Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad \sin^2\theta = 1 - \cos^2\theta$

$-\cos^2 \quad -\cos^2$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Reciprocal IDs

$\tan\theta = \dfrac{1}{\cot\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

6 $(\csc x + \cot x)^2 = (1)^2$

$(\csc x+\cot x)(\csc x+\cot x)$

	csc	+ cot
csc	$\csc^2 x$	csc cot
+ cot x	csc cot	$\cot^2 x$

$\csc^2 x + 2\csc x\cot x + \cot^2 x = 1$ Pyth. ID.

$1+\cot^2 x + 2\csc x\cot x + \cot^2 x = 1$ Combine like

$2\cot^2 x + 2\csc x\cot x + 1 = 1$ GCF

$-1 \quad -1$

$2\cot^2 x + 2\csc x\cot x = 0$

$2\cot x(\cot x + \csc x) = 0$

$\frac{2\cot x}{2} = \frac{0}{2}$ $\cot x + \csc x = 0$

$\cot x = \frac{0}{1}$ $\frac{x}{y}$

Verify.

5. $$3\sin^2 x + 4\cos^2 x = 3 + \cos^2 x$$

$$3(1-\cos^2 x) + 4\cos^2 x$$ Pyth. ID

$$3 - 3\cos^2 x + 4\cos^2 x$$ Distribute

$$3 + 1\cos^2 x = 3 + \cos^2 x$$ combine like terms

$\checkmark$

Solve.

2 trig terms
3 terms altogether
Factor

4) $2\sin^2 x + 3\sin x + 1 = 0$

$2m^2 + 3m + 1$

AC method
a · c

$\left(\sin x + \frac{1}{2}\right)\left(\sin x + \frac{2}{2}\right) = 0$

2

1 2

3

b

+

$\sin x + \frac{1}{2} = 0$

$-\frac{1}{2} \quad -\frac{1}{2}$

$\sin^{-1}(\sin x) = \sin^{-1}\left(-\frac{1}{2}\right)$

$X = 210^\circ, 330^\circ$

$\sin x + 1 = 0$

$-1 \quad -1$

$\sin^{-1}(\sin x) = \sin^{-1}(-1)$

$X = 270^\circ$

$$\cos(u - v)$$

$$= \cos u \cos v + \sin u \sin v$$

$$\left(\frac{4}{5}\right)\left(\frac{-4}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)$$

$$\frac{-16}{25} + \frac{9}{25}$$

$$\frac{-7}{25}$$

6

$$\csc x + \cot x = 1$$

Square both sides.

I. Trig function on each side.

- 2. Square both sides.

$$\underline{\quad - \cot x \qquad - \cot x}$$

$$(\csc x)^2 = (1 - \cot x)^2$$

$$\csc^2 x = 1 - 2\cot x + \cot^2 x$$

Pyth. Id.

$$1 + \cot^2 x = 1 - 2\cot x + \cot^2 x$$

$$\underline{\quad -\cot^2 x \qquad\qquad\qquad\qquad -\cot^2 x}$$

$$1 = 1 - 2\cot x$$

$$\underline{-1 \qquad -1}$$

$$\frac{0}{-2} = \frac{-2\cot x}{-2}$$

$$x = 90^\circ, 270^\circ$$

$$\cot x = \frac{0}{1} \quad \frac{x}{y}$$

Check!!

$\cancel{90^\circ} \rightarrow \frac{1}{\sin(90)} + \frac{1}{\tan(90)}$

$\cancel{270^\circ} \rightarrow$

no solution

$$(1-\cot x)^2$$

$$(1-\cot x)(1-\cot x)$$

$$\begin{array}{c|c|c}
 & 1 & -\cot x \\
\hline
1 & 1 & -\cot x \\
\hline
-\cot x & -\cot x & +\cot^2 x
\end{array}$$

$$1-2\cot x+\cot^2 x$$

7

$$(\sin\theta + \cos\theta)^2$$

$$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$$

	$\sin\theta$	$+\cos\theta$
$\sin\theta$	$\sin^2\theta$	$\sin\theta\cos\theta$
$+\cos\theta$	$\sin\theta\cos\theta$	$\cos^2\theta$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$

$$(\sin\theta - \cos\theta)^2$$

12 $\sin x(\cot x + \tan x) = \sec x$

$\frac{\sin x}{1}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$ Rewrite

$\frac{\cancel{\sin x}\cos x}{\cancel{\sin x}} + \frac{\sin^2 x}{\cos x}$ Distribute

cos x out factors

$\frac{\cos x}{\cos x} \cdot \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} \cdot \frac{1}{1}$ Add fract.

