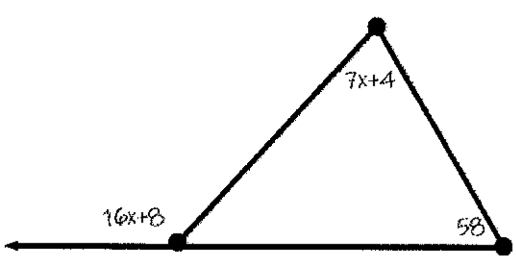
- The sum of the two non-adjacent angles in the triangle will be equal to the exterior angle.
- The non-adjacent angles are called the ________ angles.
- Equation: ________ + _______ = _________

$49 + 86 = \textit{Exterior}$

$\boxed{135 = \textit{Exterior}}$

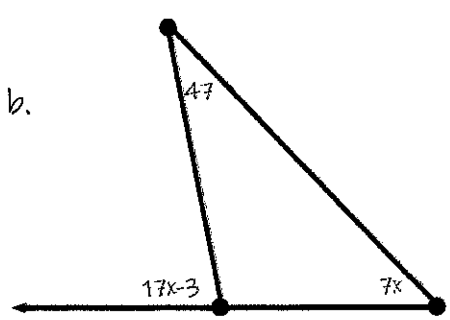
Examples: Solve for x.

a.



x = _______________

b.

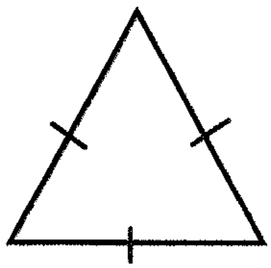


x = _______________

4. Classifying Triangles

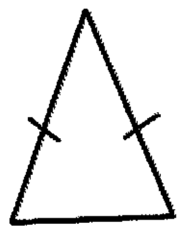
By Their Sides:

a. Three Congruent Sides:



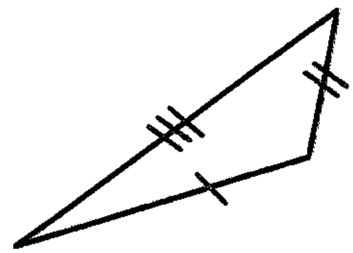
____

b. Two Congruent Sides



____

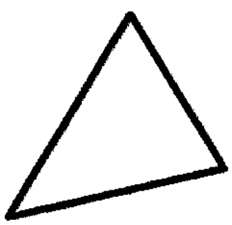
c. No Congruent Sides



____

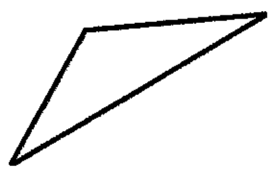
By Their Angles

d. All Acute Angles



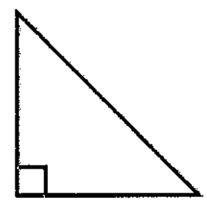
____

e. One Obtuse Angle



____

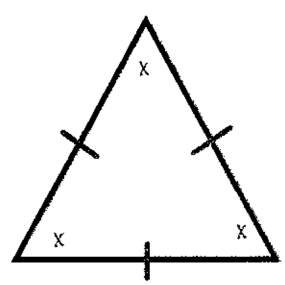
f. One Right Angle



____

5. An __________ triangle will also be __________ meaning that all three angles will be congruent too.

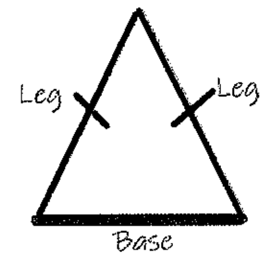
- If all angles add be 180, and they are all the same...what would the measure of each angle have to be?



- This will always be true. Each angle in an equilateral triangle will always be __________.

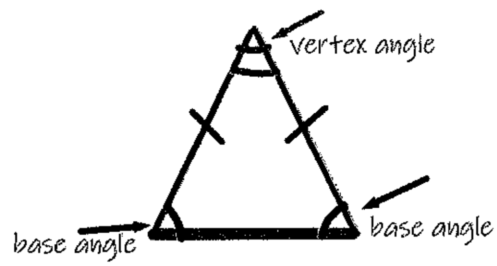
6. The Base Angles Theorem

- In an **Isosceles** triangle, the two congruent sides are called the _______ of the triangle, and the third side is called the _________.



- The two angles on the base are called the _______ _______, and they are always congruent. They are the angles opposite the congruent sides.

- The third angle is called the vertex angle.

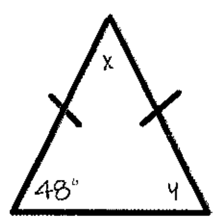


- If you know two angles are congruent, that proves the sides opposite the angles are ___________.

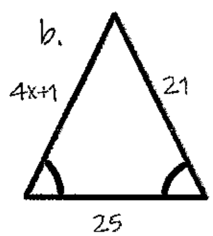
- If you know that the sides are congruent, that proves that the angles opposite them are _____________.

Examples: Solve for x and y.

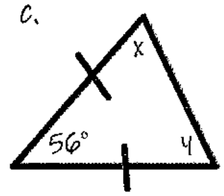
a.



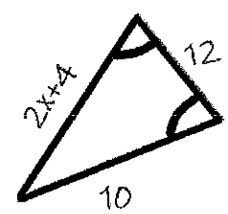
b.



c.



d.



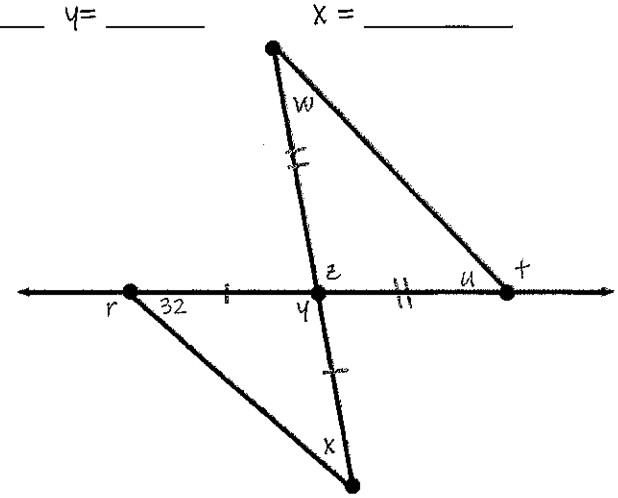
x= ______ y= ______ x= ________ x=______ y= ______ x = _________

e. **Challenge Section!**

w = __________ x = __________ y = ____________

z=____________ t = _________ u= ____________

r = ____________



7. <u>**The Triangle Inequality Theorem**</u>

- The sum of any two sides of a triangle must be ________ than the length of the ________ side.
- The largest angle is opposite of the ______________, and the smallest angle is opposite the ______________.

State if the three numbers can be the side lengths of a triangle.

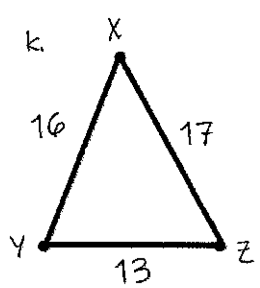
a. 10,7,20	b. 3,6,9	c. 3,4,5
d. 7,10,3	e. 12,18,7	f. 8,15,17

Two sides of a triangle have the following measures. Find the Range of possible measures of the third side.

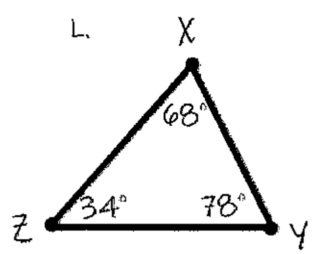
g. 9,9	h. 10,7
i. 10,9	j. 8,12

For each triangle below, if the sides are given: state which angle is the largest one, and which angle is the smallest one. IF the angles are given, state which side is the longest, and which side is the shortest.

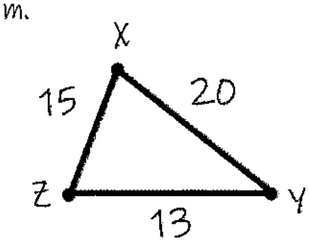
k.



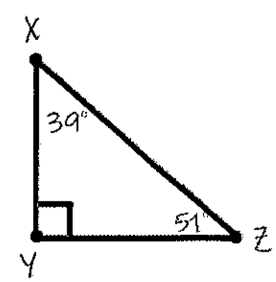
L.



m.



n.

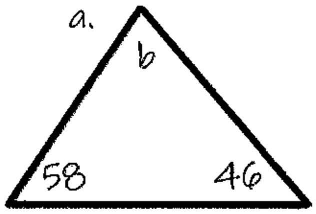


Triangles Practice Page

Part 1: The sum of the interior angles of a triangles is 180°.

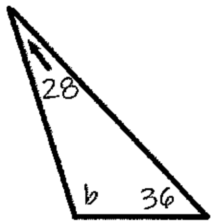
1. Find the measure of angle "b" in each diagram below.

a.



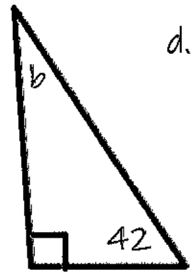
b=

b.



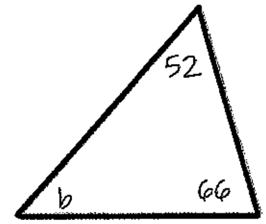
b =

c.



b =

d.



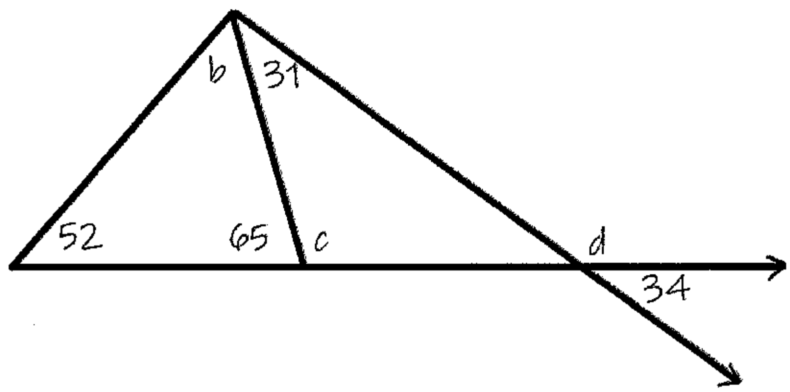
b=

e. solve for b, c, and d.

b =

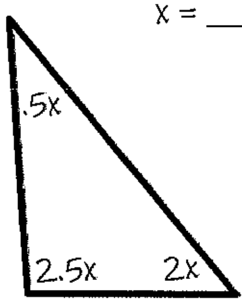
C =

D =



2. Solve for x

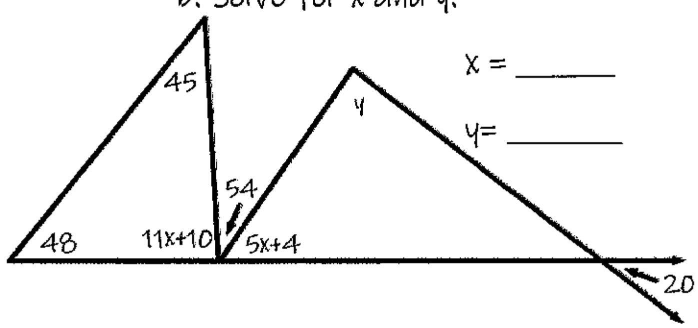
a. x =



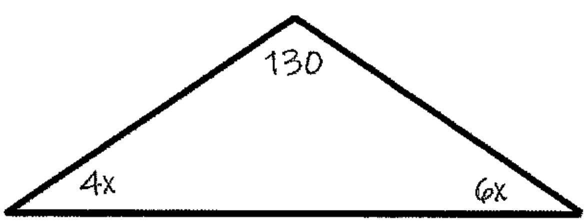
b. Solve for x and y.

x =

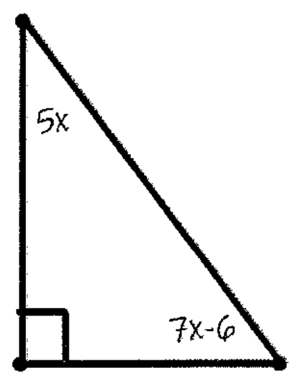
y=



c. x =



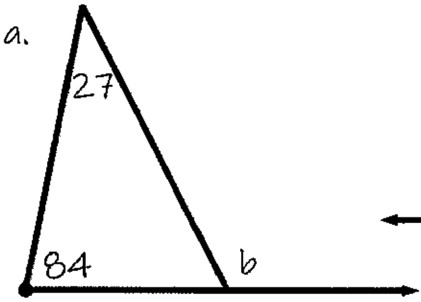
d. x =



Part 2: **Exterior angle theorem**: The sum of the two non-adjacent (remote) angles will be equal to the exterior angle.

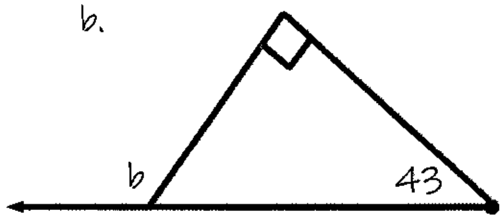
3. Solve for the missing angle, b.

a.



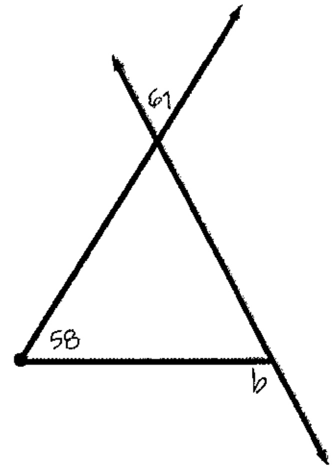
b = ____________

b.



b = ____________

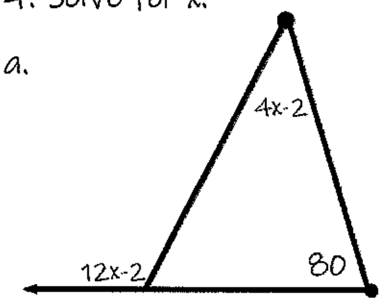
c.



b = ____________

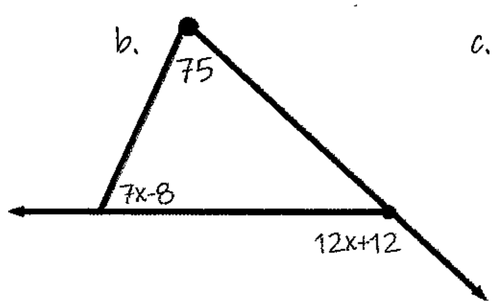
4. Solve for x.

a.



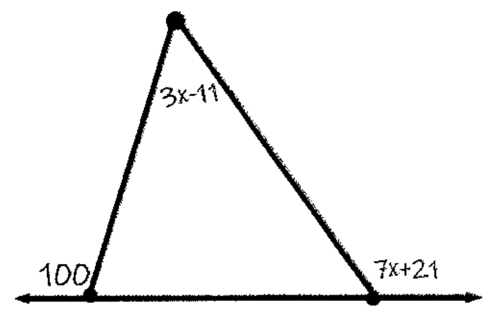
x = __________

b.



x = ____________

c.

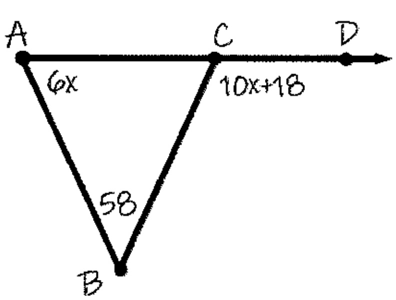


x = ______________

5. More exterior angle theorem.

a. Solve for x, then determine the requested angle measure.

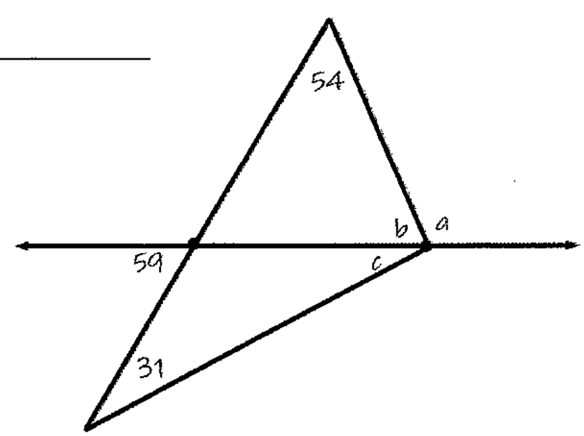
$m\angle BCD$ = ___________________



b. Find the missing angles.

a = ___________ b= ________________

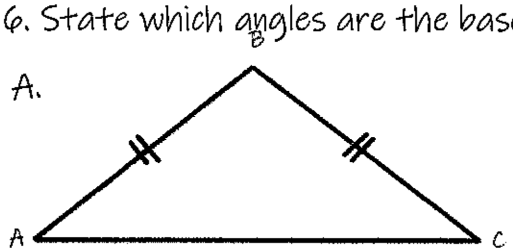
c = _____________



Part 3: The Base Angles Theorem: a. If two sides of a triangle are congruent, the angles opposite them must be congruent. b. The converse is also true: if two angles of a triangle are congruent, then the sides opposite the angles must be congruent.

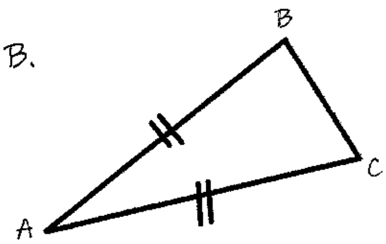
6. State which angles are the base angles.

A.



______ and ______

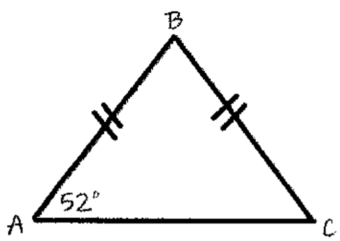
B.



