

TWO COLUMN PROOFS OF TRIG IDENTITIES

Listed below are proofs of trigonometric identities. It is your job to determine the theorem, property, or identity used in each step. (Think of this as a two-column proof similar to the ones you LOVED to do in Geometry.) Not all steps will have trig reasons.

Proof	Reason
$\begin{aligned} \sin x + \cos x &= \frac{\cot x + 1}{\csc x} \\ &= (\cot x + 1) \cdot \csc x \\ &= (\cot x + 1) \cdot \frac{1}{\csc x} \\ &= (\cot x + 1) \cdot \sin x \\ &= \cot x \sin x + \sin x \\ &= \frac{\cos x}{\sin x} \sin x + \sin x \\ &= \cos x + \sin x \end{aligned}$	

2. Proof	Reason
$\begin{aligned} \frac{1}{\sec x - \tan x} &= \sec x + \tan x \\ \frac{1}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} &= \\ \frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} &= \\ \frac{\sec x + \tan x}{1 + \tan^2 x - \tan^2 x} &= \\ \sec x + \tan x &= \end{aligned}$	

3. Proof

Reason

$$\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$$

$$\frac{\sec^2 x}{\sec^2 x} - \frac{1}{\sec^2 x} =$$

$$1 - \frac{1}{\sec^2 x} =$$

$$1 - \cos^2 x =$$

$$\sin^2 x + \cos^2 x - \cos^2 x =$$

$$\sin^2 x =$$

Proof

Reason

$$\begin{aligned}
 2 \sec x &= \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \\
 &= \frac{(1 + \sin x)^2}{\cos x(1 + \sin x)} + \frac{\cos^2 x}{\cos x(1 + \sin x)} \\
 &= \frac{1 + 2 \sin x + \sin^2 x}{\cos x(1 + \sin x)} + \frac{\cos^2 x}{\cos x(1 + \sin x)} \\
 &= \frac{1 + 2 \sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} \\
 &= \frac{1 + 2 \sin x + 1}{\cos x(1 + \sin x)} \\
 &= \frac{2 + 2 \sin x}{\cos x(1 + \sin x)} \\
 &= \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} \\
 &= \frac{2}{\cos x} \\
 &\equiv 2 \cdot \frac{1}{\cos x} \\
 &= 2 \sec x
 \end{aligned}$$