AN OKEFENOKEE FOOD WEB Learning Task:
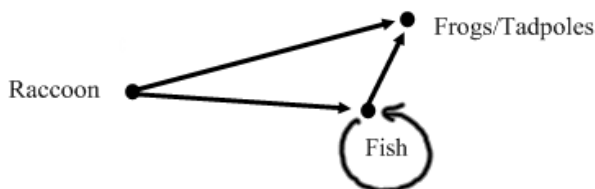
Recent weather conditions have caused a dramatic increase in the insect population of the Okefenokee Swamp area. The insects are annoying to people and animals and health officials are concerned there will be an increase in disease. Local authorities want to use an insecticide that would literally wipe out the entire insect population of the area. You, as an employee of the Environmental Protection Agency, must determine how detrimental this would be to the environment. Specifically, you are concerned on the effects on the food web of six animals known to populate the swamp.

Consider the following digraph of a food web for the six animals and the insects that are causing the problem.
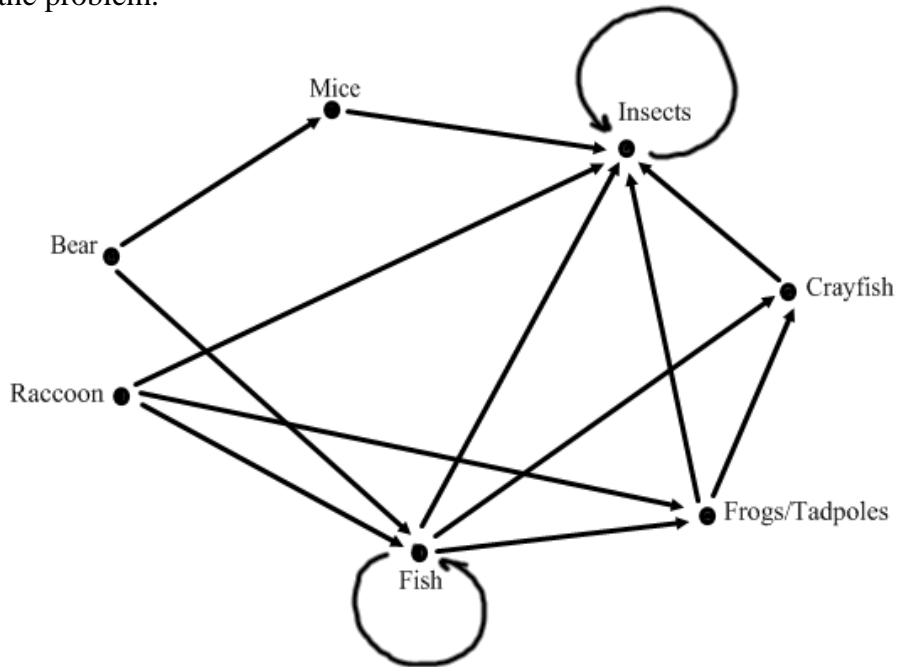
A **digraph** is a directed vertex edge graph. Here each vertex represents an animal or insects. The direction of the edges indicates whether an animal preys on the linked animal. For example, raccoons eat fish. (Note: the food web shown is simplified. Initial producers of nutrients, plants, have not been included.)

Adjacency matrices can be used in conjunction with digraphs. If we consider just the relationships between raccoons, fish, and frogs in the food web shown, an adjacency matrix would be

$$\begin{array}{c c} & \begin{matrix} R & FT & F \end{matrix} \\ \begin{matrix} R \\ FT \\ F \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

1. Label the rows and columns in alphabetical order and construct the associated matrix F to represent this web. What does a row containing a single one indicate? What does a column of zeros indicate?

2. Which animals have the most direct sources of food? How can this be determined from the matrix? Find the number of direct food sources for each animal.

3. The insect column has the most ones. What does this suggest about the food web?

4. The matrix F^2 denotes indirect (through one intermediary) sources of food. For example, the fish relies on insects for food, and the bear relies on the fish for food, so the insect is an indirect source of food for the bear. Find F^2. Notice that insect column contains all nonzero numbers. What does this indicate?

	Bear	*Crayfish*	*Fish*	*Frogs*	*Insects*	*Mice*	*Raccoons*
Bear							
Crayfish							
Fish							
Frogs							
Insects							
Mice							
Raccoons							

$F^2 =$

5. Compute additional powers of the food web matrix to represent the number of direct and indirect sources of food for each animal. Which animal has the most food sources?

$F^3 =$ $F^4 =$

If an insecticide is introduced into the food web, killing the entire insect population, several animals will lose a source of food.

6. Construct a new matrix G to represent the food web with no insects. What effect does this have on the overall animal population? What has happened to the row sums? Compare these with those of the original matrix. What does a row sum of zero indicate?

7. Will all the animals be affected by the insecticide? Which animal(s) will be least affected?

8. Organize and summarize your findings in a brief report to the health officials. Take and support a position on whether using an insecticide to destroy the insect population is harmful to the environment.

Extension Problem:

Within a small group of associates, different people are willing to share secrets selectively. Allen will share with Elyse. Brett will share with Chloe and Allen. Chloe will share with Elyse and Dora. Dora will share with Fiona and Elyse. Elyse will share with Allen and Brett. Fiona will share with Allen and Brett.

1. Show this information with a digraph and an adjacency matrix named S.

2. What do the zeros on the diagonal of the adjacency matrix indicate?

3. Find S^2 and S^3.

4. Is it possible for Fiona to share a secret and that secret reach Chloe? What is the minimum number of times the secret is shared when Charles knows the secret?

5. How many ways is it possible for a secret to get from Brett to Elyse in 3 or fewer secret sharing episodes?