Putting Vectors to Use

Mathematical Goals:

- Use vectors to represent and solve problems.

GSE Standards:

- **MGSE9-12.N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v,** $|\mathbf{v}|$**,** $\|\mathbf{v}\|$**,** v).
- **MGSE9-12.N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **MGSE9-12.N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.
- **MGSE9-12.N.VM.4** Add and subtract vectors.
- **MGSE9-12.N.VM.4a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **MGSE9-12.N.VM.4b** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- **MGSE9-12.N.VM.4c** Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $(-\mathbf{w})$ is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **MGSE9-12.N.VM.5** Multiply a vector by a scalar.
  - **MGSE9-12.N.VM.5a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v_x}, \mathbf{v_y}) = (c\cdot\mathbf{v_x}, c\cdot\mathbf{v_y})$.
  - **MGSE9-12.N.VM.5b** Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along **v** (for $c > 0$) or against **v** (for $c < 0$).

Standards for Mathematical Practice:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.

Putting Vectors to Use

Many different problems can be solved using vectors. Any situation that involves quantities with both magnitude and direction can be represented using vectors. For each problem below, draw and label a diagram, then use what you know about vectors to answer the question.
Show all work on a separate piece of paper.

1. A ship leaves port and travels 49 miles at a standard position angle of 30°. The ship then travels for 89 miles in a standard position angle of 70°. At that point, the ship drops anchor. A helicopter, beginning from the same port, needs to join the ship as quickly as possible. Tell the helicopter's pilot how to get to the ship.

2. You jump into a river intending to swim straight across to the other side. But when you start swimming, you realize the current is traveling 4 miles per hour due south. You are trying to swim due East at 1 mile per hour, but the current is pulling on you. If you don't make any adjustment for the current, how far from your starting point will you be in 15 minutes?

3. A plane is traveling at 400 mph along a path 40° North of East. A strong wind begins to blow at 50 mph from North to South. If no adjustment is made for the wind, what are the resulting bearing and groundspeed of the plane?

4. A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.
   a. What is the resultant velocity?
   b. If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?
   c. What distance downstream does the boat reach the opposite shore?

5. A ship sails 12 hours at a speed of 8 knots (nautical miles per hour) at a heading of 68° south of east. It then turns to a heading of 75° north of east and travels for 5 hours at 15 knots.
   a. Find the resultant displacement vector. Give your answer in component form.
   b. Convert your answer to magnitude-direction form.

6. In three-person tug-of-war, three ropes are tied at a point. Adam is pulling due East with a force of 600 Newtons, Barry is pulling due North with a force of 400 Newtons, and Cal is pulling the third rope. The knot in the middle is not moving. What are the direction and magnitude of Cal's effort?

Problems #4 and #5 adapted from Arizona Department of Education: Standards and Assessment Division