______ and ______

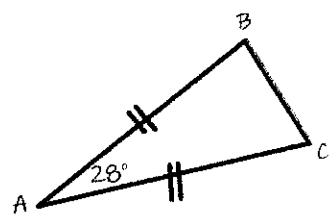
7. Find both missing angles in each triangle.

a.



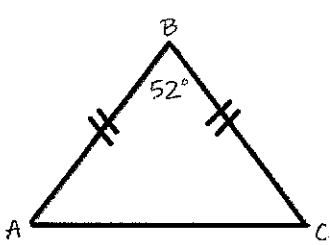
∠B = ______ ∠C = ______

b.



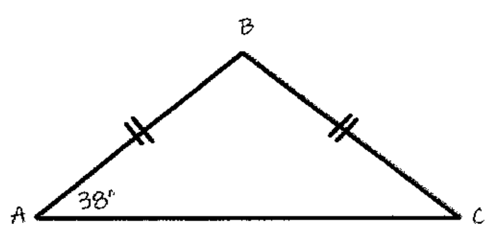
∠B = ______ ∠C = ______

c.



∠A = ______ ∠C = ______

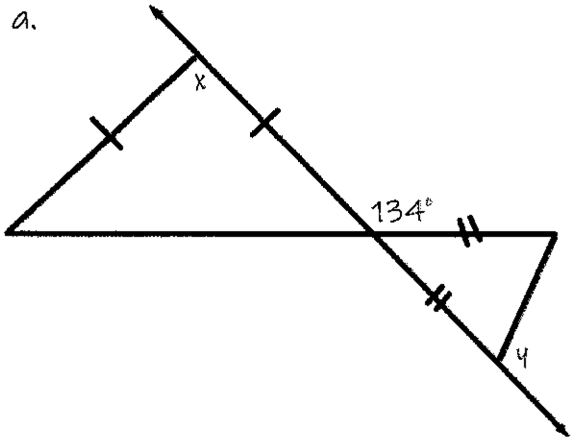
d.



∠B = ______ ∠C = ______

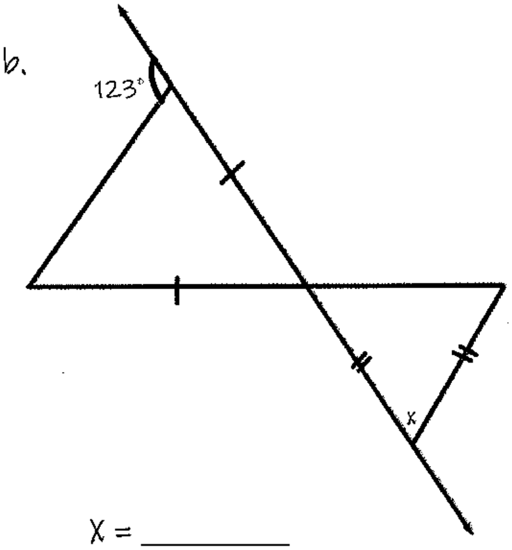
8. Challenge problems with Isosceles Triangles

a.



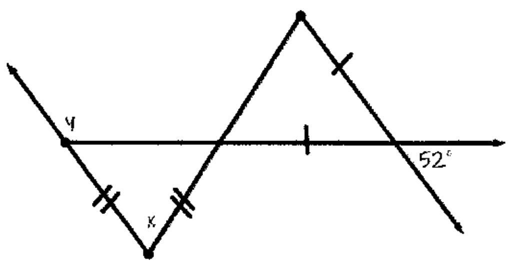
X = ________ y = ________

b.



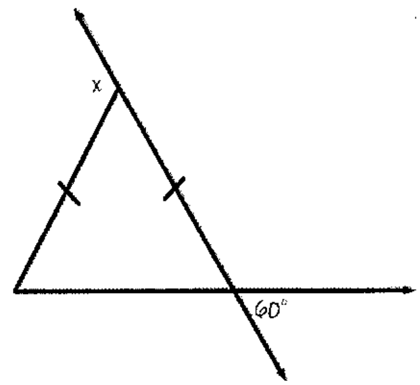
X = ________

c.



x = ________ y = ________

d.



x = ________

Part 4: **Triangle inequality Theorem:** The sum of any two sides of a triangle must be greater than the third side.

Determine if the following could be side lengths of a triangle.

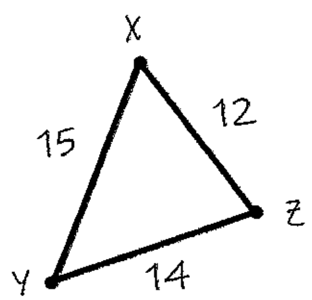
9. a. 3,4,6 b. 5,7,12 c. 5,7, 8 d. 12,15,17 e. 20, 40, 61

10. Given two sides of a triangle, find the interval of values for the third side.

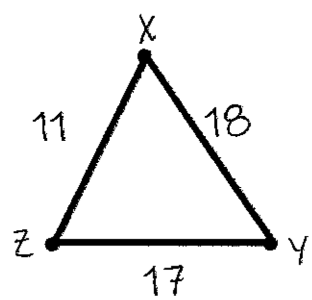
a. 9,11 b. 5, 8 c. 18, 22 d. 14, 29

11. Given the sides lengths, state which angle must be the largest, medium, and smallest. List them in Greatest to Least order.

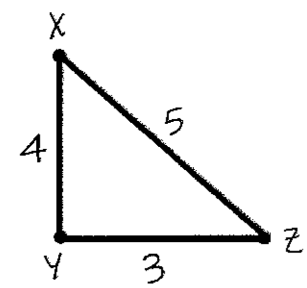
a.



b.

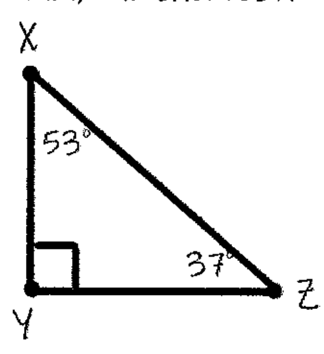


c.

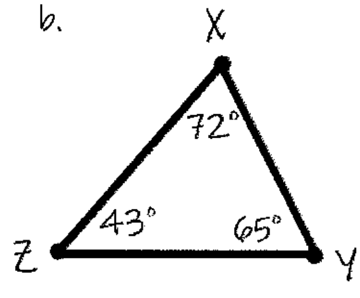


12. Given the measures of the angles, determine which side must be the longest, medium, and shortest. List them in Greatest to Least order.

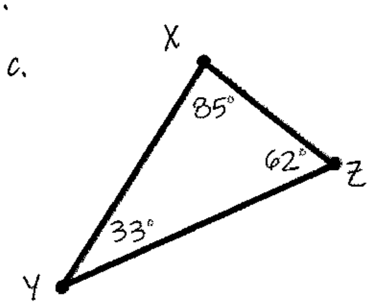
a.



b.



c.



Transformation Rules

Translation- moves every point of a figure by the same distance in a given direction. We can "slide" a point or a figure left, right, up or down.

- Right: $(x,y) \rightarrow (x+a, y)$ This will shift the point "a" units **right**
- Left: $(x,y) \rightarrow (x-a, y)$ This will shift a point "a" units **left**.
- Up: $(x,y) \rightarrow (x, y+b)$ This will shift a point "b" units **up**
- Down: $(x,y) \rightarrow (x, y-b)$ This will shift a point "b" units **down**.

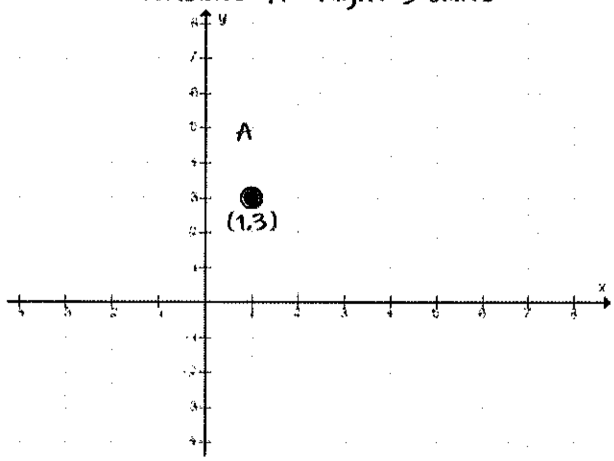
Define:

Pre-Image- ______________________________.

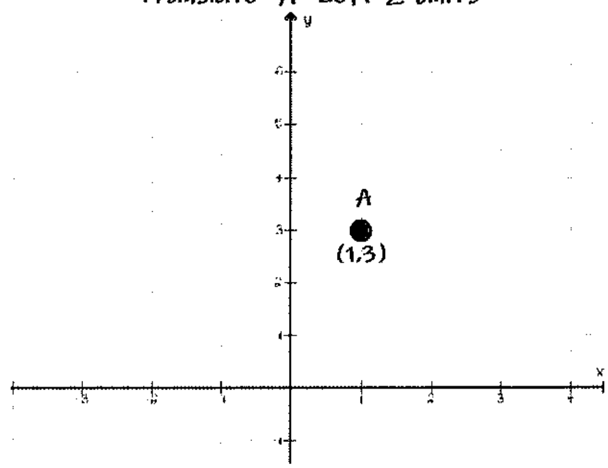
Image- ______________________________.

Examples:

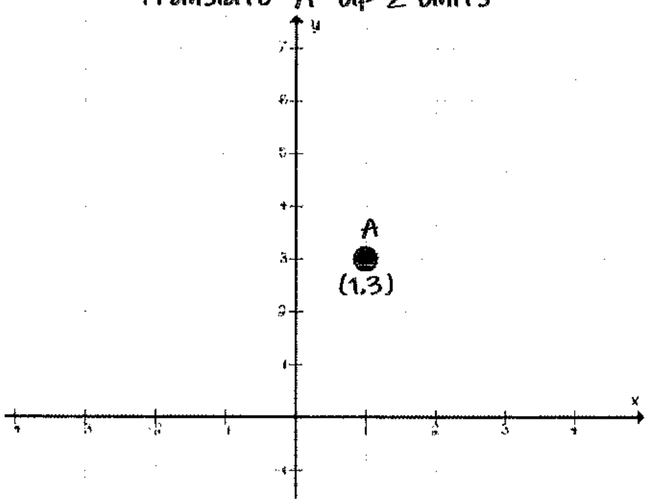
Translate "A" Right 3 Units



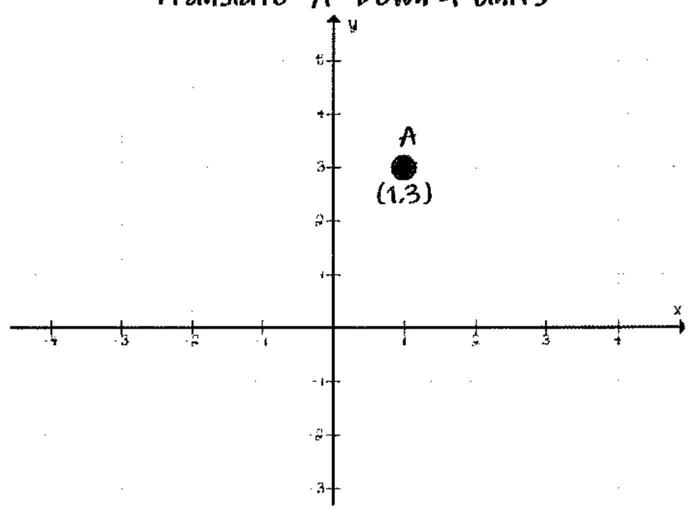
Translate "A" Left 2 Units



Translate "A" Up 2 Units

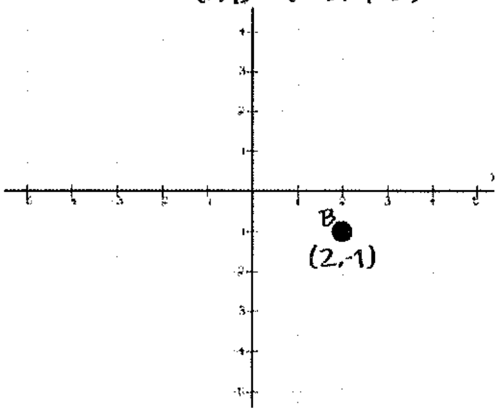


Translate "A" Down 4 Units

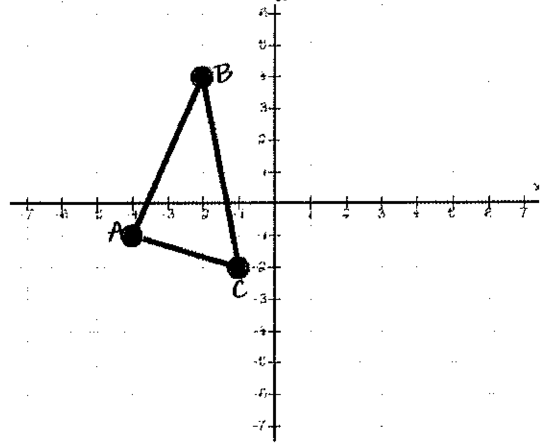


You Try!

1. Translate: $(x,y) \rightarrow (x-3, y+2)$



2. Translate: $(x,y) \rightarrow (x+2, y-5)$



3. **Working Backwards:** The coordinates shown were translated by the rule $(x,y) \rightarrow (x+5, y-2)$. What were the coordinates of the pre-image?

A (,) → A'(2,5)

B (,) → B' (4, 7)

C (,) → C' (5,-1)

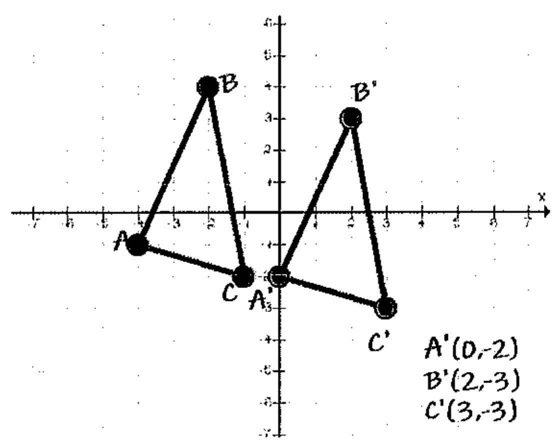
4. **Writing a rule**: Write a rule that would produce the translation shown below.

a. A (3,7) → A' (-5, 4) Rule: (x,y)→ __________

b. B (4, 5) → B' (9, -2) Rule: (x,y)→ __________

c. Using the figure, determine the rule for the translation that has occurred.

Rule: (x, y) → __________

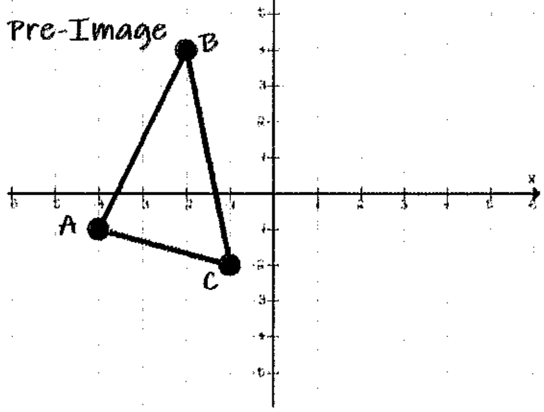


A'(0,-2)
B'(2,-3)
C'(3,-3)

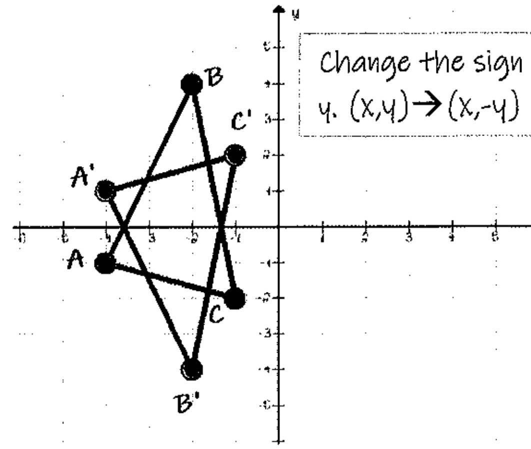
Reflections: A reflection "flips" a point or a figure over a given line. All the points of the image will be the same distance away from the line of reflection as the pre-image, just on the opposite side of the line.