$\frac{[\cos^2 x + \sin^2 x]}{\cos x}$ Pyth ID

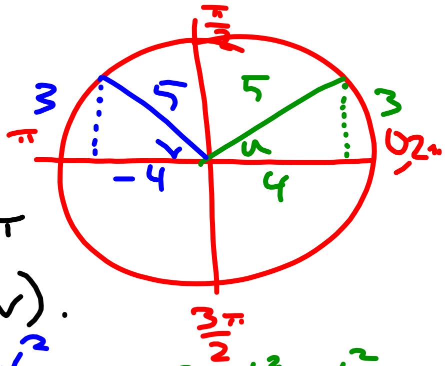
$\frac{1}{\cos x} = \sec x$ Reciprocal

$= \sec x$ ✓

11. $\tan u = \frac{3}{4}\ \frac{O}{A},\quad 0 < u < \frac{\pi}{2}$

$\cos v = -\frac{4}{5}\ \frac{A}{H},\quad \frac{\pi}{2} < v < \pi$

Find $\sin(u+v)$ and $\cos(u-v)$.



$a^2+b^2=c^2$
$(-4)^2+b^2=5^2$
$16+b^2=25$
$-16 \quad -16$
$\sqrt{b^2}=\sqrt{9}$
$b=3$

$a^2+b^2=c^2$
3^2+4^2
$9+16$
$\sqrt{25}\ \sqrt{c^2}$
$5=c$

$\sin(u+v)$

$= \sin u \cos v + \cos u \sin v$

$\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$\frac{-12}{25} + \frac{12}{25}$

$\frac{0}{25} = 0$

15

$\sin(-285^\circ)$

$\sin(45^\circ - 330^\circ) = \sin^{45}\cos^{330} - \cos^{45}\sin^{330}$

Diff of sine

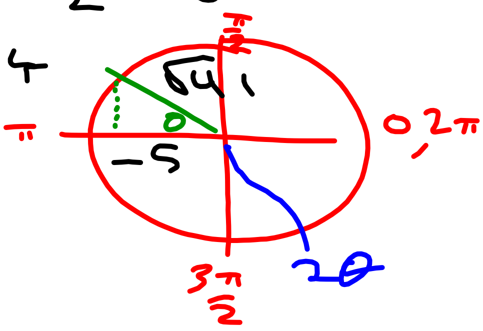
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{-\sqrt{2}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\tan\theta = -\frac{4}{5}$

$\frac{\pi}{2} < \theta < \pi$



Find $\cos 2\theta$.

$A^2 + B^2 = C^2$

$(-4)^2 + (5^2) = C^2$

$16 + 25 = C^2$

$\sqrt{c^2} = \sqrt{41}$

$C = \sqrt{41}$

$$\cos 2\theta =$$

$$\cos^2\theta - \sin^2\theta$$

$$\left(\frac{-5}{\sqrt{41}}\right)^2 - \left(\frac{4}{\sqrt{41}}\right)^2$$

$$\left(\frac{25}{41}\right) - \left(\frac{16}{41}\right)$$

$$\frac{9}{41}$$

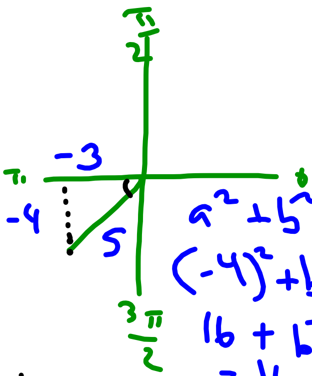
9) $\sin u = \dfrac{-4^{\,O}}{5_{\,H}}$ $\pi < u < \dfrac{3\pi}{2}$, find $\cos(2u)$

