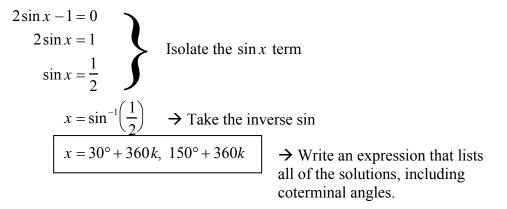
*Trig Identities*, like the one to the right, are true for ALL angles:  $\sin^2 x + \cos^2 x = 1$ 

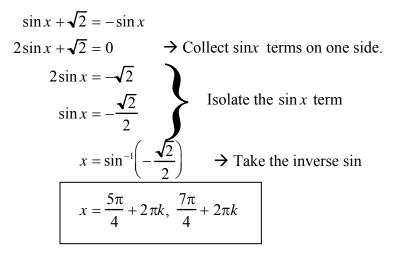
*Trig Equations*, like the one to the right, are only true for SOME angles:  $\sin x = 1$ 

When we *solve* trig equations, we must find **the values of x** that make the statement true. The equation,  $\sin x = 1$ , is true when  $x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots$  A general solution would be  $x = \frac{\pi}{2} + 2\pi k$ .

Example: Solve  $2\sin x - 1 = 0$ .



<u>Example</u>: Solve  $\sin x + \sqrt{2} = -\sin x$ .

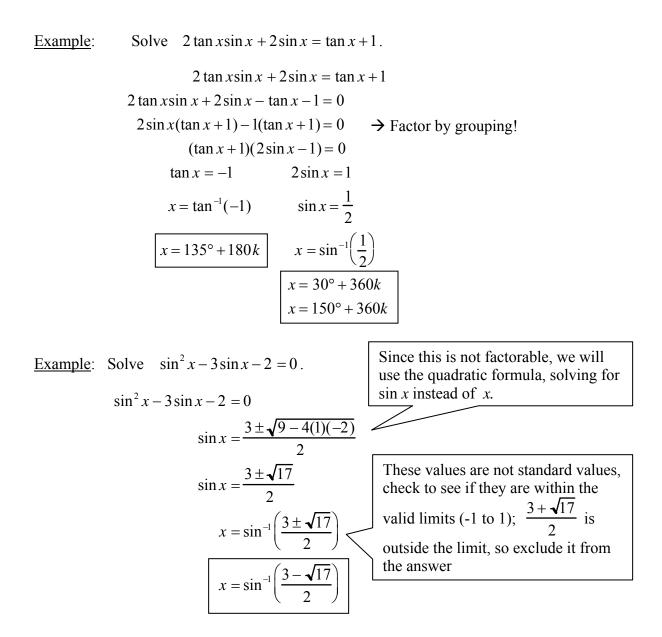


Example: Solve  $3\tan^2 x - 1 = 0$ .  $3\tan^{2} x - 1 = 0$   $3\tan^{2} x = 1$   $\tan^{2} x = \frac{1}{3}$ Isolate the  $\tan^{2} x$  term  $\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$   $\rightarrow$  Square root both sides  $\tan x = \pm \frac{\sqrt{3}}{3} \rightarrow \text{Don't forget the plus/minus}$  $x = 30^{\circ} + 180^{\circ}k, \ 150^{\circ} + 180^{\circ}k$ Solve  $2\cos 3x - 1 = 0$ . Example:  $2\cos 3x - 1 = 0$ 3x = 1  $\cos 3x = \frac{1}{2}$ When the equation involves a *multiple* angle, don't divide by the coeffient of the angle, isolate the  $\cos 3x$  term  $3x = \cos^{-1}\frac{1}{2}$  $3x = 60^\circ + 360k, \ 300^\circ + 360k$  $x = 20^{\circ} + 120k, \ 100^{\circ} + 120k$  $\rightarrow$  Divide both sides by the coefficient of the angle.

Sometimes the trig equation is in the form of a quadratic equation, and can be solved by either factoring or by using the quadratic formula.

Example: Solve 
$$2\sin^2 x - \sin x - 1 = 0$$
.  
 $2\sin^2 x - \sin x - 1 = 0$   
 $(\sin x - 1)(2\sin x + 1) = 0$   
 $\sin x = 1$  and  $2\sin x = -1$   
 $x = \sin^{-1} 1$   $\sin x = -\frac{1}{2}$   
 $x = 90^\circ + 360k$   $x = \sin^{-1}\left(-\frac{1}{2}\right)$   
 $x = 210^\circ + 360k, 330^\circ + 360k$ 

Example: Solve  $\cot x \cos^2 x = 2 \cot x$ .  $\cot x \cos^2 x = 2 \cot x$   $\cot x \cos^2 x - 2 \cot x = 0$   $\cot x (\cos^2 x - 2) = 0$   $\cot x = 0$  and  $\cos^2 x = 2$   $x = \cot^{-1} 0$   $\sqrt{\cos^2 x} = \sqrt{2}$   $x = 90^\circ + 180k$   $\cos x = \pm \sqrt{2}$   $\cos x \cosh^2 x = 2$   $\cos x \cosh^2 x = 2$  $\cos x \cosh^2 x = 2$ 



Sometimes we need to use an identity to rewrite the equation into a form we can solve. If the equation is in the form of a quadratic trinomial, it also helps to write all the trig expressions in terms of the same trig function.