- Reflect over x-axis: Change the sign of y. $(x,y) \rightarrow (x,-y)$
- Reflect over y-axis: Change the sign of x. $(x,y) \rightarrow (-x, y)$
- Reflect over the line y = x: Change the order. $(x,y) \rightarrow (y,x)$
- Reflect over the line y = -x: Change the order and the signs. $(x,y) \rightarrow (-y,-x)$

Examples:



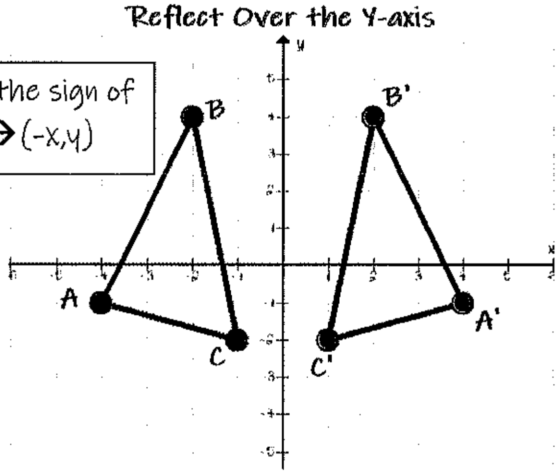
Reflect Over the X-axis



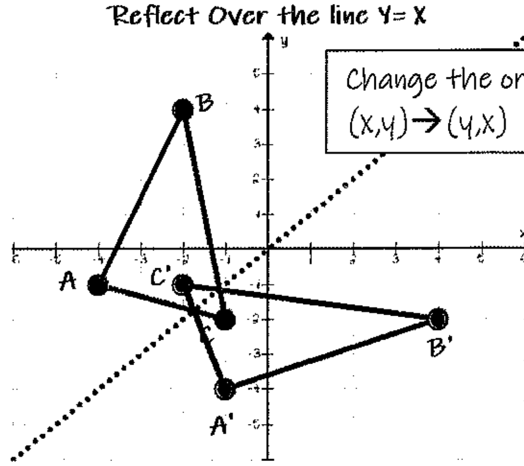
Change the sign of y. $(x,y) \rightarrow (x,-y)$

Reflect Over the Y-axis

Change the sign of x. $(x,y) \rightarrow (-x,y)$



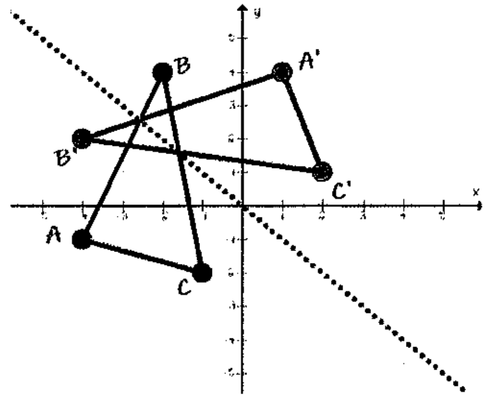
Reflect Over the line Y= X



Change the order. $(x,y) \rightarrow (y,x)$

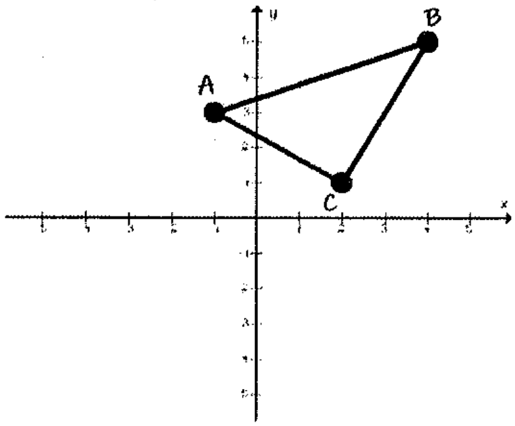
Reflect Over the line Y= - X

Change the order and sign. $(x,y) \rightarrow (-y,-x)$

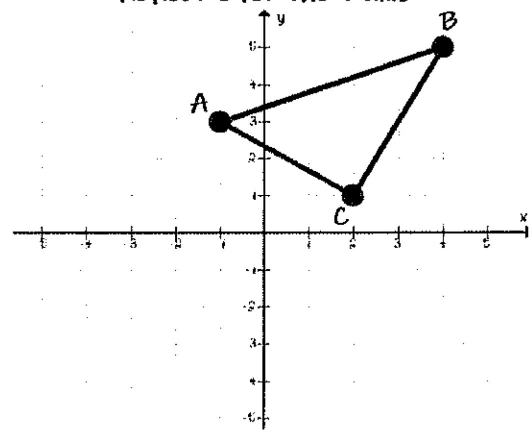


You Try!

1. Reflect Over the X-axis



2. Reflect Over the Y-axis



3. Apply the given reflection to the coordinates below.

a. Reflect over y = x

A(1,2) → A' ________

b. Reflect over y = -x

B(3,-4) → B' ________

c. Reflect over x-axis

C(-3,-2)→ C' ________

4. Determine the line of reflection:

a. Given the coordinate:

A(1,2)→A'(-2,-1)

____

b. Given the coordinate:

B(3,-4)→B'(-3,-4)

____

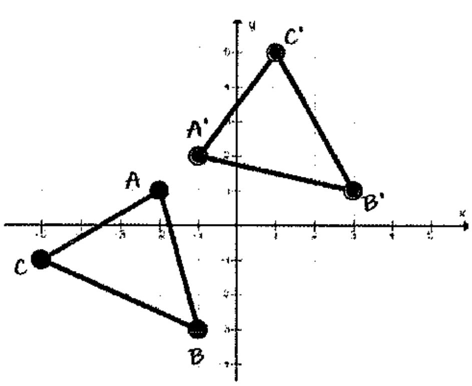
c. Given the coordinate:

C(-3,-2)→ C'(-2,-3)

____

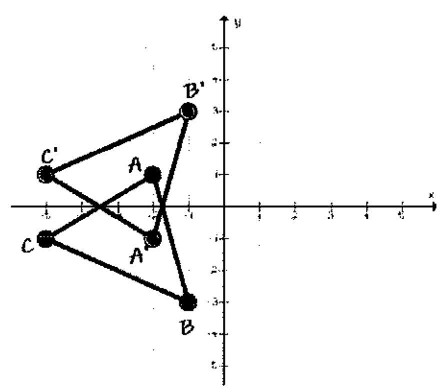
5. Determine the line of reflection from the figures:

a.



____

b.



____

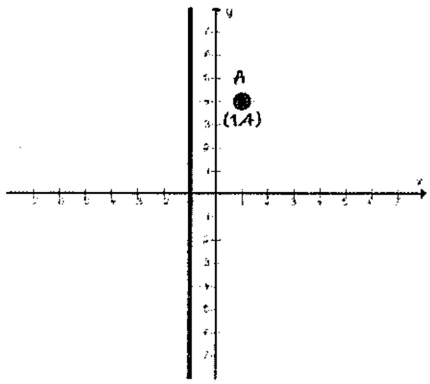
Reflecting over a given line: Mirror the points the same distance away on the other side

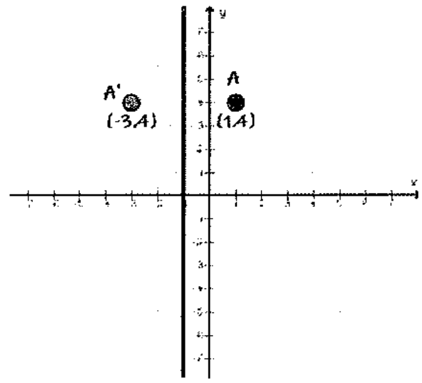
X = # is always a vertical line!

Y = # is always a horizonal line!

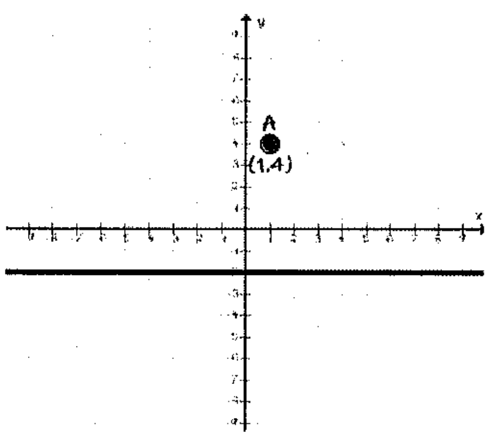
Examples:

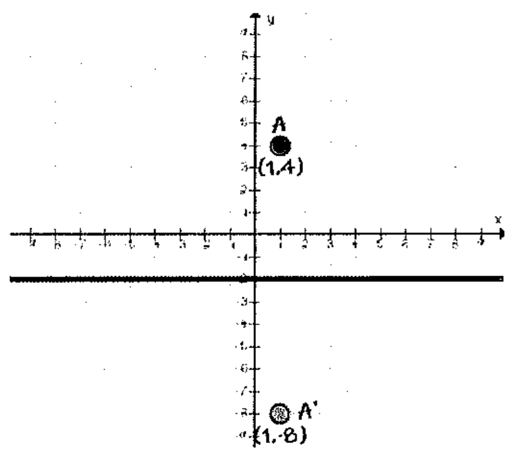
a. Reflect the point A over the line **x = -1**. "A" is two units away from the line **x = -1**, so we place A' two units away from **x = -1**, on the opposite side of the line.



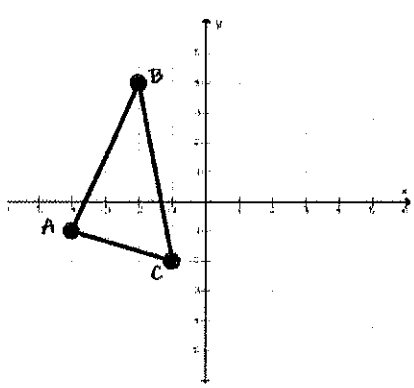


b. Reflect the point A over the line **y = -2**. The point A is six units from the line **y = -2**, so we place A' six units away from **y = -2** on the opposite side.

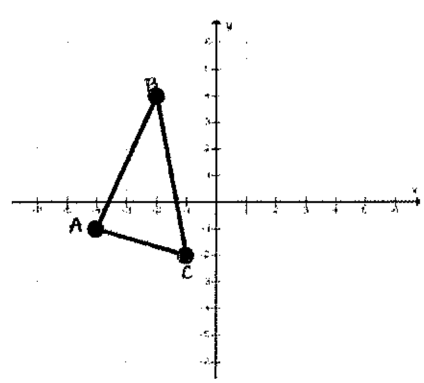




You Try! A. Reflect $\triangle ABC$ over the line y = 1.



B. Reflect $\triangle ABC$ over the line x = 1.



Rotations: When we rotate a point or figure, we are turning it about a fixed point called the **center of rotation.** We will assume that the center of rotation is the origin unless otherwise specified.

- When the center of rotation is the origin, we have a set of rules we can apply to our coordinate.
- The direction of rotation is understood to be **counter-clockwise** unless otherwise specified.

90 Degrees CCW is the same as 270 CW

- Use the rule $(x,y) \rightarrow (-y,x)$

270 Degrees CCW is the same as 90 CW

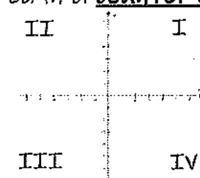
- Use the rule $(x,y) \rightarrow (y,-x)$

180 Degrees is the same in both directions

- Use the rule $(x,y) \rightarrow (-x,-y)$

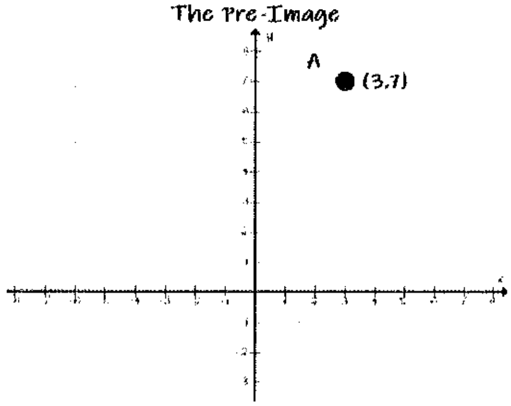
Why **Counter Clockwise??**

The quadrants of the coordinate plane are *numbered* in a **counter clockwise** direction.

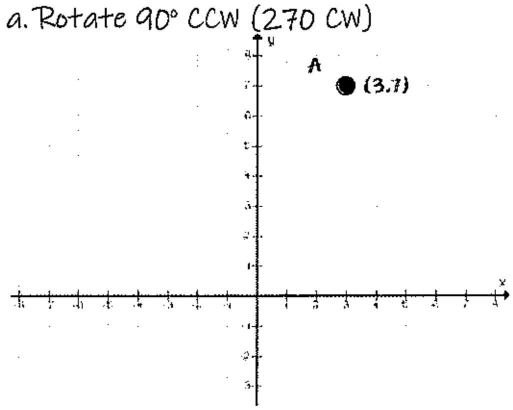


Examples with one point: A is the point (3,7). Let's look at what happens to it as we rotate.

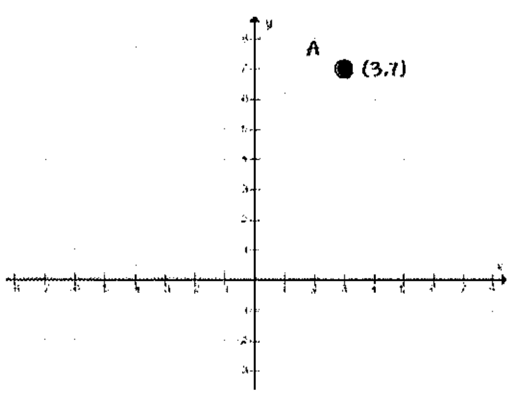
The Pre-Image



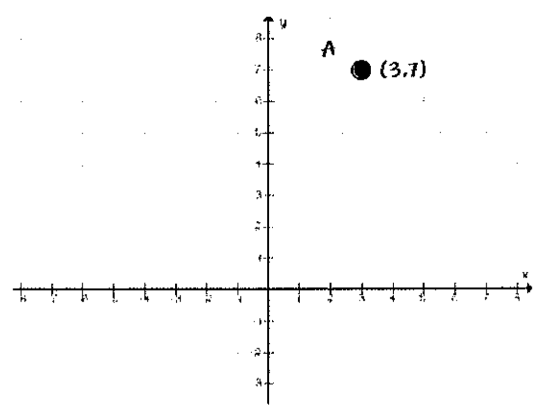
a. Rotate 90° CCW (270 CW)



b. Rotate 270° CCW (90CW)

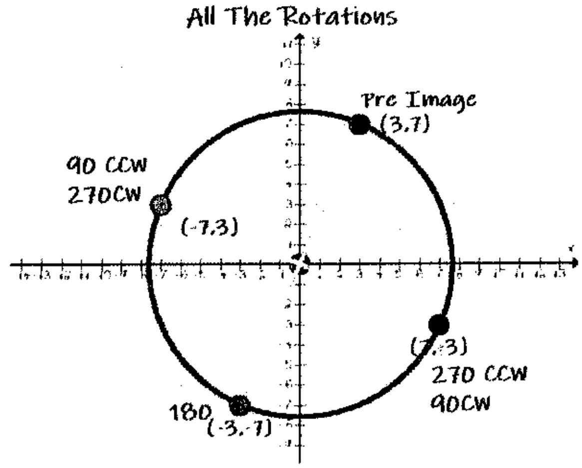


c. Rotate 180°



Look in Depth: What's really happening as we rotate?

- We are dragging our point along an imaginary circle! Each time we preform a rotation, the point will be the same distance away from the center of rotation, just a different spot!
- If we want to rotate about a point that is not the origin, the process is a little bit different….



When the center of rotation is NOT the origin, here's what we can do:

1. Subtract the center of rotation from your coordinate. This shifts the center of rotation back to the origin, allowing us to use our rules.
2. Apply the rule.
3. Add the center of rotation back to your coordinate. This shifts the center of rotation back to the right spot.

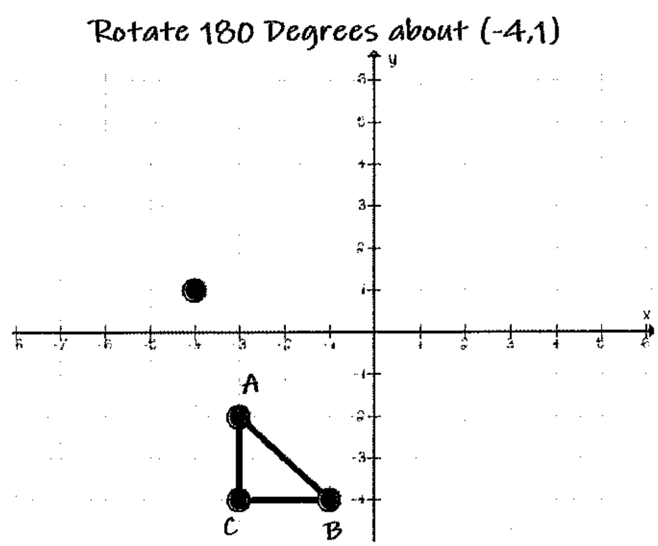
Take a Look: Rotate $\triangle ABC$ 180° about the point (-4,1)

1. Subtract the center of rotation from each coordinate:

A (-3,-2) becomes (-3 – -4 , -2 – 1) = A*(_______)

B(-1,-4) becomes (-1 – -4 , -4 – 1) = B*(_______)

C (-3,-4) becomes (-3 – -4 , -4 – 1) = C*(_______)



2. Apply the Rule: 180 degrees (x,y)→(-x,-y)

A*(______) becomes A**(________)

B*(______) becomes B**(________)

C*(______) becomes C**(________)

3. Add the Center of Rotation back in!

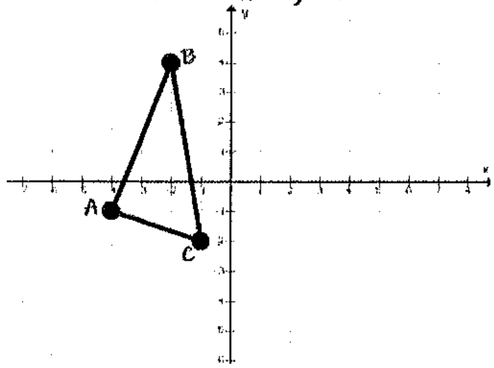
A**(________) becomes (____+ -4, ____+ 1)= A'(_______)

B**(________) becomes (____+ -4, ____+1)= B'(_______)

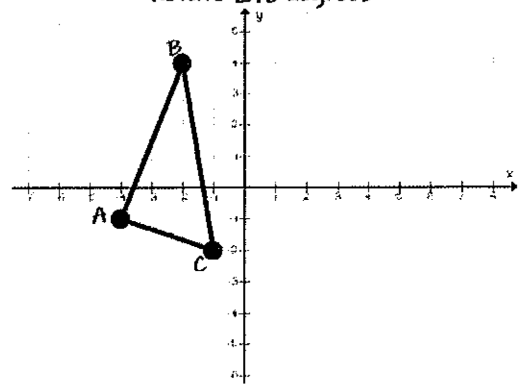
C** (________) becomes (____+-4, ____+1) = C'(_______)

You Try!