$$\cos(2u) = \cos(u+u) = \cos(u)\cos(u) - \sin(u)\sin(u)$$

$$\left(\frac{-3}{5}\right)\left(\frac{-3}{5}\right) - \left(\frac{-4}{5}\right)\left(\frac{-4}{5}\right)$$

$$\frac{9}{25} - \frac{16}{25}$$

$$\frac{-7}{25}$$



$a^2 + b^2 = c^2$

$(-4)^2 + b^2 = 25$

$16 + b^2 = 25$

$-16 \qquad -16$

$\sqrt{b^2} = \sqrt{9}$

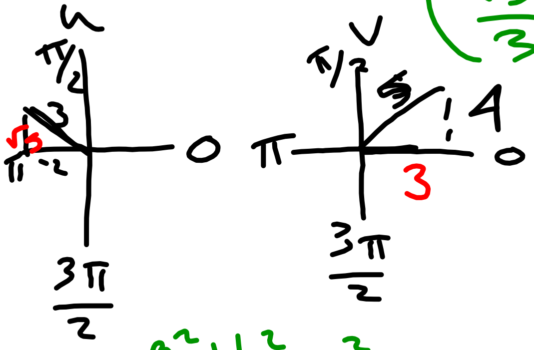
$b = 3$

1. Fall ↑ or ↓
2. Angle at origin
3. Hyp is pos.

(10) $\cos(u) = \frac{-2^{\,A}}{3_{\,H}}$, $\frac{\pi}{2} < u < \pi$, $\sin(v) = \frac{4_{\,O}}{5_{\,H}}$, $0 < v < \frac{\pi}{2}$

$$\sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v)$$

$$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{5}\right) - \left(\frac{-2}{3}\right)\left(\frac{4}{5}\right)$$



$$\frac{3\sqrt{5}}{15} + \frac{+8}{15}$$

$$\frac{3\sqrt{5} + 8}{15}$$

$a^2 + b^2 = c^2$

$(-2)^2 + b^2 = 3^2$

$4 + b^2 = 9$

$b = \sqrt{5}$

$$(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$$

1. multiply

2. Pyth. ID.

	sec	−1
sec	$\sec^2$	$-\sec$
+1	sec	−1

$\sec^2 - 1$

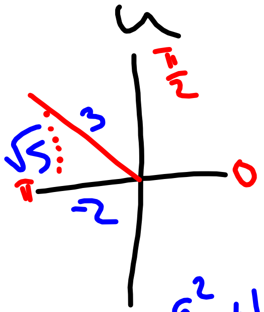
$1 + \tan^2\theta \quad -1$

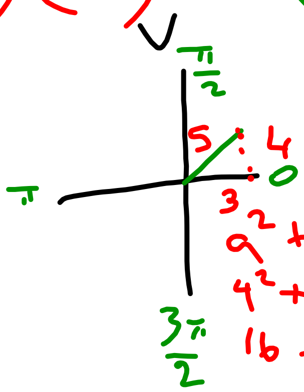
$\tan^2\theta = \tan^2\theta$

⑩ $\cos u = \frac{-2}{3}\frac{A}{H},\ \frac{\pi}{2} < u < \pi\ ;\ \sin v = \frac{4}{5}\frac{O}{H},\ 0 < v < \frac{\pi}{2}$

$\sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v)$

$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{5}\right) - \left(\frac{-2}{3}\right)\left(\frac{4}{5}\right)$





1. Fall ↑ or ↓
2. Angle at origin
3. Hyp is pos.

$a^2 + b^2 = c^2$

$(-2)^2 + b^2 = 3^2$

$4 + b^2 = 9$

$-4 \quad -4$

$\sqrt{b^2} = \sqrt{5}$

$b = \sqrt{5}$

$a^2 + b^2 = c^2$

$4^2 + b^2 = 5^2$

$16 + b^2 = 25$

$-16 \quad -16$

$\sqrt{b^2} = \sqrt{9}$

$b = 3$

$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{5}\right) - \left(\frac{-2}{3}\right)\left(\frac{4}{5}\right)$

