# Law of Cosines SAS, SSS

$$a = \sqrt{(b^2 + c^2 - 2bc \cdot \cos A)}$$

$$b = \sqrt{(a^2 + c^2 - 2ac \cdot \cos B)}$$

$$c = \sqrt{(a^2 + b^2 - 2ab \cdot \cos C)}$$

$$A = \cos^{-1}\left(\frac{(b^2+c^2-a^2)}{(2bc)}\right)$$

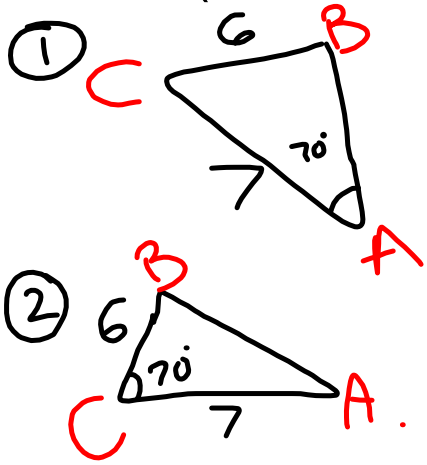
$$B = \cos^{-1}\left(\frac{(a^2+c^2-b^2)}{(2ac)}\right)$$

$$C = \cos^{-1}\left(\frac{(a^2+b^2-c^2)}{(2ab)}\right)$$

# Warm-up

## May 2, 2017

Find c. (There could be more than one solution.)



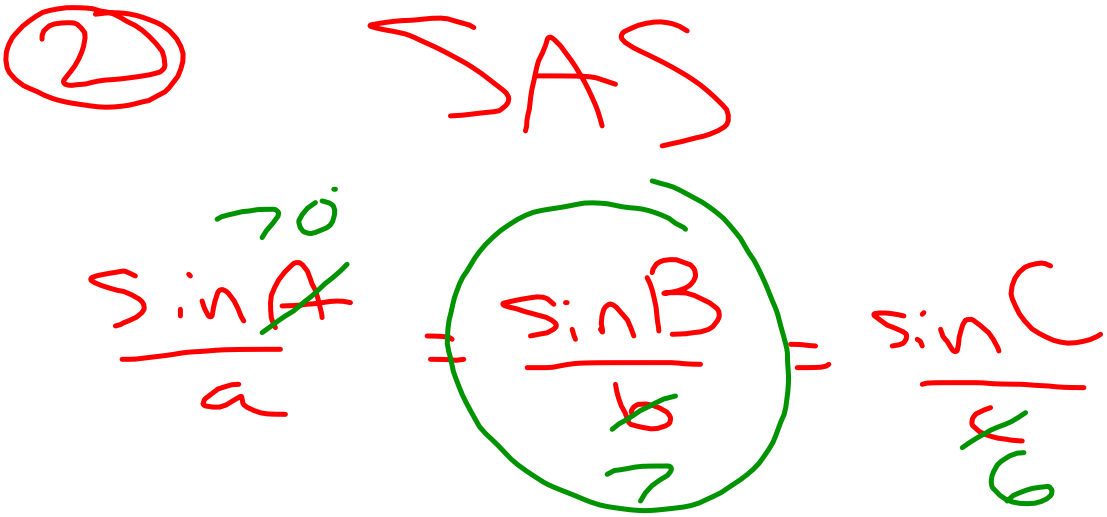




SAS

$\frac{\sin A}{6} = \frac{\sin B}{7} = \boxed{\frac{\sin 70^\circ}{c}}$

There is a solution, this.