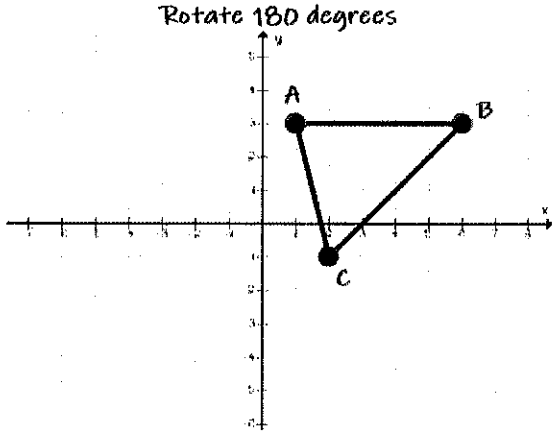
1. Rotate 90 degrees



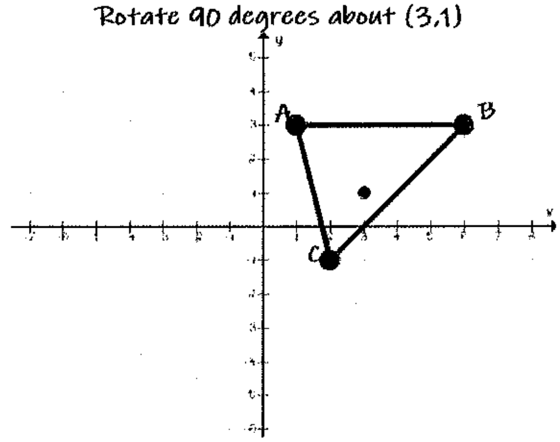
2. Rotate 270 degrees



3. Rotate 180 degrees



4. Rotate 90 degrees about (3,1)



5. Determine the transformation that has occurred from the coordinates:

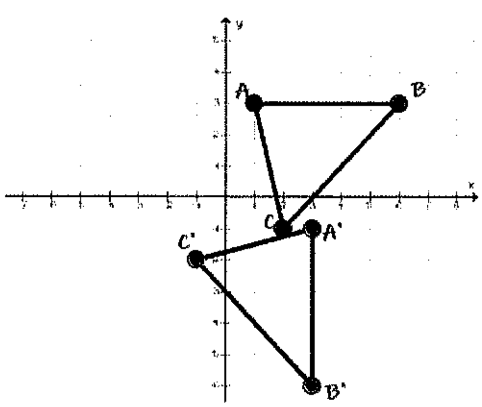
a. A(1,7) → A'(-7,1) ____________

b. B(-2,5) → B'(5,2) ____________

c. C (-2,-3) → C'(2,3) ____________

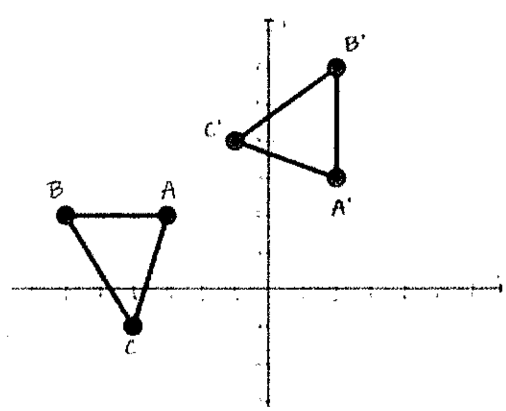
6. Determine the transformation that has occurred from the figures:

a.



____

b.



____

Practice Translations

1. Translate the image by (x + 4, y – 6)

A(–2,4) → A'________
B(0,–8) → B'________
C(–3,5) → C'________

2. Translate the image by (x – 1, y + 5)

D(1,2) → D'________
E(–3,–5) → E'________
F(4,–1) → F'________

3. Find the pre-image (x – 9, y + 13)

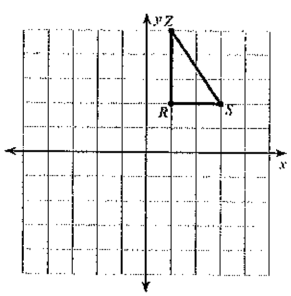
G________ → G'(5,–29)
H________ → H'(20,–19)
I________ → I'(21,–4)

4. Find the pre-image (x + 7, y – 19)

G________ → G'(2,18)
H________ → H'(13,29)
I________ → I'(24,37)

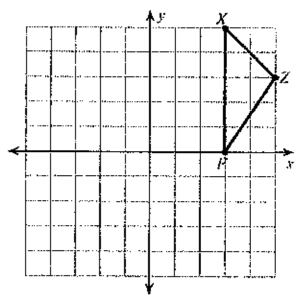
5. Translate the image.

translation: $(x, y) \to (x - 3, y - 5)$

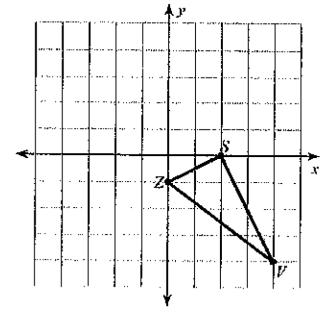


6. Translate the image.

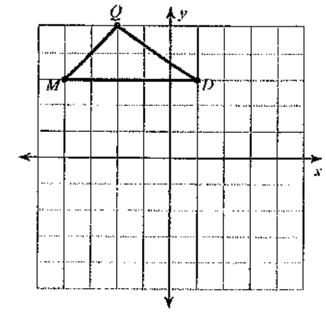
translation: $(x, y) \to (x - 2, y - 4)$



7. Translate the image.
translation: $(x, y) \to (x - 5, y + 1)$

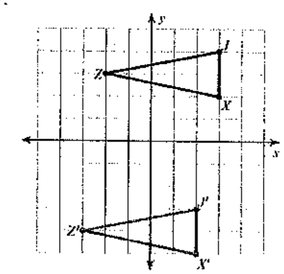


8. Translate the image.
translation: $(x, y) \to (x + 3, y - 4)$

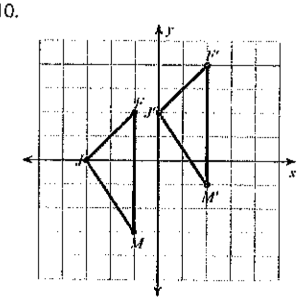


Write a rule for the given translation.

9.

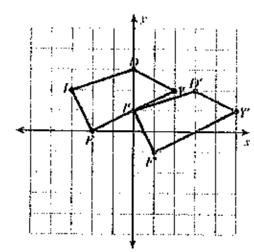


10.

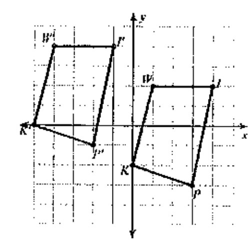


11.

a.



b.



Reflections Practice

12. Reflection across the x-axis.

R(−2,2) →
J(−1,4) →
G(3,4) →

13. Reflect across the y-axis.

H(1,−3) →
Z(1,2) →
W(4,1) →

14. Reflect across the line y = x.

E(−4,−2) →
N(−1,0) →
A(1,−3) →

15. Reflect across the line y = -x.

N(−4,2) →
L(−1,3) →
R(−1,2) →

16. Reflect across the y-axis.

R(1,−5) →
Y(0,−3) →
U(2,0) →
V(4,−2) →

17. Reflect across the line y = -x.

Z(−5,−2) →
P(−5,2) →
N(−3,3) →
A(−2,0) →

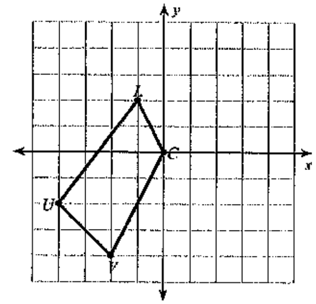
18. Reflect across the x-axis.

C(0,0) →
A(1,4) →
T(2,4) →
H(4,0) →

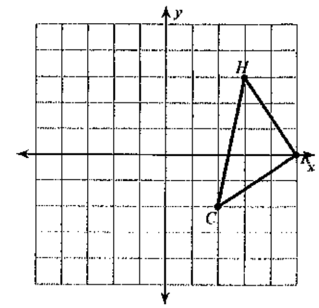
19. Reflect across the line y = x.

J(−3,1) →
L(−1,3) →
B(0,1) →
M(−2,−4) →

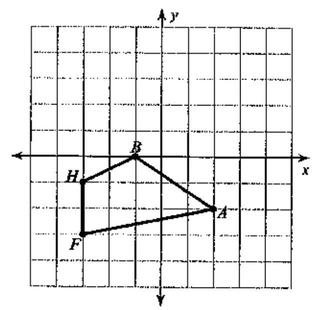
20. Reflect the image.
reflection across the y-axis



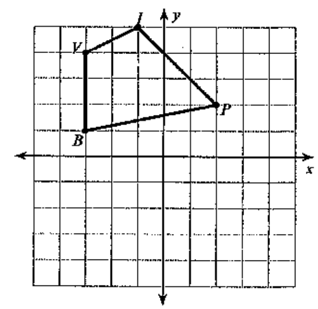
21. Reflect the image.
reflection across the x-axis



22. Reflect the image.
reflection across the y-axis



23. Reflect the image.
reflection across the x-axis



Write a rule to describe each transformation.

24. Z(0,−4) → Z'(0,4)
W(1,0) → W'(1,0)
S(3,0) → S'(3,0)

25. Q(−4,−3) → Q'(4,−3)
S(−5,1) → S'(5,1)
L(−2,−1) → L'(2,−1)

26. N(1,2) → N'(1,−2)
E(1,5) → E'(1,−5)
C(5,2) → C'(5,−2)

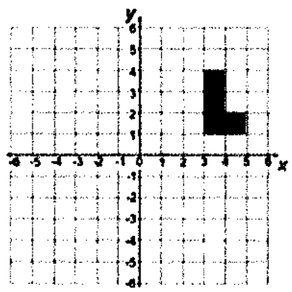
27. J(1,2) → J'(−1,2)
S(1,5) → S'(−1,5)
X(5,2) → X'(−5,2)

Name: ______________________________ Date: ______________

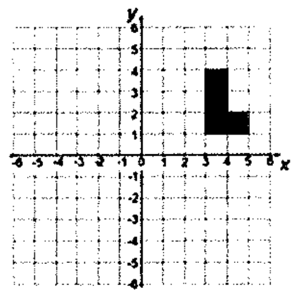
Rotations Practice

1. Where will the L-Shape be if it is...

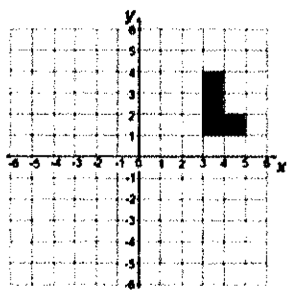
a. rotated 180° around the origin?



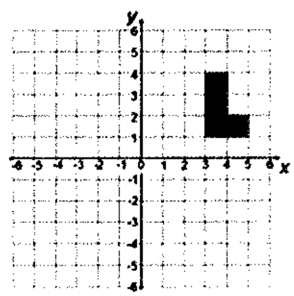
b. rotated 90° clockwise around the origin?



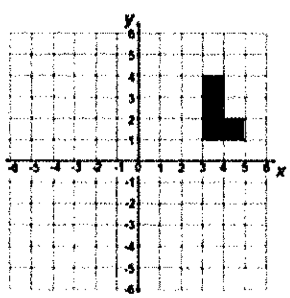
c. rotated 90° counterclockwise around the origin?



d. rotated 270° clockwise around the origin?

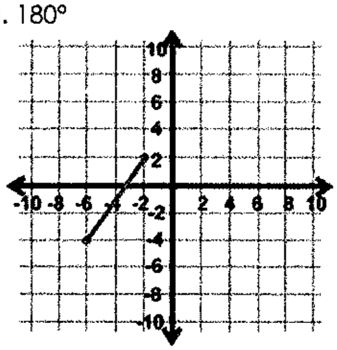


e. rotated 90° counterclockwise around the point (3, 0)?

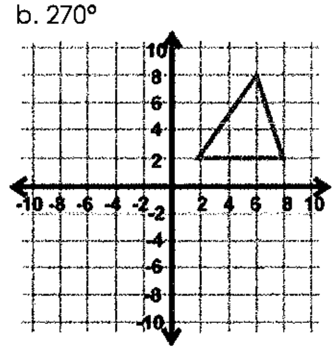


2. Rotate each figure about the origin using the given clockwise angle.

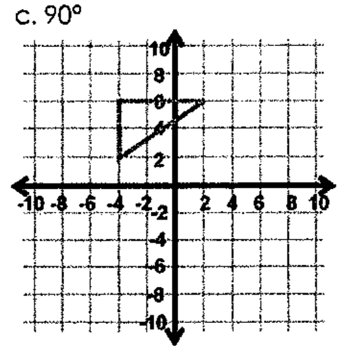
a. 180°



b. 270°



c. 90°



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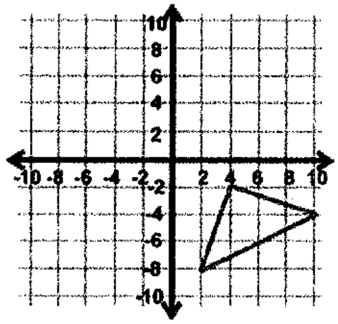
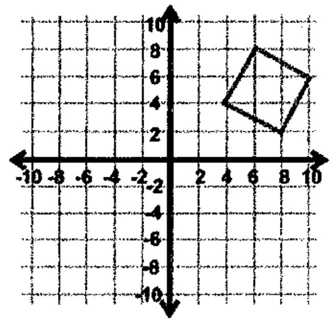
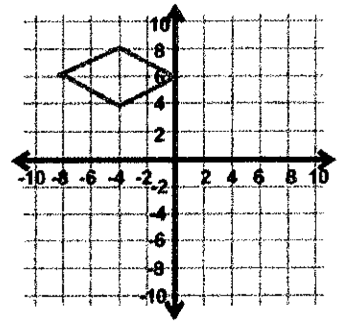
d. 270°



e. 180°

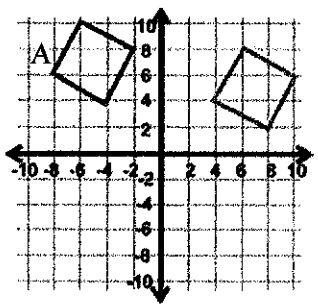


f. 90°

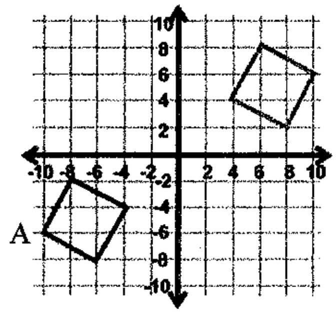


3. Find the angle of rotation for the graphs below. The center of rotation is the origin, and the image labeled A is the preimage. Your answer will be 90°, 180°, or 270°.

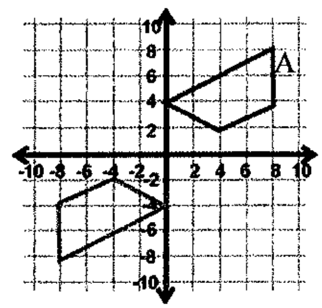
a.



b.



c.



Adapted from: Mathematics Vision Project

Composition of Transformations

All the transformations we have done so far can be called **isometries** or **rigid motions.**

a. An **isometry** is a ____________ where the pre-image and the __________ are **congruent**. When we perform the transformation, all the side lengths and angles stay the same length and measure. Its just the location and orientation of the figure that has changed. **Rigid Motion** is a ____________ for isometry.

Our three isometries are ____________, ____________, and ____________.

Compositions of Transformations: a combination of transformations that happens when we apply multiple transformations to the same figure.

Example 1:

Recall, what's the rule for reflect over x-axis?

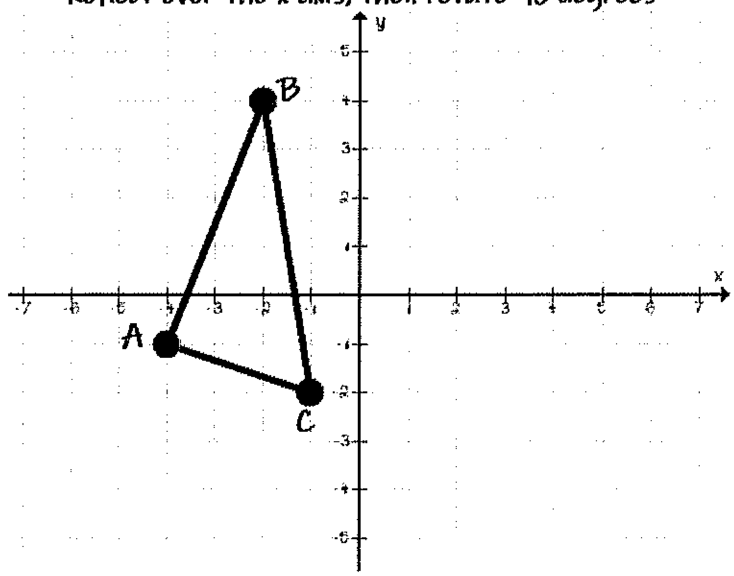
Recall? What's the rule for rotating 90 degrees?

A (,) → A' ______ → A'' _______

B (,) → B' ______ → B'' _______

C (,) → C' ______ → C'' _______

Reflect over the x-axis, then rotate 90 degrees



Identify the single reflection that could have produced this combination in one step.

Reflection over ________________.