$\frac{3\sqrt{5}}{15} + \frac{8}{15} = \frac{3\sqrt{5}+8}{15}$

$$\sin(45^\circ + x) - (\sin(45 - x))$$

$$\sin(45)\cos(x) + \cos(45)\sin(x) + (-\sin(45)\cos(x) + \cos(45)\sin(x))$$

$$\boxed{\cancel{\frac{\sqrt{2}}{2}\cos(x)}} + \frac{\sqrt{2}}{2}\sin(x)\ \boxed{\cancel{-\frac{\sqrt{2}}{2}\cos(x)}} + \frac{\sqrt{2}}{2}\sin(x)$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\sin(x)$$

$$\frac{\cancel{2}\sqrt{2}\sin x}{\cancel{2}}$$

$$\sqrt{2}\sin(x)$$

② $\cos^2 x - 6\cos x + 8 = 0$

Big X

8

$-2 \qquad -4$

-6

$(\cos x - 2)(\cos x - 4) = 0$

$\cos x - 2 = 0$
$\quad +2 \quad +2$
$(\cos x) = (2)$
$\cos^{-1} \qquad \cos^{-1}$

$x = \varnothing$

no solution

$\cos x - 4 = 0$
$\quad +4 \quad +4$
$(\cos x) = (4)$
$\cos^{-1} \qquad \cos^{-1}$

$\varnothing$

Solving

7) $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$

$+1 \quad +1$

$\frac{3\tan^2 x}{3} = \frac{1}{3}$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan^{-1}(\tan x) = \tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right) \quad \frac{y}{x}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\tan^2 x - 3 = 0$

$+3 \quad +3$

$\sqrt{\tan^2 x} = \sqrt{3}$

$\tan^{-1}(\tan x) = \tan^{-1}\left(\pm\frac{\sqrt{3}}{1}\right)$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Sum / Diff

11) $\tan u = \frac{3}{4}\frac{O}{A}$, $0 \le u \le \frac{\pi}{2}$

$\cos v = -\frac{4}{5}\frac{A}{H}$, $\frac{\pi}{2} \le v \le \pi$

Find $\sin(u+v)$

$= \sin(u)\cos(v) + \cos(u)\sin(v)$

$\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

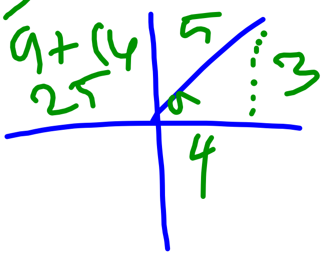
$a^2 + b^2 = \sqrt{c^2} = \sqrt{25}$

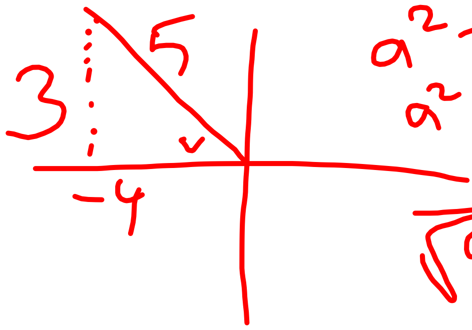
$3^2 + 4^2$

$9 + 16$

25

$c = 5$





$a^2 + (-4)^2 = 5^2$

$a^2 + 16 = 25$

$-16 \quad -16$

$\sqrt{a^2} = \sqrt{9}$

$a = 3$

$$\frac{-12}{25} + \frac{12}{25}$$

$$\frac{-12+12}{25}$$

$$\frac{0}{25} = 0$$

Verify L2

7) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

	$\sin x$	$+\cos x$
$\sin x$	$\sin^2 x$	$\sin x \cos x$
$+\cos x$	$\sin x \cos x$	$\cos^2 x$

	$\sin x$	$-\cos x$
$\sin x$	$\sin^2 x$	$-\sin x \cos x$
$-\cos x$	$-\sin x \cos x$	$\cos^2 x$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x \cos x + \underbrace{\sin^2 x + \cos^2 x}_{1} - 2\sin x \cos x$