Example: 
$$\sin^2 x + \cos 2x - \cos x = 0$$
  
 $\sin^2 x + \cos 2x - \cos x = 0$   
 $\sin^2 x + \cos^2 x - \sin^2 x - \cos x = 0$   
 $\cos^2 x - \cos x = 0$   
 $\cos x (\cos x - 1) = 0$   
 $\cos x = 0$   
 $\cos x = 0$   
 $\cos x = 1$   
 $x = \cos^{-1}0$   
 $x = 0^{\circ} + 180k$   
 $x = 0^{\circ} + 360k$   
 $\sin^2 x + \cos^2 x - \cos x = 0$   
 $\cos x = 0$   
 $\sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x + \sin^2 x$ 

Example:  $2\sin^2 x + 3\cos x - 3 = 0$ 

 $2\sin^{2} x + 3\cos x - 3 = 0 \quad \Rightarrow \text{Replace } \sin^{2} x \text{ with } 1 - \cos^{2} x$   $2(1 - \cos^{2} x) + 3\cos x - 3 = 0$   $2 - 2\cos^{2} x + 3\cos x - 3 = 0$   $2\cos^{2} x - 3\cos x + 1 = 0$   $(2\cos x - 1)(\cos x - 1) = 0$   $\cos x = \frac{1}{2} \qquad \cos x = 1$   $x = 60^{\circ} + 360k, \quad x = 0^{\circ} + 360k$   $x = 300^{\circ} + 360k$ 

Example:  $\sec^2 x - 2\tan x = 4$ 

$$1 + \tan^{2} x - 2 \tan x = 4 \qquad \Rightarrow \text{Replace sec}^{2} x \text{ with } 1 + \tan^{2} x$$

$$\tan^{2} x - 2 \tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x = 3$$

$$\tan x = -1$$

$$x = \tan^{-1} 3$$

$$x = \tan^{-1} 3$$

$$x = \tan^{-1} (-1)$$

$$x = \tan^{-1} 3 + 180k$$

$$x = 135^{\circ} + 180k$$

## Example:

 $\sin x = \tan x$ 

 $\sin x = \tan x$ 

$$\sin x - \frac{\sin x}{\cos x} = 0$$

$$\Rightarrow \text{ Replace } \tan x \text{ with } \sin x/\cos x. \text{ This expression still has 2 different trig functions, but it is still factorable. We don't always need to rewrite the equation in terms of a single trig function.}$$

$$\sin x (1 - \sec x) = 0$$

$$\sin x = 0 \qquad \sec x = 1$$
  

$$x = \sin^{-1} 0 \qquad x = \sec^{-1} 1$$
  

$$x = 0^{\circ} + 180k \qquad x = 0^{\circ} + 360k \qquad \rightarrow$$
  

$$x = 0^{\circ} + 180k$$

Both factors have the same solution, we only need to write it once.

In the next example, none of our identities will help us get the equation into a form we can solve, so we will **square both sides of the equation**. NOTE: When we square both sides of an equation, we subject ourselves to the possibility of *extraneous solutions*! We will need to check our solutions before moving on!

Example:

 $\cos x + 1 = \sin x$ 

$$\cos x + 1 = \sin x$$

$$(\cos x + 1)^{2} = \sin^{2} x$$

$$\cos^{2} x + 2\cos x + 1 = 1 - \cos^{2} x$$

$$2\cos^{2} x + 2\cos x = 0$$

$$2\cos x(\cos x + 1) = 0$$

$$\cos x = 0 \qquad \cos x = -1$$

$$x = \cos^{-1} 0 \qquad x = \cos^{-1}(-1)$$

$$x = 90^{\circ} + 180k \qquad x = 180^{\circ} + 360k$$

Test both sets of solutions.

$\cos 90^\circ + 1 = \sin 90^\circ$		
0+1	=1 (yes)	$\cos 180^\circ + 1 = \sin 180^\circ$
$\cos 270^\circ + 1 = \sin 270^\circ$		-1+1=0 (yes)
0 + 1 = -1 (no)		
Final solution:	$x = 90^\circ + 360k$	$x = 180^\circ + 360k$