Example 2:

A(,)→A'________ → A''_________

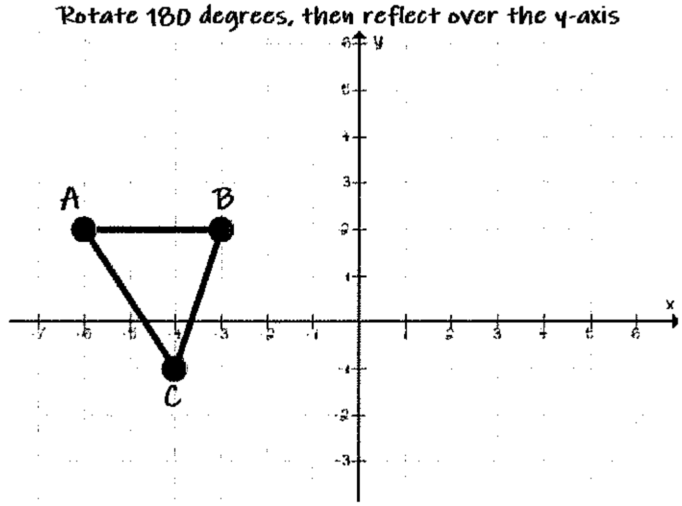
B(,)→B'________ → B''_________

C(,)→C'________ → C''_________

- What one transformation could have produced this combination in one step?

  ____

Rotate 180 degrees, then reflect over the y-axis



Another notation: For Compositions, there is a special type of notation that tells us how to work a problem.

Example 3:

a. $T_{x,y}$ denotes a ____________. The ______ value tells you to go right when it's __________ and left when it's _____________. The ______ value tells you to go ___ when it's positive, and ______ when it's negative.

b. R_θ denotes a ___________. There will be a 90, 270, or 180 instead of the θ. The default direction for a rotation is always ______________.

c. r_{line} denotes a __________. The line of reflection will be give where you see the word "line". We often reflect over the following lines: ______________, _____________, ___________, ______________, _________________, _________________.

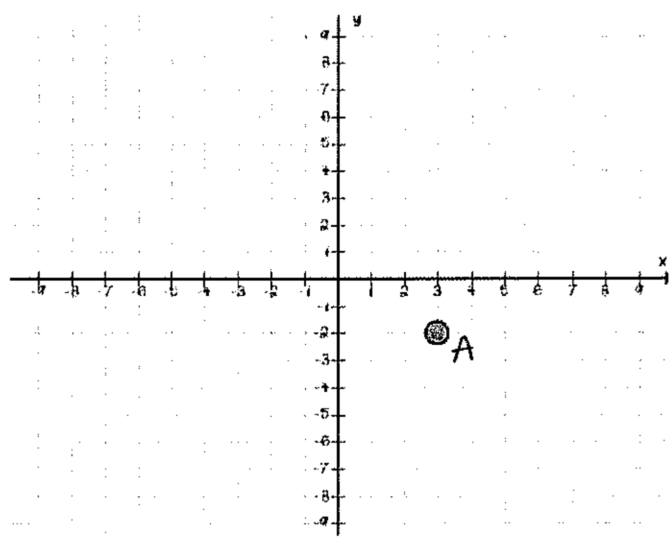
d. When working in composition notation we have to work from _________ to __________, which is the opposite of what we are used to!

Example 4:

What is the image of the point A(3, -2) under the transformation $R_{90°} \circ T_{-4,3}$?

- Step 1: Work from Right to left! So first we will ________________ the point, and then we will _____________ it.

A (3,-2) will be moved ______ to the left, and ______ up. To become A' _________.



- Step 2: Now we will ______ the point ________ degrees **counterclockwise,** using the rule (x,y)→________

A' __________ becomes A'' _____________.

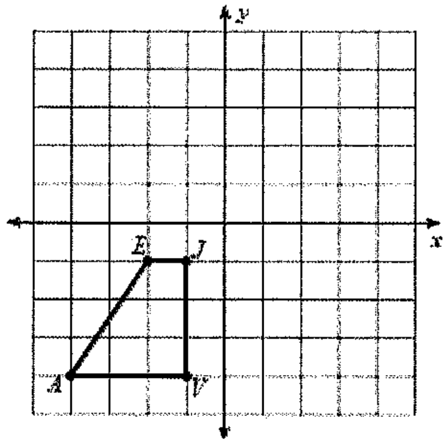
Remember we work **right to left** in this notation only!

Name: ______________________________________ Date: ____________________

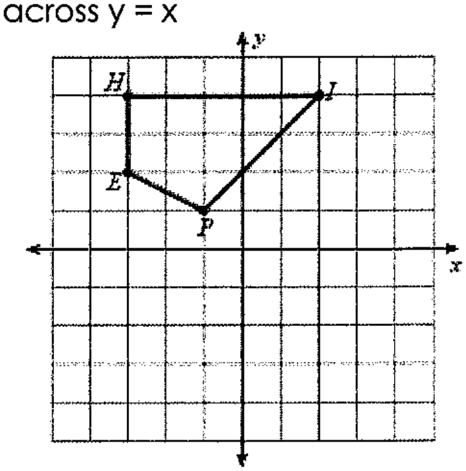
Composition of Transformations

Draw each of the figures after each of the composition is performed.

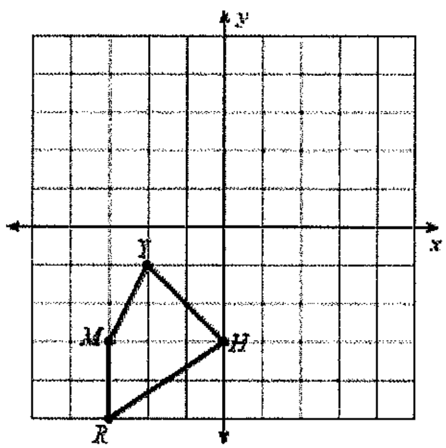
1) Translate by the rule $(x,y)\rightarrow(x+4,y+4)$, then rotate 90^0 clockwise about the origin



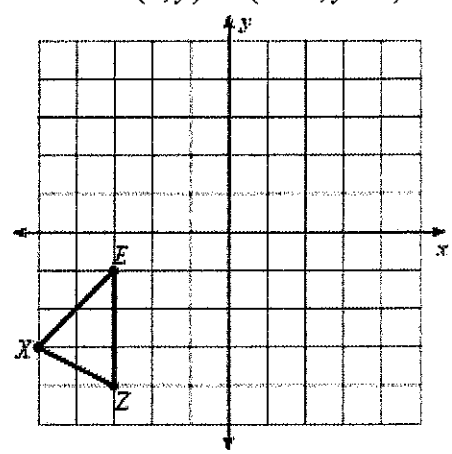
2) Rotate 180^0 about the origin, then reflect across y = x



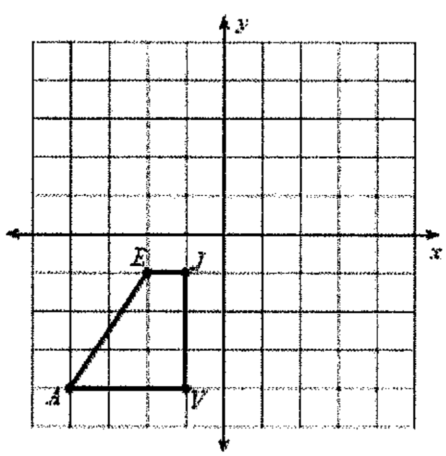
3) Reflect across the x-axis, then translate by the rule $(x,y)\rightarrow(x-1,y-3)$



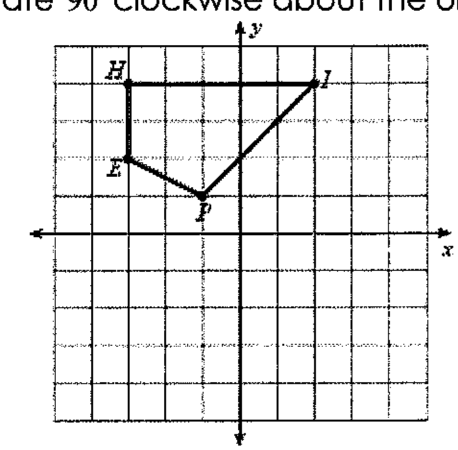
4) Translate by the rule $(x,y)\rightarrow(x+6,y+3)$, then by the rule $(x,y)\rightarrow(x-2,y-4)$



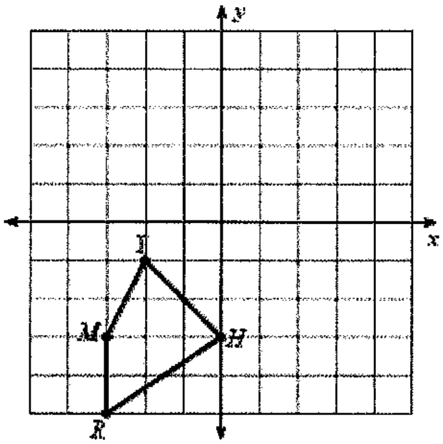
5) Reflect over x = -2, then reflect over the y-axis



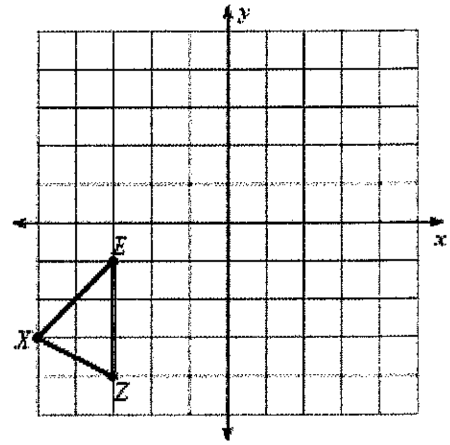
6) Translate by the rule $(x,y)\rightarrow(x-2,y-5)$, then rotate 90^0 clockwise about the origin



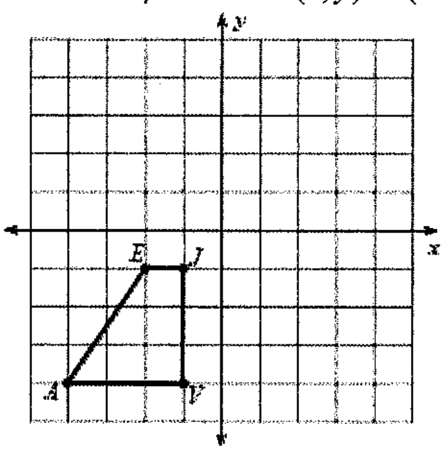
7) Translate by the rule $(x,y)\rightarrow(x+1,y+5)$, then reflect over the line y = x



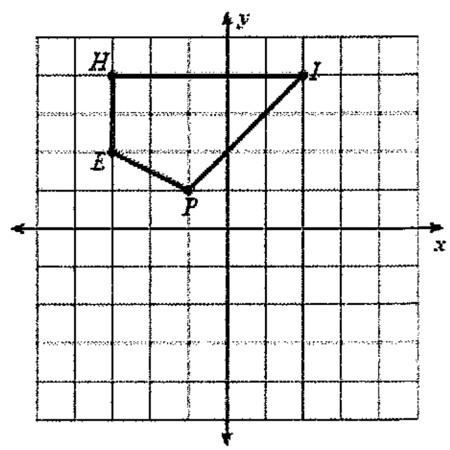
8) $r_{x-axis} \circ R_{90}$



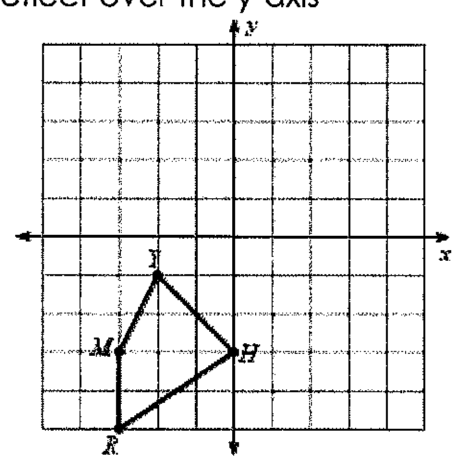
9) Rotate 90^0 clockwise about the origin, then translate by the rule $(x,y)\rightarrow(x+5,y)$



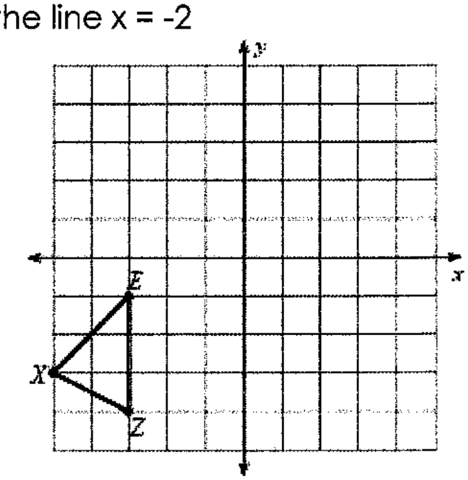
10) $R_{180} \circ r_{y=-x}$



11) Translate by the rule $(x,y)\rightarrow(x+4,y+1)$, then reflect over the y-axis



12) Reflect over the x-axis, then reflect over the line x = -2



Corresponding Parts of Congruent Triangles are Congruent

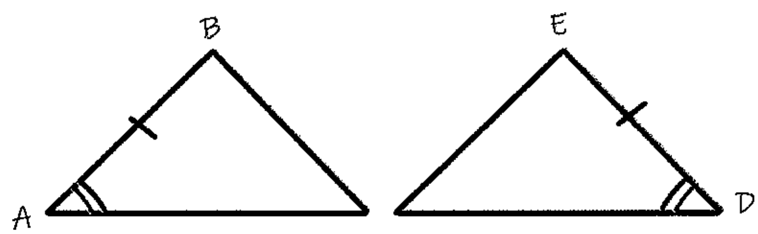
A **Congruence Statement** tells us how the parts of one triangle match up with another triangle. The order of the letters is super important.

Example: Without even having a picture, if we have the statement $\triangle ABC \cong \triangle DEF$ then we know…

$\angle A \cong \angle D$	$\overline{AB} \cong \overline{DE}$
$\angle B \cong \angle E$	$\overline{BC} \cong \overline{EF}$
$\angle C \cong \angle F$	$\overline{BC} \cong \overline{EF}$

Congruence Markings:

- In a diagram, when two angles are congruent, they will be marked with the same number of arches.
- When two side lengths are congruent, they will have the same number of tick marks.



- In the diagram to the right, the markings show that $\angle A \cong \angle D$, and $\overline{AB} \cong \overline{DE}$.

<u>You Try!</u> Based on the congruence statements or markings in the figure, determine the pairs of corresponding congruent angles, and corresponding congruent sides.

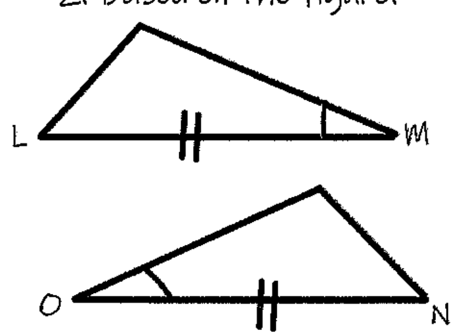
1. $\triangle CAT \cong \triangle DOG$

$\angle C \cong$ ______ $\overline{CA} \cong$ ______

$\angle A \cong$ ______ $\overline{AT} \cong$ ______

$\angle T \cong$ ______ $\overline{TC} \cong$ ______

2. Based on the figure:



$\angle M \cong$ ________

$\overline{ON} \cong$ ________

You Try!

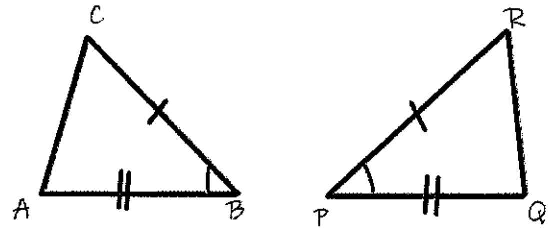
3. $\triangle IJK \cong \triangle LMN$

$\angle I \cong$ ______ $\overline{LM} \cong$ ______

$\angle J \cong$ ______ $\overline{MN} \cong$ ______

$\angle K \cong$ ______ $\overline{LN} \cong$ ______

2. Based on the figure:



$\angle B \cong$ ______

$\overline{AB} \cong$ ______

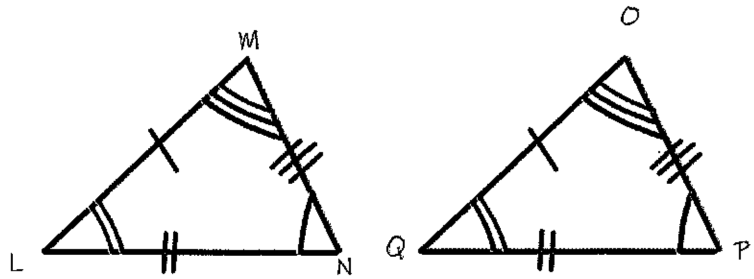
4. If $\triangle XYZ \cong \triangle TAC$...

$\angle X \cong$ ______ $\overline{TA} \cong$ ______

$\angle Y \cong$ ______ $\overline{AC} \cong$ ______

$\angle Z \cong$ ______ $\overline{TC} \cong$ ______

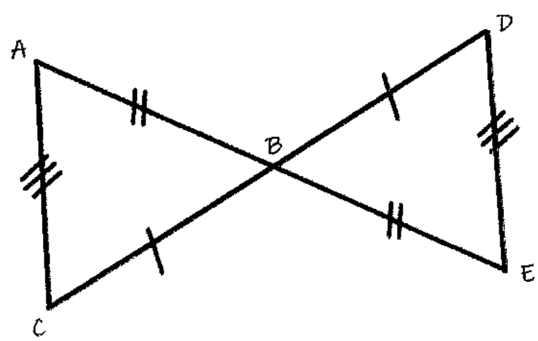
5. Based on the figures, write a congruence statement



$\triangle LMN \cong$ ______________

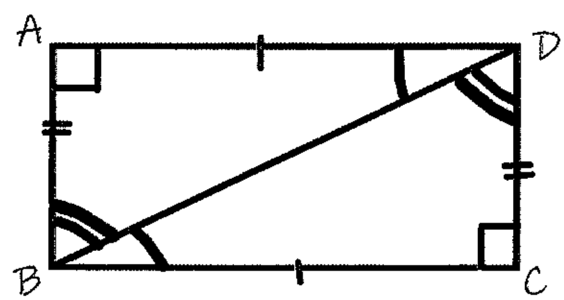
6. Based on the figures, write a congruence statement.

a.