$1 + 1 + 2\sin x \cos x - 2\sin x \cos x$

$2 = 2$

HEY! ♥

10) $$\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2\tan x$$

$$\frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} \quad \frac{\sin x}{1}$$ Rewrite

Divide fraction

$$\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}$$ Add fraction

$$2\,\frac{\sin x}{\cos x}$$ Quotient ID

$2\tan x$

$2\tan x \quad =$ ✓

Solve

2 $\sqrt{3}\sec x - 1 = 0$

$+1 \quad +1$

$\frac{\sqrt{3}\sec x}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\sec x = \frac{1}{\sqrt{3}}$

$\cancel{\cos^{-1}}\left(\cos x\right) = \left(\frac{\sqrt{3}}{1}\right)$
$\cos^{-1}$

$x = \emptyset$ no solution

Sum/Diff

10 $\cos u = \frac{-2}{3}$ A H, $\frac{\pi}{2} < u < \pi$

$\sin v = \frac{4}{5}$ O H, $0 < v < \frac{\pi}{2}$

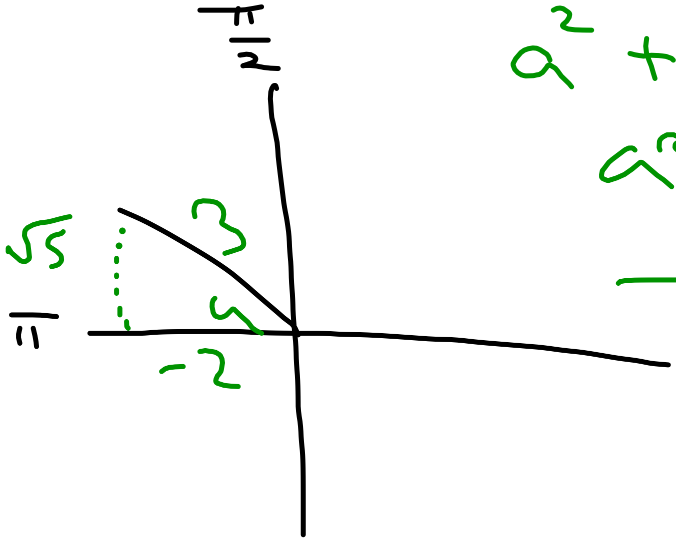
Find $\sin(u-v)$

$= \sin(u)\cos(v) - \cos(u)\sin(v)$

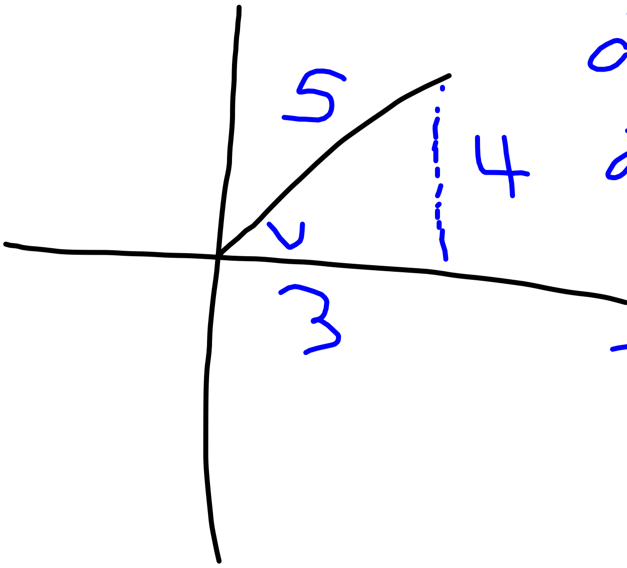
$\left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{5}\right) - \left(\frac{-2}{3}\right)\left(\frac{4}{5}\right)$