2
SAS
70°
sin A / a = sin B / b = sin C / c
7
6

① SSA Donkey

$h = b \cdot \sin A$

$= 7 \sin(70)$

$= 6.58$

$6 < 6.58 < 7$

$a < h < b$

no solution

$a^2+b^2=c^2$

$k^2+(c-x)^2=b^2$

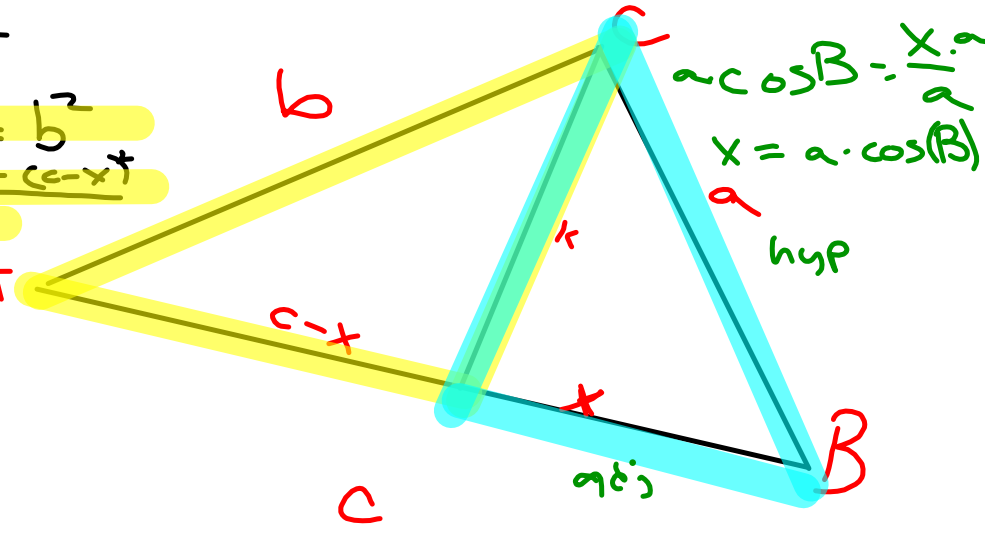
$-(c-x)^2 \quad -(c-x)^2$

$k^2=b^2-(c-x)^2$

$k^2+x^2=a^2$

$-x^2-x^2$

$k^2=a^2-x^2$




$\cos B = \frac{x}{a}$

$x = a \cdot \cos(B)$

$k^2=a^2-x^2$

$k^2=b^2-(c-x)^2$

$b^2-(c-x)^2=a^2-x^2$

$b^2-(c-x)^2=a^2-x^2$

$b^2-(c^2-2cx+x^2)=a^2-x^2$

$b^2-c^2+2cx-x^2=a^2-x^2$

$+c^2-2cx+x^2 \quad +c^2-2cx+x^2$

$b^2=a^2+c^2-2cx$

| | $c$ | $-x$ |
|---|---|---|
| $c$ | $c^2$ | $-cx$ |
| $-x$ | $-cx$ | $x^2$ |

$c^2-2cx+x^2$

$x=a\cdot\cos B$

$b^2=a^2+c^2-2c\cdot a\cos B$

$b=\sqrt{(a^2+c^2-2ac\cdot\cos(B))}$

$c=\sqrt{(a^2+b^2-2ab\cdot\cos(C))}$

$a=\sqrt{(c^2+b^2-2cb\cdot\cos(A))}$

$a^2 + b^2 = c^2$

$k^2 + (c-x)^2 = b^2$

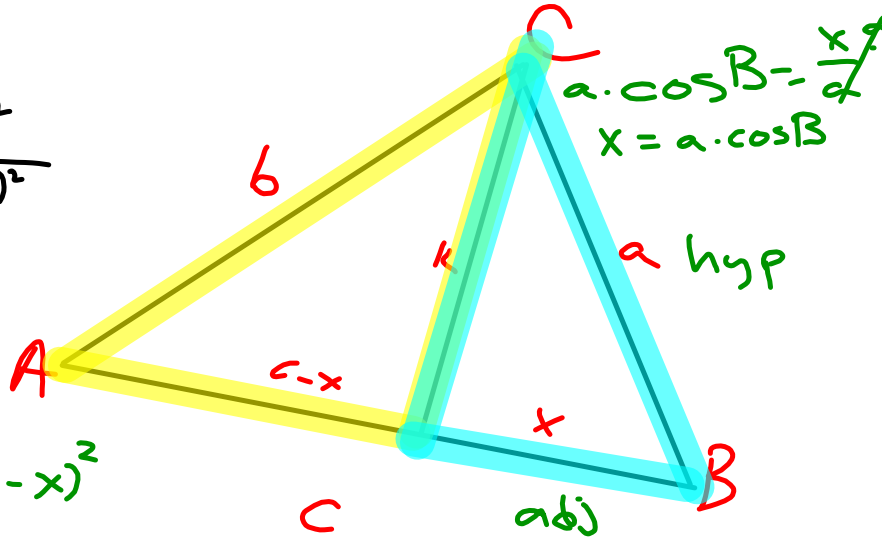
$-(c-x)^2 \quad -(c-x)^2$

$k^2 = b^2 - (c-x)^2$

$k^2 + x^2 = a^2$

$-x^2 \quad -x^2$

$k^2 = a^2 - x^2$




$a \cdot \cos B = \frac{x}{a}$

$x = a \cdot \cos B$

$k^2 = b^2 - (c-x)^2$

$k^2 = a^2 - x^2$

$a^2 - x^2 = b^2 - (c-x)^2$

$a^2 - x^2 = b^2 - (c-x)^2$

$a^2 - x^2 = b^2 - (c^2 - 2cx + x^2)$

$a^2 - x^2 = b^2 - c^2 + 2cx - x^2$

$+c^2 - 2cx + x^2 \quad +c^2 - 2cx + x^2$

| | $c$ | $-x$ |
|---|---|---|
| $c$ | $c^2$ | $-cx$ |
| $-x$ | $-cx$ | $x^2$ |

$c^2 - 2cx + x^2$

$b^2 = a^2 + c^2 - 2cx$

$\sqrt{b^2} = \sqrt{a^2 + c^2 - 2 \cdot c \cdot a \cdot \cos B}$