$\triangle ABC \cong$ ______________

b.



$\triangle ABD \cong$ ______________

Proving Triangles Congruent

So far, we have answered questions about triangles that we have been told are congruent. **But what if we are not told whether or not they are the same?** There are a few ways that we can show that the triangles MUST be the same. These ways are called **theorems** or **postulates**. If we have enough information to show that one of the theorems or postulates is reprented, **we can PROVE that two triangles are congruent.**

Congruence Postulate #1

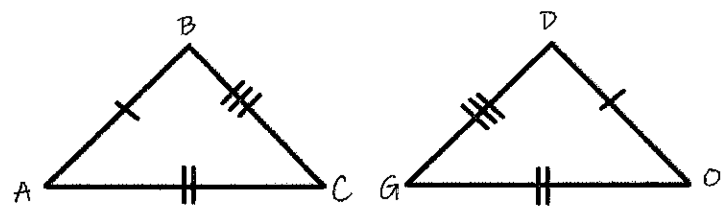
<u>Side Side Side (SSS)</u>: If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

<u>Example 1:</u>

<u>a.</u> From the diagram we see:

$\overline{AB} \cong \overline{OD}$ and

$\overline{BC} \cong \overline{DG}$ and $\overline{CA} \cong \overline{OG}$



Therefore... $\triangle ABC \cong \triangle ODG$ by SSS

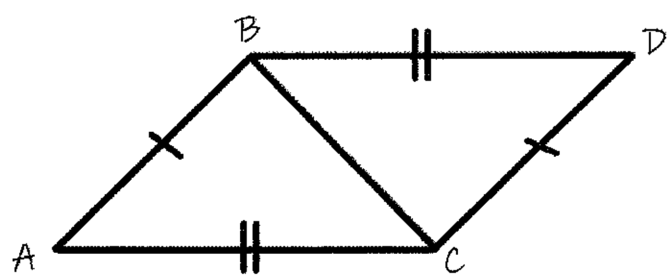
<u>b. Let's take a look:</u>

From the diagram, we know that

$\overline{AB} \cong$ ______ and $\overline{AC} \cong$ ______

But that's only 2 sides, and we need three.

What do you notice about the third side of each triangle? ______________________________

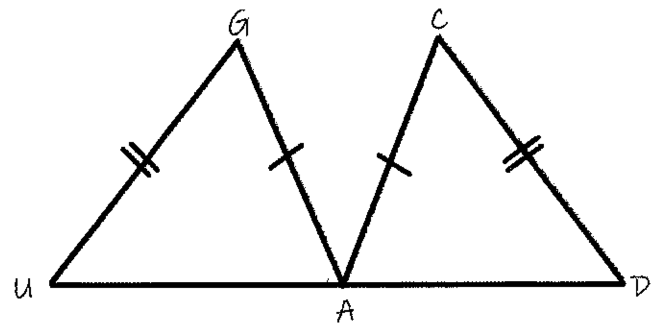


Anytime we add something to a diagram, we **must** have a property or justification!

When two triangles are sharing a side length we can use the ____________ property to show that it is congruent to itself! Therefore: ______________

<u>Now we can prove</u>: $\triangle ABC \cong$ _______ by the ____________ postulate.

c. Another property we'll see with side lengths...



Given: **A is the midpoint of $\overline{DU}$**, Can we prove that $\triangle UGA \cong \triangle DCA$?

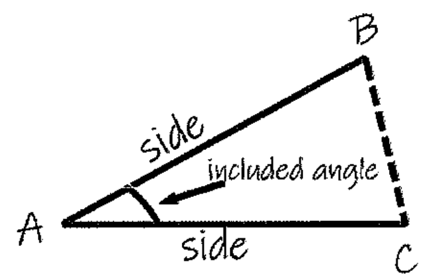
So... we know that: $\overline{UG} \cong$ ____ and $\overline{GA} \cong$ ____. That's only two sides, so we are going to need another side length. The **given** information in the problem says that **A** is the **midpoint** of $\overline{DU}$.

Define Midpoint: ____

Based on this definition, know we know __________ ≅ ____________.

Therefore $\triangle UGA \cong \triangle DCA$ by ____________.

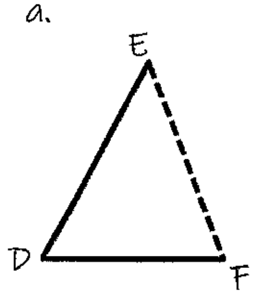
Our next postulate with involve using some angles, so we need to understand some vocabulary first. When two sides of a triangle meet, they form an angle. The angle where two sides meet is called their "**included**" angle.



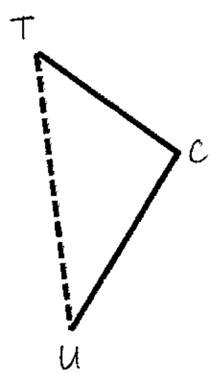
- In the diagram to the right, angle A is the included angle of sides _______ and ________.

You try: Identify the included angle of the solid sides of each triangle.

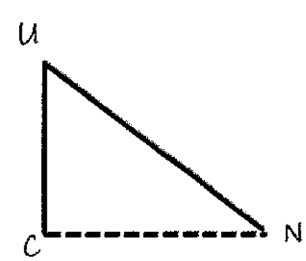
2. a.



b.

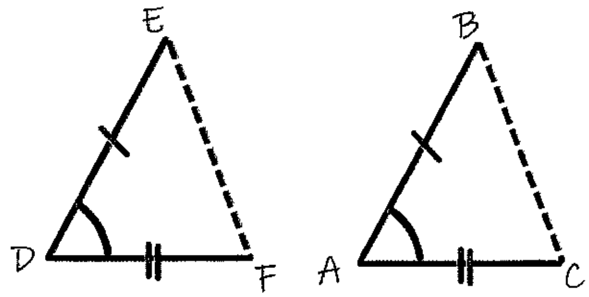


c.



____________________ ____________________ ____________________

Let's Take a closer look:



- From the diagram we can see that $\overline{DE} \cong \overline{AB}$ and $\overline{DF} \cong \overline{AC}$
- We can also see that the **Included angle** between the sides is marked as congruent.
- Anytime you have **two fixed distances** (your solid sides) bound by the **same angle** (the included angle) the distances it takes to connect those endpoints (E to F) or (B to C) will always be the same!

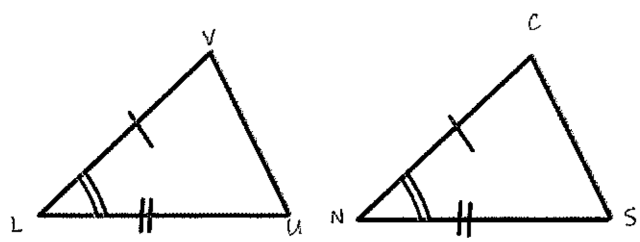
This means anytime you see two **congruent sides** with **congruent included angles** you know that the two triangles MUST be congruent.

When we prove triangles congruent this way, we are using our second postulate:

Congruence Postulate #2

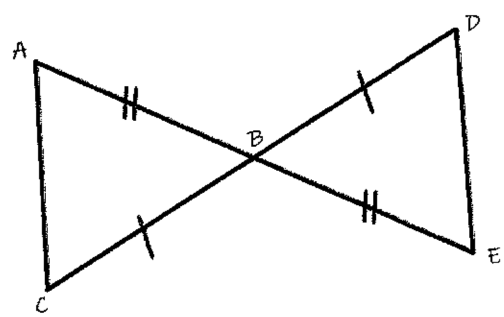
Side Angle Side (SAS): If two sides and the included angle of one triangle are congruent two sides and the included angle of another triangle, the two triangles will be congruent.

3. **a.** From the diagram we see: $\overline{LV} \cong \overline{NC}$ and $\overline{LU} \cong \overline{NS}$ and the included angles: $\angle L \cong \angle N$



Therefore... $\triangle LVU \cong \triangle NCS$ by SAS

b. Lets Take a Look:



From the diagram, we know that

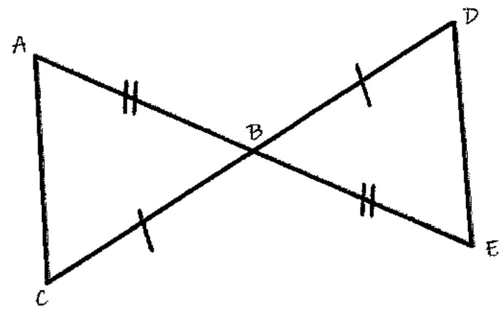
$\overline{AB} \cong$ ______ and $\overline{BC} \cong$ ______

But that's only 2 sides. We either need another side for SSS or the included angle for SAS.

Remember, we MUST have a property or justification to add anything to our diagram.... Do you notice anything about the included angles that we know from previous lessons? ______________________________

3b Continued:

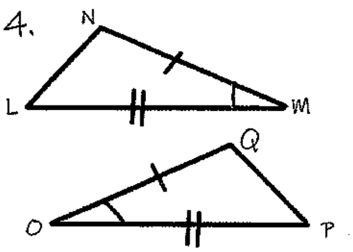
Any time you see __________ __________ you can add a marking for them into your diagram!



Now we know that ∠__________ ≅ ∠____________.
Since these are the included angles, we can now say that $\triangle ABC \cong$ ______ by the ________ Postulate.

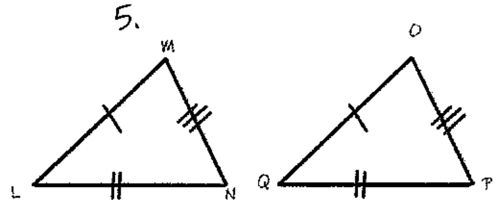
You Try: Decide which congruence postulate can be used for each pair of triangles below. If they are congruent, write a congruence statement. If neither postulate can be used, put an "X" in each blank.

4.



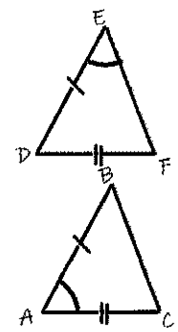
$\triangle LMN \cong$ _______ by ______

5.



$\triangle LMN \cong$ _______ by ______

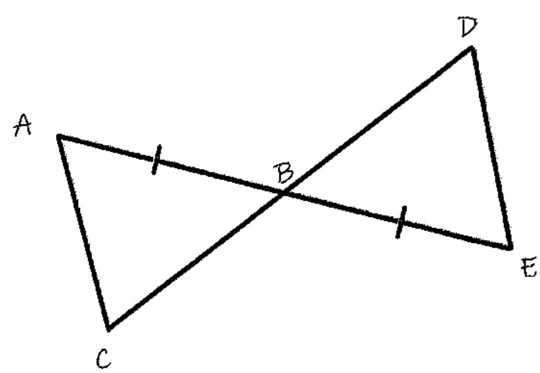
6.



$\triangle DEF \cong$ _______ by ______

Challenge Problem! Putting it all together ☺

7. Given: **B** is the **Midpoint** of $\overline{CD}$.



From the diagram we know.... $\overline{AB} \cong$ _____

- What can we mark because of the midpoint?

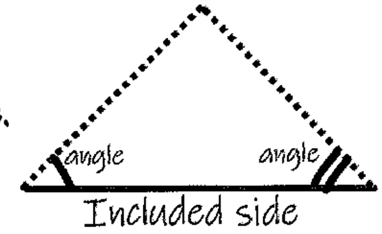
  _________ ≅ _________

- We can mark _________ ≅ _________ because they are ________ __________.

Therefore: $\triangle ABC \cong \triangle$_______ by _______

Vocabulary to help us with our next postulate:

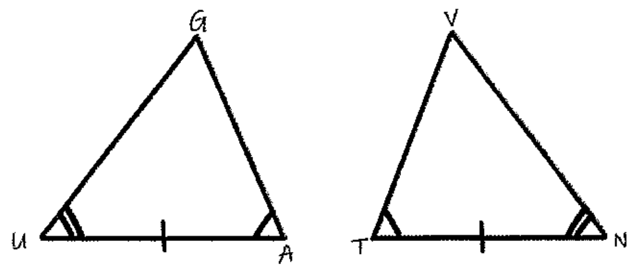
The side length that is between two angles is called the **included side.**



Postulate #3

Angle Side Angle (ASA)- If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

8. Example:

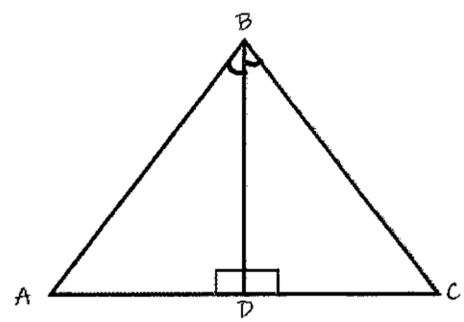


From the diagram we see: $\angle U \cong \angle N$ and

$\angle A \cong \angle T$ and the included sides $\overline{UA} \cong \overline{NT}$

Therefore... $\triangle UGA \cong \triangle NVT$ by ASA

9. . **Lets Take a Look:**



From the diagram, we know that

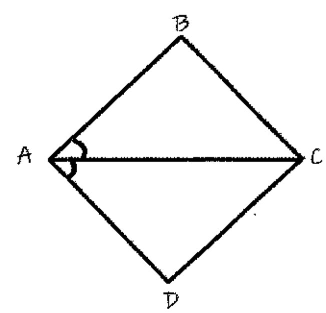
$\angle ABC \cong \angle$ ______ and $\angle ADB \cong$ ______

But that's only two angles. We the included sides to be congruent for ASA.

Remember, we **MUST** have a property or justification to add anything to our diagram...Do you see anything we are allowed to mark?

Since ________ $\cong$ _________ by the ________ ________,

Therefore $\triangle ABD \cong \triangle$ ________ by ________.



10. Another Property we might see dealing with angles:

Given: $\overline{AC}$ is an **angle bisector** for $\angle BCD$.

Can you prove the two triangles are congruent? We know....

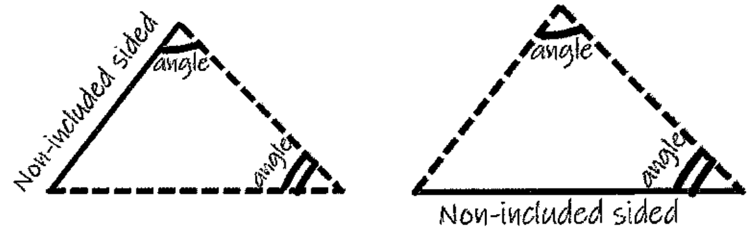
$\angle BAC \cong \angle$ ____ from the diagram, and ________ $\cong$ _________ by the reflexive Property.

Define Angle Bisector: ____

Therefore, $\angle BCA \cong \angle$ ____ and that means that $\triangle ABC \cong \triangle$ _______ by ________

Vocabulary for our next postulate:

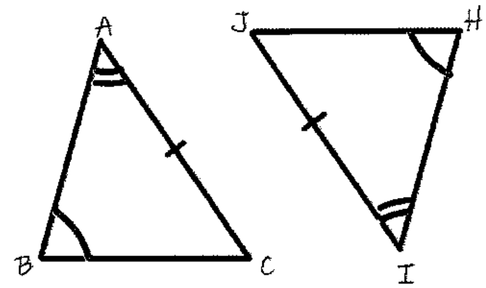
A side length that is not directly in between to angles is called a **non-included** side.



Theorem #4

Angle Angle Side(AAS): If two angles and a non-included side of one triangle are congruent the two angles and the corresponding non-included side of another triangle, the two triangles are congruent.

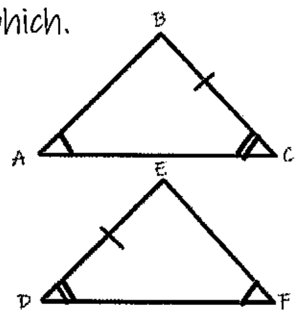
Example:



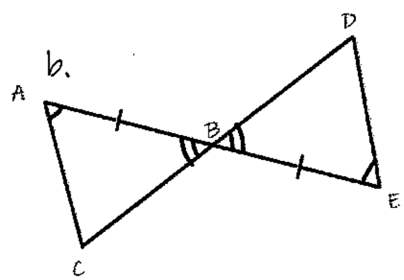
11. From the diagram we see: $\angle A \cong \angle I$ and $\angle B \cong \angle H$ and the corresponding **non-included** sides $\overline{AC} \cong \overline{IJ}$ Therefore... $\triangle ABC \cong \triangle IJH$ by AAS

12. Two of the examples below are examples of AAS, one is an example of ASA. Decide which is which.

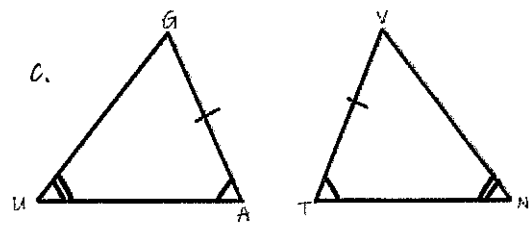
a.



b.

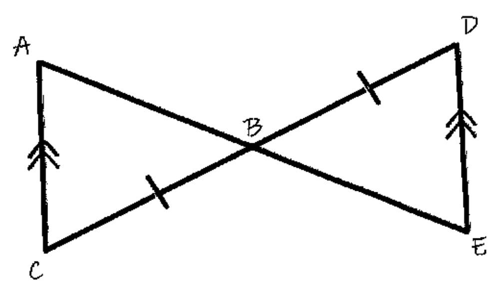


c.



$\triangle ABC \cong$ ______ by ______ $\triangle ABC \cong$ ______ by ______ $\triangle UGA \cong$ ______ by ______

13. Another property we may see with angles...



Given: $\overline{AC} \parallel \overline{DE}$, prove the two triangles congruent.