$\frac{3\sqrt{5}}{15} - \frac{(-8)}{15}$

$\frac{3\sqrt{5}+8}{15}$



$$a^2 + (-2)^2 = 3^2$$
$$a^2 + 4 = 9$$
$$-4 \quad -4$$
$$\sqrt{a^2} = \sqrt{5}$$
$$a = \sqrt{5}$$



$$a^2 + b^2 = c^2$$
$$a^2 + (4)^2 = 5^2$$
$$a^2 + 16 = 25$$
$$-16 \quad -16$$
$$\sqrt{a^2} = \sqrt{9}$$
$$a = 3$$

Sum/DIFF/Double

14 $\cos(255^\circ)$ $\cos(300-45)$

$\cos 300 \cos 45 + \sin 300 \sin 45$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

Verify L2

8

$$\frac{(1+\sin x)}{(1+\sin x)}1 - \frac{\cos^2 x}{(1+\sin x)} = \sin x$$

$$\frac{1+\sin x}{1+\sin x} - \frac{\cos^2 x}{1+\sin x}$$

$$\frac{1+\sin x - (\cos^2 x)}{1+\sin x}$$

$$\frac{1+\sin x - 1(1-\sin^2 x)}{1+\sin x}$$

Pyth. (1)

$$\sin^2 x + \cos^2 x = 1$$
$$-\sin^2 x \qquad -\sin^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

$$\frac{\cancel{1} + \sin x - \cancel{1} + \sin^2 x}{1 + \sin x}$$

$$\frac{\overset{\therefore \sin x}{\sin x} + \overset{\therefore \sin x}{\sin^2 x}}{1 + \sin x}$$

$$\frac{\sin x \cancel{(1 + \sin x)}}{\cancel{(1 + \sin x)}}$$

$$\sin x = \sin x$$

Solve

(3) $3\tan^3 x = \tan x$

$-\tan x \quad -\tan x$

$3\tan^3 x - \tan x = 0$ GCF

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ $\quad$ $3\tan^2 x - 1 = 0$

$\tan^{-1} \quad \tan^{-1}$ $\quad$ $+1 \quad +1$

$x = \pi, 0$ $\quad$ $\frac{3\tan^2 x}{3} = \frac{1}{3}$

$\sqrt{\tan^2 x} = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ $\quad$ $\frac{y}{x}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$$\frac{y}{x} \qquad \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \qquad \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Solve

$$\sec x \csc x = 2\csc x$$
$$-2\csc x \quad -2\csc x$$
$$\sec x \csc x - 2\csc x = 0$$

Get 0 on right side

$$\csc x(\sec x - 2) = 0$$

GCF

$\csc x = 0$

$\sin x = \frac{1}{0}$

undefined

$\varnothing$

$\sec x - 2 = 0$

$+2 \quad +2$

$\sec x = 2$

$(\cos x) = \left(\frac{1}{2}\right)$

$\cos^{-1} \quad \cos^{-1}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

4) $2\sin^2 x + 3\sin x + 1 = 0$

$(\sin x + \frac{2}{2})(\sin x + \frac{1}{2}) = 0$

2, 3, 2, 1

$\sin x + 1 = 0$
$-1 \quad -1$

$(\sin x) = (-1)$
$\sin^{-1} \quad \sin^{-1}$

$x = \frac{3\pi}{2}$

$\sin x + \frac{1}{2} = 0$
$-\frac{1}{2} \quad -\frac{1}{2}$

$(\sin x) = (-\frac{1}{2})$
$\sin^{-1} \quad \sin^{-1}$

$x = \frac{11\pi}{6}, \frac{7\pi}{6}$

Verify

$$\frac{\csc x}{\sec x - \tan x} - \frac{\csc x}{\sec x + \tan x} = 2\sec x$$

$(s+t)$ $(s-t)$

$$\frac{\csc x(\sec x + \tan x) - \csc x(\sec x - \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$$

$$\frac{\csc x\left[\sec x + \tan x + 1(-\sec x + \tan x)\right]}{(\sec^2 x - \tan^2 x)}$$

$$\frac{\csc x(\tan x+\tan x)}{(\sec^2 x-\tan^2 x)}$$

$$\frac{\csc x\,(2\tan x)}{\underbrace{\sec^2 x}_{1+\cancel{\tan^2 x}}-\cancel{\tan^2 x}}$$