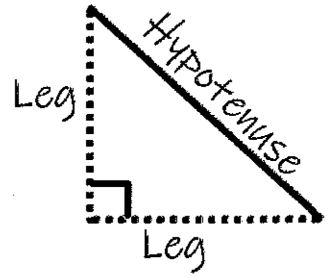
We know.... $\overline{CB} \cong \overline{DB}$ from the diagram, and ________ $\cong$ __________ because they are vertical angles. Since $\overline{AC} \parallel \overline{DE}$, what kind of angles are $\angle A$ and $\angle E$? ________________________

Therefore, $\angle A \cong \angle$ ___ and that means that $\triangle ABC \cong \triangle$ ______ by ________.

$\angle C$ and $\angle D$ are also ________ _________, so there is more than one correct way to do this one. ☺

Recall: Right Triangles

In a Right triangle, the side lengths that form the right angle are called the _______ of the triangle, and the side opposite the right angle is called the __________.

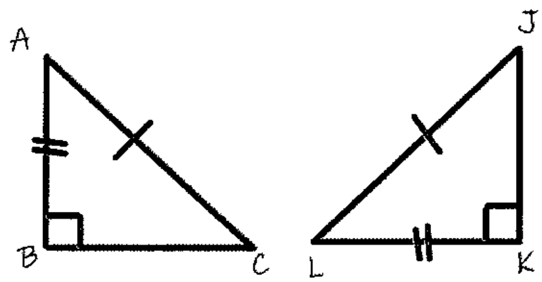


- Right triangles have many special properties! We have a triangle congruence theorem that works ONLY for right triangles!
- All our other postulates and theorems work for right triangles too! Right triangles just have an extra on that is special just for them.

Hypotenuse Leg(HL): If the hypotenuse and one leg in a right triangle are congruent to the hypotenuse and one leg of another right triangle, the two triangles are congruent.

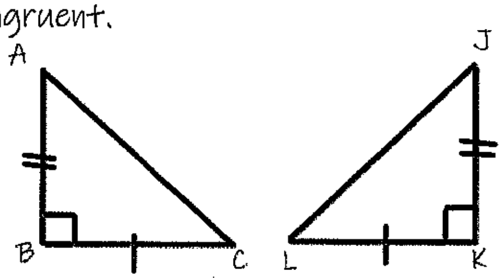
Example:

14. From the diagram we see: $\angle B$ and $\angle K$ are both right angles, making these right triangles. $\overline{AB} \cong \overline{LK}$. These are the ______ of the right triangles. $\overline{AC} \cong \overline{LK}$ These segments are the __________ of the right triangles. Therefore... $\overline{\triangle ABC \cong \triangle LKJ}$ by HL

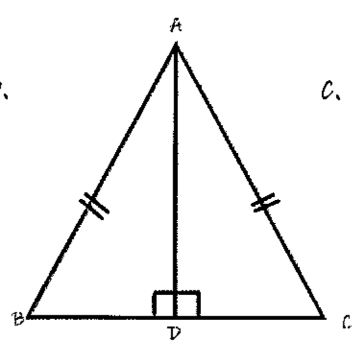


15. **You Try!** Determine which postulate or theorem you can use to prove the triangles congruent.

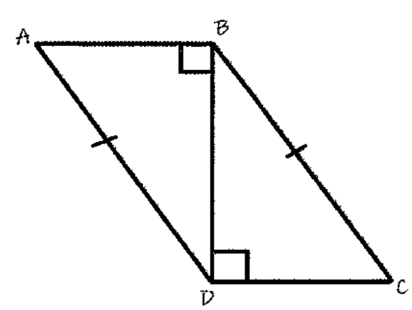
a.



b.



c.

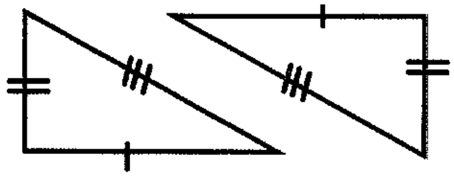


d. **True or False**: HL is the only method to prove that two right triangles are congruent.

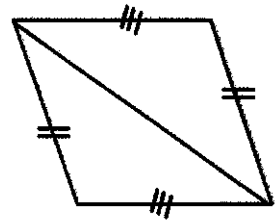
Triangle Congruence Practice - SSS and SAS

1. Which of the following examples does **NOT** show **SSS** congruence?

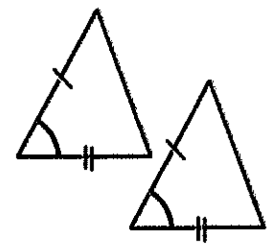
a.



b.

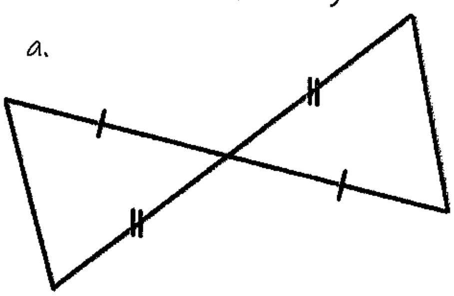


c.

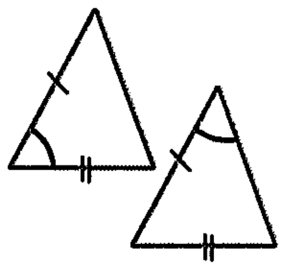


2. Which of the following does **NOT** show SAS congruence?

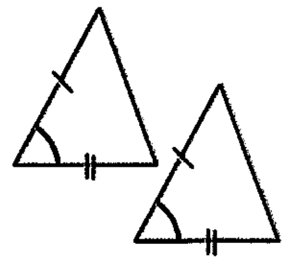
a.



b.

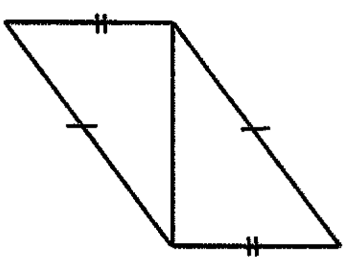


c.



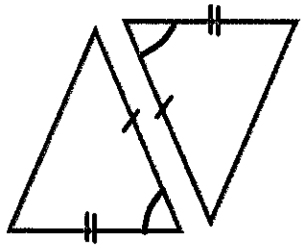
3. Determine if you can use SSS or SAS to prove the pairs of triangles below congruent. If it does not fit one of those postulates, write "neither."

a.



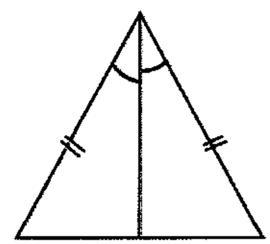
____

b.



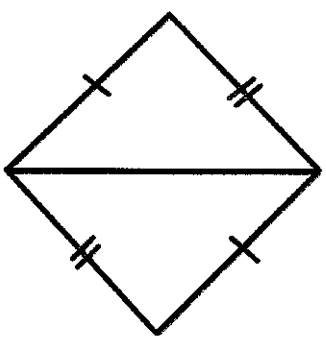
____

c.



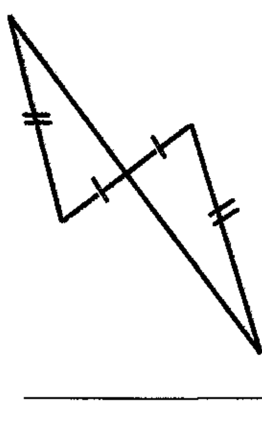
____

d.



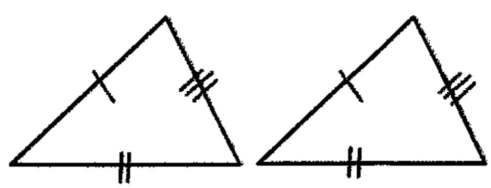
____

e.



____

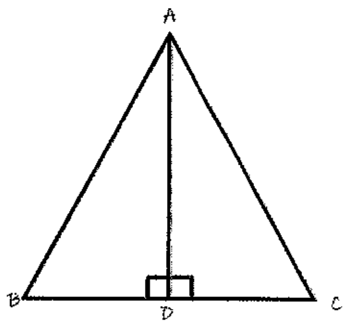
f.



____

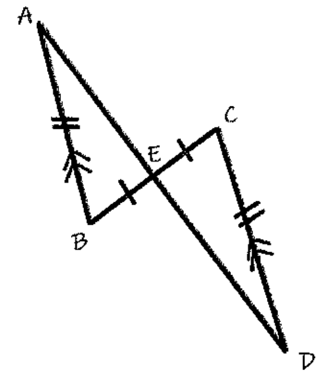
4. Given the information, determine which postulate you can use to prove the triangles congruent.

a. Given: D is the midpoint of BC.



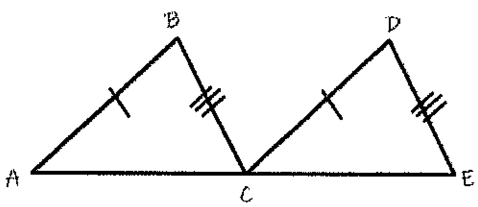
△BAD ≅△_______ by ____________

b. Given: AB || CD.



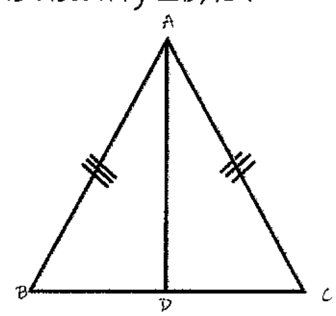
△ABE ≅△_______ by ____________

c. Given: C is the midpoint of AE.



△ABC ≅△_______ by ____________

D. Given $\overline{AD}$ is bisecting ∠BAC.



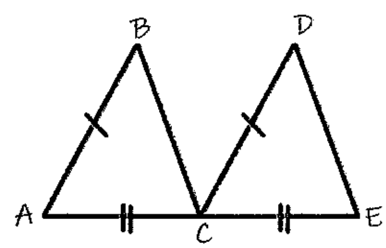
△BAD ≅△_______ by ____________

Challenge Section, TEST PREP:

5. What **additional** information is needed to prove....

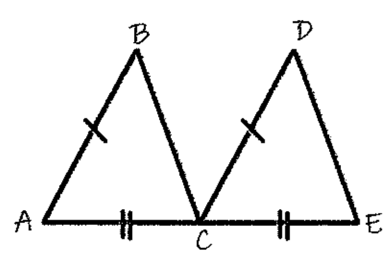
a. △ABC ≅△CDE SSS?

If _______ is congruent to _______ then that would meet the criteria for SSS.



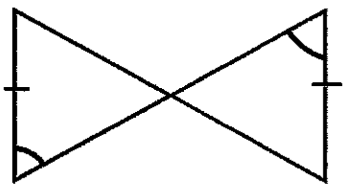
b. △ABC ≅△CDE SAS?

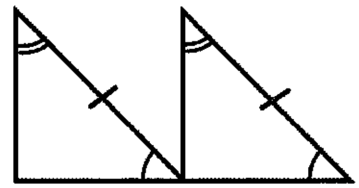
If _______ is congruent to _______ then that would meet the criteria for SAS.

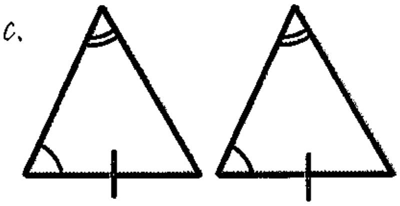


ASA, AAS, and HL Congruence Practice

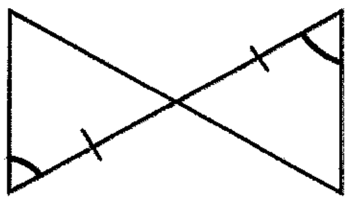
1. Determine which of the following is NOT an example of AAS congruence, then state which type of congruence it is.

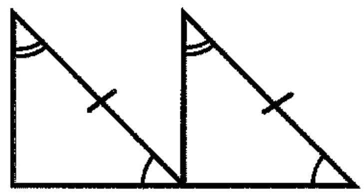
a.


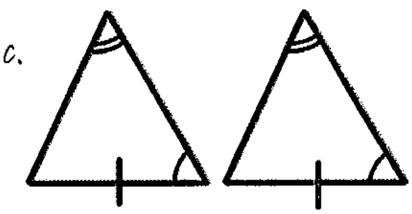
b.


c.


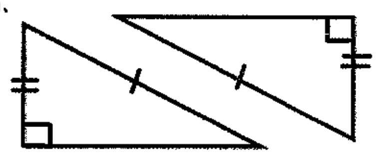
2. Determine which of the following is NOT an example of ASA Congruence, then state which type it is.

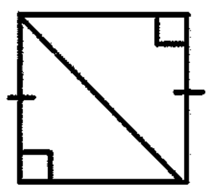
a.


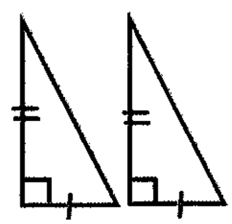
b.


c.


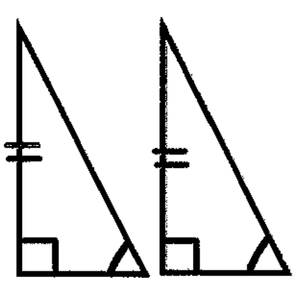
3. Determine which of the following is NOT an example of HL, then state which type it is.

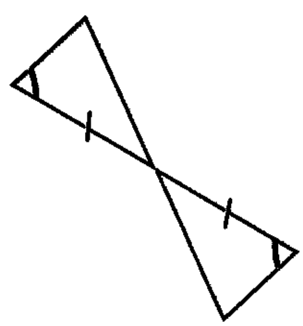
a.


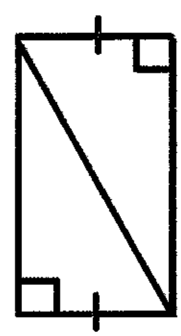
b.


c.


4. Match each triangle to the correct postulate/theorem. There will be one of each of the following: ASA, AAS, HL.

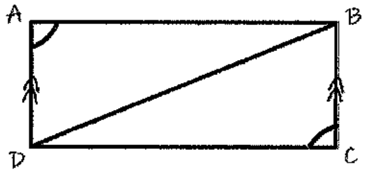
a.


b.


c.


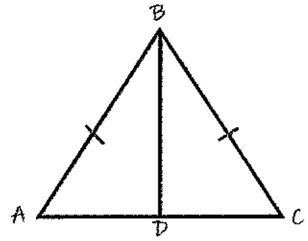
5. Given the information, determine which postulate you can use to prove the triangles congruent.

a. $\overline{AD} \parallel \overline{BC}$



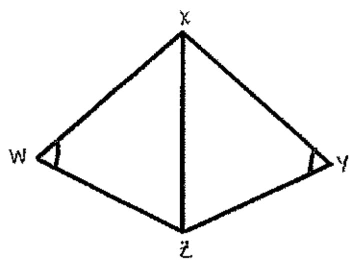
$\triangle ABD \cong \triangle$_______ by ___________

b. $\overline{BD} \perp \overline{AC}$; D is the midpoint of $\overline{AC}$



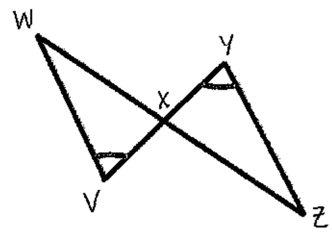
$\triangle ADB \cong \triangle$_______ by ___________

c. $\overline{XZ}$ is bisecting $\angle WXY$



$\triangle WXZ \cong \triangle$_______ by ___________

d. X is the midpoint of $\overline{YV}$



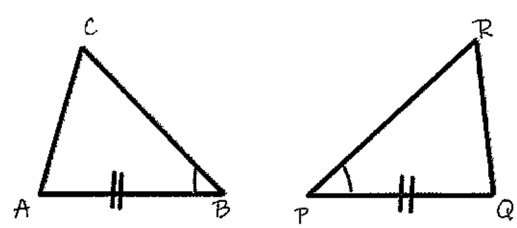
$\triangle WXV \cong \triangle$_______ by ___________

Challenge Section, TEST PREP:

5. What **additional** information is needed to prove....

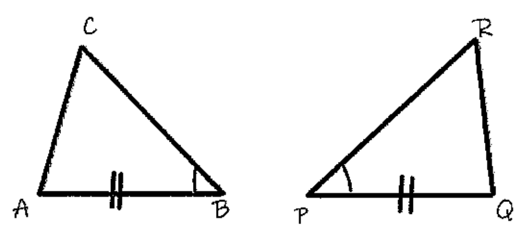
a. $\triangle ABC \cong \triangle QPR$ by **ASA**?

If _______ is congruent to _______ then that would meet the criteria for ASA.



b. $\triangle ABC \cong \triangle QPR$ **AAS**?

If _______ is congruent to _______ then that would meet the criteria for AAS.



Two Column Proof

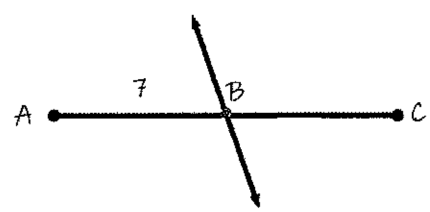
- A Two Column Proof is just a way to organize an argument. On the left side of the table, we put true ____________, and on the right side, we put the __________/__________ for that statement.
- Each line of the proof is one of the __________ we take towards proving our argument.
- The __________ that we use might be given in the problem, a definition, postulate, or theorem.

	Reasons/Justification
If something is marked in the diagram, or in the given information.....	
If you see that the triangles are sharing a side.....	
If you see parallel lines... • •	
If you see vertical angles....	
When you write a congruence statement for two triangles...	
If the proof involves triangles, but is asking you to prove a pair of side or angles for you final answer.....	

Let's review some definitions, and how we can use them in two column proof.

Example 1: Prove that the length of $\overline{BC} = 7$

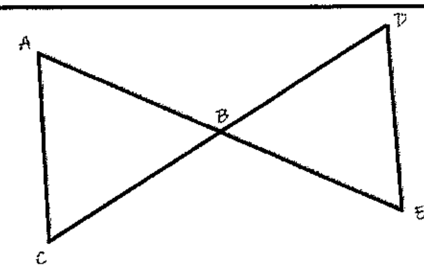
Given: $\overline{AB} = 7$, and B is the midpoint of $\overline{AC}$.



- The **Midpoint** of a segment ____________ the segment into two __________ pieces.

Statements	Reasons/Justification
1. $\overline{AB} = 7$	
2. B is the midpoint of $\overline{AC}$.	
3. $\overline{AB} \cong \overline{BC}$	
4. $\overline{BC} = 7$	**Transitive Property of Equality**

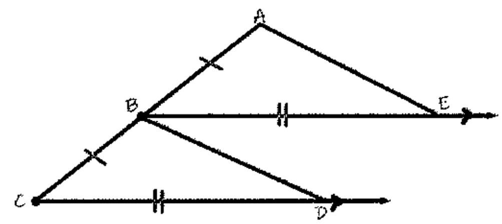
Now, lets see how it works with **Triangles.**



Example 2: **Given:** B is the midpoint of $\overline{AE}$, $\angle A \cong \angle E$

PROVE: $\triangle ABC \cong \triangle EBD$

Statements	Reasons/Justifications
1. B is the Midpoint of $\overline{AE}$	
2.	Definition of Midpoint
3. $\angle A \cong \angle E$	
4.	Vertical Angles are congruent
5. $\triangle ABC \cong \triangle EBD$	



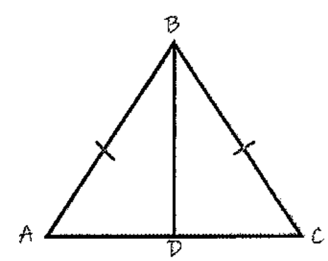
Example 3: Let's look at how parallel lines can help us with a proof.

Given: $\overline{CB} \cong \overline{BA}$, $\overline{CD} \cong \overline{BE}$, $\overline{CD} \parallel \overline{BE}$

PROVE: $\triangle ABE \cong \triangle BCD$

Statements	Reasons/Justifications
1. $\overline{CB} \cong \overline{BA}$	
2. $\overline{CD} \cong \overline{BE}$	
3. $\overline{CD} \parallel \overline{BE}$	
4.	Corresponding Angles are Congruent
5. $\triangle ABE \cong \triangle BCD$	

Example 4: Reminder...Which Property do we use when triangles share a side? ______________________

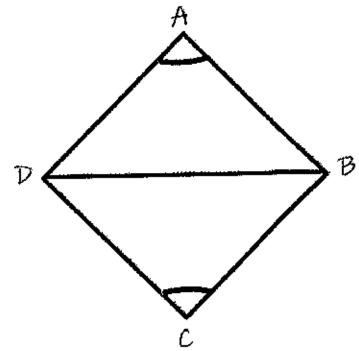


Given: $\overline{AB} \cong \overline{CB}$, D is the midpoint of $\overline{AC}$.

PROVE: $\triangle ADB \cong \triangle CDB$

Statements	Reasons/Justifications
1. $\overline{AB} \cong \overline{CB}$	
2. D is the midpoint of $\overline{AC}$	
3.	Definition of Midpoint
4. $\overline{BD} \cong \overline{BD}$	
5. $\triangle ADB \cong \triangle CDB$	

Example 6: Recall, an **angle bisector** divides one angle into _______ congruent ______.

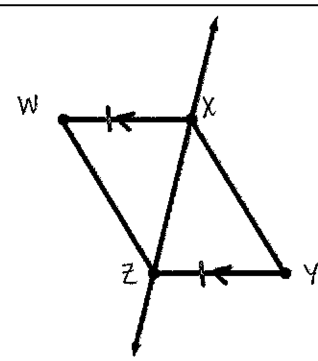


Given: $\angle A \cong \angle C$, and $\overline{DB}$ is bisecting $\angle ABC$.

PROVE: $\triangle DAB \cong \triangle DCB$

Statements	Reasons/Justifications
1.	Given
2.	Given
3. $\angle ABD \cong \angle CBD$	Definition of _________
4. $\overline{BD} \cong \overline{BD}$	
5. $\triangle DAB \cong \triangle DCB$	

7. **You Try!**



Given: $\overline{WX} \cong \overline{YZ}$, and $\overline{WX} \parallel \overline{YZ}$

PROVE: $\triangle WXZ \cong \triangle YZX$

Statements	Reasons/Justifications
1.	Given
2. $\overline{WX} \parallel \overline{YZ}$	
3.	Alternate Interior Angles are Congruent
4. $\overline{XZ} \cong \overline{XZ}$	
5. $\triangle WXZ \cong \triangle YZX$	

Lets talk about **right triangles**!

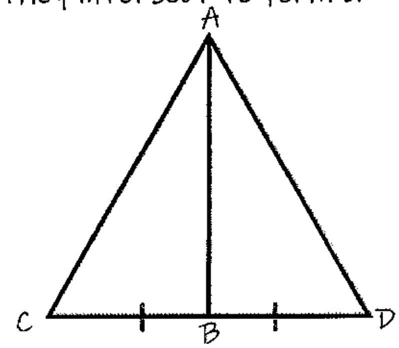
QUICK QUIZ....

8. **True or False:** Hypotenuse leg is the only theorem/postulate that can be used to show that two right triangles are congruent. ______________

9. Recall, if two lines are __________ to each other then they intersect to form a right angle.

Let's look at a proof that uses this property!

Given: $\overline{AB} \perp \overline{CD}$, $\overline{CB} \cong \overline{DB}$



PROVE: $\triangle ABC \cong \triangle ABD$

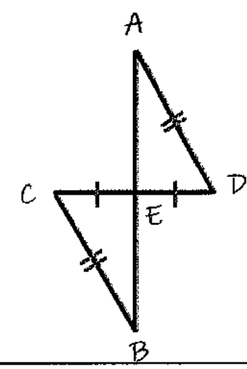
Statements	Reasons/Justifications
1. $\overline{CB} \cong \overline{DB}$	
2.	Given
3. $\angle ABC$ and $\angle ABD$ are $90°$	Definition of __________.
4.	ALL Right Angles are Congruent
5.	
6. $\triangle ABC \cong \triangle ABD$	

** When using SSS, SAS, ASA, or AAS for right triangles, we must state that our $90°$ angles are congruent**

HL Proofs are a little bit different! Lets take a look.

Quick Question...What is the only kind of triangle that we can use **HL** on? __________

So as we are writing our proof, we will need a statement and a justification that what we are working with is actually a right triangle.



Example 10:

Given: $\overline{AB} \perp \overline{CD}$, $\overline{CB} \cong \overline{DA}$, $\overline{CE} \cong \overline{DE}$

PROVE: $\triangle ECB \cong \triangle EDA$

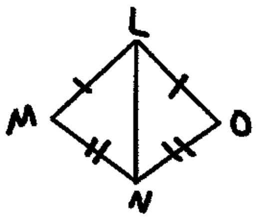
Statements	Reasons/Justifications
1. $\overline{CB} \cong \overline{DA}$	
2.	Given
3. $\overline{AB} \perp \overline{CD}$	
4. _____ and _____ are $90°$	Definition of Perpendicular
5. $\triangle ECB$ and $\triangle EDA$ are right triangles.	
6. $\triangle ECB \cong \triangle EDA$	

Name:

Triangle Proofs Practice

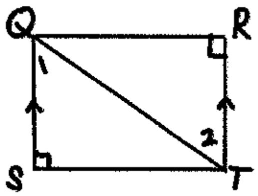
Matching: Use the choices listed at the bottom in the box for problems #1 – 4

Problem 1:



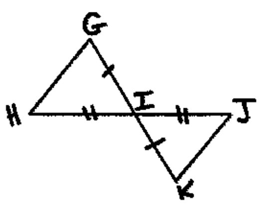
Statement	Reason
1. $\overline{LM} \cong \overline{LO}$	1.
2. $\overline{MN} \cong \overline{ON}$	2.
3. $\overline{LN} \cong \overline{LN}$	3.
4. $\Delta LMN \cong \Delta LON$	4.

Problem 2:



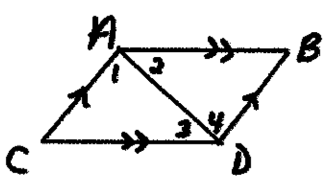
Statement	Reason
1. $\overline{QS} \parallel \overline{RT}$	1.
2. $\angle R \cong \angle S$	2.
3. $\angle 1 \cong \angle 2$	3.
4. $\overline{QT} \cong \overline{QT}$	4.
5. $\Delta QST \cong \Delta TRQ$	5.

Problem 3:



Statement	Reason
1. $\overline{GI} \cong \overline{KI}$	1.
2. $\overline{HI} \cong \overline{JI}$	2.
3. $\angle GIH \cong \angle KIJ$	3.
4. $\Delta GIH \cong \Delta KIJ$	4.

Problem 4:



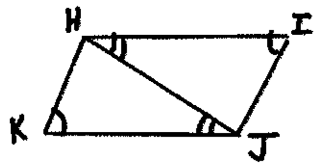
Statement	Reason
1. $\overline{AC} \parallel \overline{BD}, \overline{AB} \parallel \overline{CD}$	1.
2. $\angle 1 \cong \angle 4, \angle 2 \cong \angle 3$	2.
3. $\overline{AD} \cong \overline{AD}$	3.
4. $\Delta ADC \cong \Delta DAB$	4.

Choices for problems #1 – 4 (some will be used more than once):

AAS
ASA
Alternate Interior Angles are ≅
Given
Reflexive Property
SAS
SSS
Vertical Angles are ≅

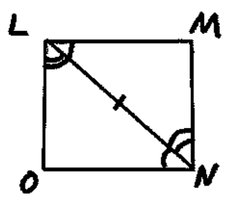
Fill in the blank proofs:

Problem 5:



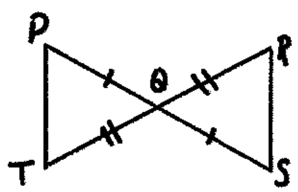
Statement	Reason
1. $\angle I \cong \angle K$	1.
2. $\angle IHJ \cong \angle KJH$	2.
3. $\overline{HJ} \cong \overline{HJ}$	3.
4. $\Delta HJK \cong \Delta JHI$	4.

Problem 6:



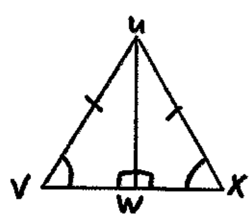
Statement	Reason
1. $\angle MLN \cong \angle ONL$	1.
2. $\angle OLN \cong \angle$ _____	2. Given
3.	3. Reflexive Property
4. $\Delta LNO \cong \Delta NLM$	4.

Problem 7:



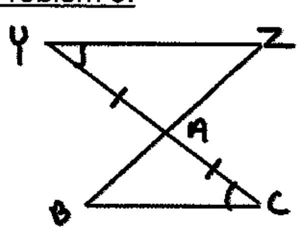
Statement	Reason
1. $\overline{PQ} \cong \overline{QS}$	1.
2.	2. Given
3. $\angle PQT \cong \angle RQS$	3.
4. $\Delta PQT \cong \Delta SQR$	4.

Problem 8:



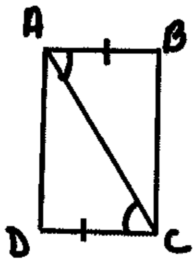
Statement	Reason
1. $\overline{UV} \cong \overline{UX}$	1.
2.	2. Right Angle Congruence
3.	3. Reflexive Property
4. $\angle V \cong \angle X$	4.
5. $\Delta UWV \cong \Delta UWX$	5.

Problem 9:



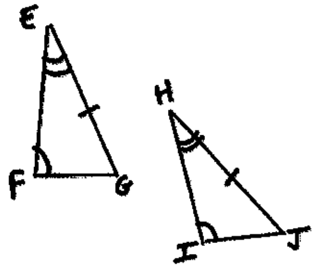
Statement	Reason
1. $\angle Y \cong \angle C$	1.
2.	2. Given
3.	3. Vertical Angles
4. $\Delta YZA \cong \Delta CBA$	4.

Problem 10:



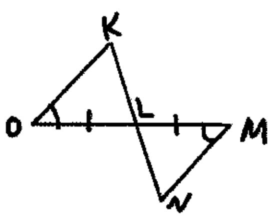
Statement	Reason
1. $\angle BAC \cong \angle DCA$	1. Given
2.	2. Given
3.	3.
4. $\Delta ABC \cong \Delta CDA$	4.

Problem 11:



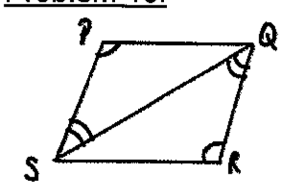
Statement	Reason
1. $\angle F \cong \angle I$	1.
2. $\angle$___ $\cong \angle$___	2.
3.	3.
4. $\Delta EFG \cong \Delta HIJ$	4.

Problem 12:



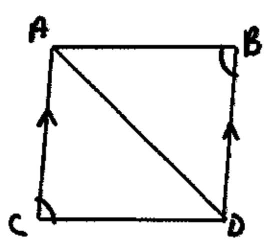
Statement	Reason
1. $\angle$___ $\cong \angle M$	1. Given
2.	2. Given
3. $\angle KLO \cong \angle$____	3.
4. $\Delta KLO \cong \Delta NLM$	4.
5. $\angle K \cong \angle N$	5. CPCTC

Problem 13:



Statement	Reason
1. $\angle P \cong \angle$___	1.
2.	2.
3.	3. Reflexive
4. $\Delta PQS \cong \Delta RSQ$	4.

Problem 14:



Statement	Reason
1. $\overline{AC} \parallel \overline{BD}$	1.
2.	2. Given
3. $\angle CAD \cong \angle BDA$	3.
4.	4. Reflexive Property
5. $\Delta ACD \cong \Delta$_____	5.

CPCTC Proofs

What does <u>CPCTC</u> stand for?

C- ______________________

P- ______________________

C- ______________________

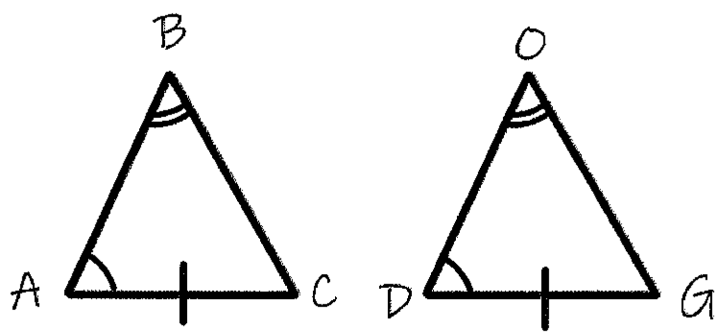
T- ______________________

C- ______________________

How is it helpful?

If we can prove two triangles congruent, we can determine which lengths and angles that were unmarked correspond. We know that the remaining corresponding lengths and angles will be congruent to each other.

Let's take a look:



1. Just looking at the diagram, is $\angle C$ marked to tell us that it is congruent to $\angle G$? __________

2. Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle DOG$?

3. Are $\angle C$ and $\angle G$ congruent? Make an argument for why or why not in at least two sentences.

4. Let's write a two column proof to summarize our argument!

<u>GIVEN:</u> $\angle A \cong \angle D$; $\angle B \cong \angle O$; $\overline{AC} \cong \overline{DG}$.

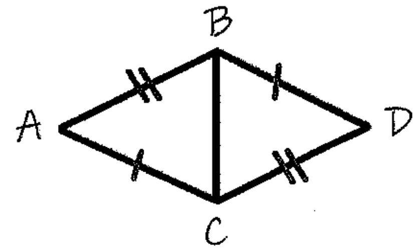
<u>PROVE:</u> $\angle C \cong \angle G$ **Notice that for CPCTC Proofs, the "Prove" line doesn't list two triangles!**

Statements	Reason/Justification
1.	Given
2. $\angle B \cong \angle O$	
3.	GIVEN
4. $\triangle ABC \cong \triangle DOG$	
5. $\angle C \cong \angle G$	

Whatever the "Prove" statement is asking you to do should always be your LAST step!

2. GIVEN: $\overline{AB} \cong \overline{DC}$; $\overline{AC} \cong \overline{DB}$.

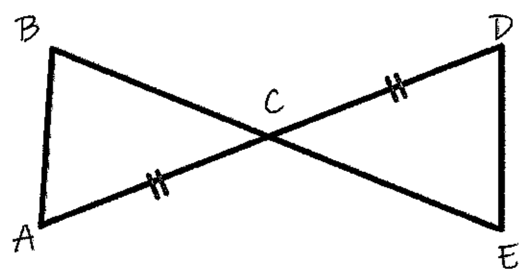
PROVE: $\angle A \cong \angle D$



STATEMENTS	REASONS/JUSTIFICATIONS
1. $\overline{AB} \cong \overline{DC}$	
2.	GIVEN
3.	
4. $\triangle ABC \cong \triangle DCB$	
5.	CPCTC

3. **GIVEN:** $\overline{AC} \cong \overline{DC}$, C is the midpoint of $\overline{BE}$.

PROVE: $\overline{AB} \cong \overline{DE}$



STATEMENTS	REASONS/JUSTIFICATIONS
1. $\overline{AC} \cong \overline{DC}$	
2.	Given
3. $\overline{BC} \cong \overline{EC}$	Definition of __________
4. $\angle BCA \cong \angle$______	_________ angles are congruent.
5. $\triangle ABC \cong \triangle$______	
6. $\overline{AB} \cong \overline{DE}$	