Pyth ID

$1+\tan^2 x=\sec^2 x$

$$\csc x\,(2\tan x)$$

$$\frac{1}{\cancel{\sin x}}\left(\frac{2\cancel{\sin x}}{\cos x}\right)$$

$$2\left(\frac{1}{\cos x}\right)=2\sec x=2\sec x$$

L1 4 Verify

$$\cos^2 x\,(\underbrace{1+\tan^2 x}) = 1$$

Pyth. ID

$1+\tan^2 x = \underline{\sec^2 x}$

$$\cos^2 x\,(\underline{\sec^2 x})$$

Reciprocal ID

$$\cancel{\cos^2 x}\left(\frac{1}{\cancel{\cos^2 x}}\right)$$

$$1 = 1$$

L1 6

$$(\sec x + \tan x)(\sec x - \tan x) = 1$$

	sec	+ tan
sec	$\sec^2 x$	$\sec x \tan x$
$-\tan$	$-\sec x \tan x$	$-\tan^2 x$

$$\sec^2 x - \tan^2 x$$

$$1 + \tan^2 x - \tan^2 x$$

$$1 = 1$$

Pyth. ID

$$1 + \tan^2 x = \sec^2 x$$

4 5

$$3\sin^2x + 4\cos^2x = 3 + \cos^2x$$

$$3(1-\cos^2x) + 4\cos^2x$$

$$3 - 3\cos^2x + 4\cos^2y$$

$$3 + \cos^2x = 3 + \cos^2x$$

Pyth. ID

$$\cos^2x + \sin^2x = 1$$

$$-\cos^2x \quad -\cos^2x$$

$$\sin^2x = \underbrace{1-\cos^2x}$$

Solve

4) $2\sin^2 x + 3\sin x + 1 = 0$

$ax^2 + bx + c$

$(\sin x + \frac{1}{2})(\sin x + 1) = 0$

$\sin x + \frac{1}{2} = 0$ $\sin x + 1 = 0$

$-\frac{1}{2}$ $-\frac{1}{2}$ -1 -1

$\sin^{-1}(\sin x) = \sin^{-1}(-\frac{1}{2})$ $\sin^{-1}(\sin x) = \sin^{-1}(-1)$

$x = \frac{11\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}$

$2\sin(\frac{3\pi}{2})^2 + 3\sin(\frac{3\pi}{2}) + 1$

$2(-1)^2 + 3(-1) + 1$

$2 - 3 + 1$

0

Check

$$2\sin\left(\overset{\frac{7\pi}{6}}{\frac{11\pi}{6}}\right)^2 + 3\sin\left(\overset{\frac{7\pi}{6}}{\frac{11\pi}{6}}\right) + 1$$

$$2\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1$$

$$2\left(\frac{1}{4}\right) + \frac{-3}{2} + 1$$

$$\frac{1}{2} - \frac{3}{2} + 1$$

$$-1 + 1$$

$$0$$

7) $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$

$+1 \quad +1$

$\frac{3\tan^2 x}{3} = \frac{1}{3}$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}} \quad \frac{y}{x}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{7\pi}{6}$

$\tan^2 x - 3 = 0$

$+3 \quad +3$

$\sqrt{\tan^2 x} = \sqrt{3}$

$\tan x = \pm\frac{\sqrt{3}}{1} \quad \frac{y}{x}$

$x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

on unit circle...

$$= \frac{y}{x}$$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$x \quad y$

$\frac{\pi}{3}$

$$\tan\theta = \frac{y}{x} = \frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{2}} \cdot \frac{\cancel{2}}{1} = \frac{\sqrt{3}}{1}$$

$$\sqrt{3}\boxed{\sec x} - 1 = 0$$

$$+1 \quad +1$$

$$\frac{\sqrt{3}\sec x}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

cos flipped $\sqrt{3}$

$$(\sec x) = \left(\frac{1}{\sqrt{3}}\right)$$

$$x = \sqrt{3}$$

no solution