$b = \sqrt{(a^2 + c^2 - 2ac \cdot \cos(B))}$

$a = \sqrt{(b^2 + c^2 - 2b \cdot c \cdot \cos(A))}$

$c = \sqrt{(a^2 + b^2 - 2a \cdot b \cdot \cos(C))}$

$a^2+b^2=c^2$

$k^2+(c-x)^2=b^2$

$-(c-x)^2 \quad -(c-x)^2$

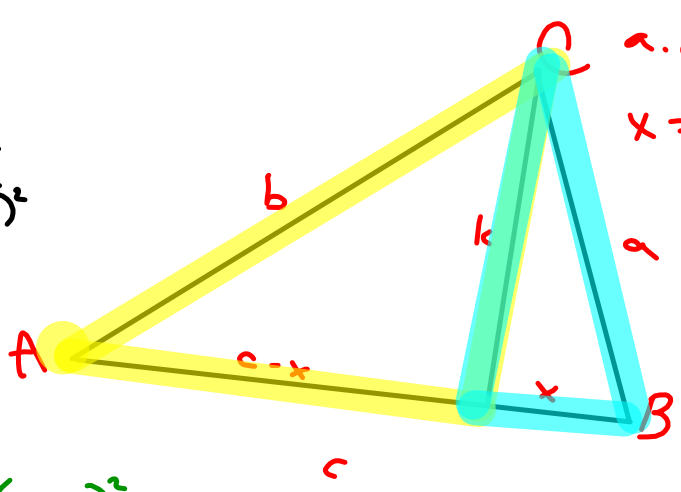
$k^2=b^2-(c-x)^2$

$k^2+x^2=a^2$

$-x^2 \quad -x^2$

$k^2=a^2-x^2$

$a^2-x^2=b^2-(c-x)^2$




$a \cdot \cos B = \frac{x}{a}$

$x = a \cdot \cos B$

$$a^2 - x^2 = b^2 - (c-x)^2$$

$$a^2 - x^2 = b^2 - (c^2 - 2cx + x^2)$$

$$a^2 - x^2 = b^2 - c^2 + 2cx - x^2$$

$$+c^2 - 2cx + x^2 \quad +c^2 - 2cx + x^2$$

$$a^2 + c^2 - 2cx = b^2$$

$$a^2 + c^2 - 2c \cdot a \cdot \cos B = b^2$$

$c - x$

| | $c$ | $-x$ |
|---|---|---|
| $c$ | $c^2$ | $-cx$ |
| $-x$ | $-cx$ | $x^2$ |

$c^2 - 2cx + x^2$

$$b = \sqrt{a^2 + c^2 - 2ac \cdot \cos(B)}$$

$$a = \sqrt{b^2 + c^2 - 2bc \cdot \cos(A)}$$

$$c = \sqrt{b^2 + a^2 - 2ba \cdot \cos(C)}$$

$$b^2 = \sqrt{a^2 + c^2 - 2ac \cdot \cos(B)}^2$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$

$$-a^2 \quad -a^2 - c^2$$

$$-c^2$$

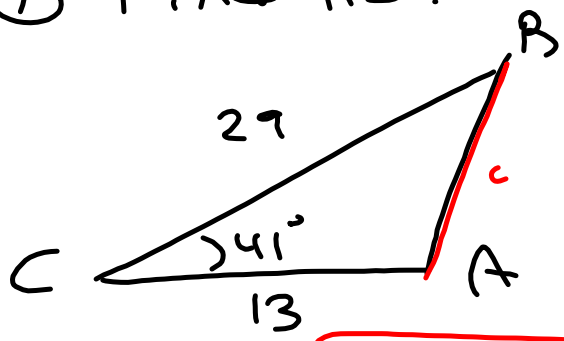
$$\frac{(b^2 - a^2 - c^2)}{-2ac} = \frac{-2ac \cdot \cos B}{-2ac}$$

$$\cos^{-1}(\cos(B)) = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$B = \cos^{-1}\left(\frac{(a^2 + c^2 - b^2)}{(2ac)}\right)$$

SAS

① Find AB.

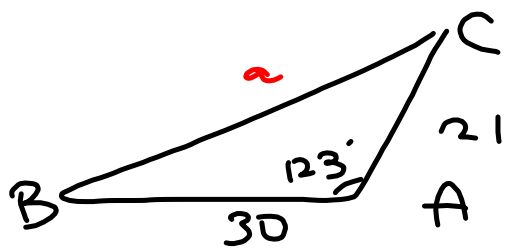



$$c = \sqrt{a^2 + b^2 - 2ab \cdot \cos(C)}$$

$$= \sqrt{(29^2 + 13^2 - 2 \cdot 29 \cdot 13 \cdot \cos(41))}$$

$$= 21.0$$

② Find BC. a

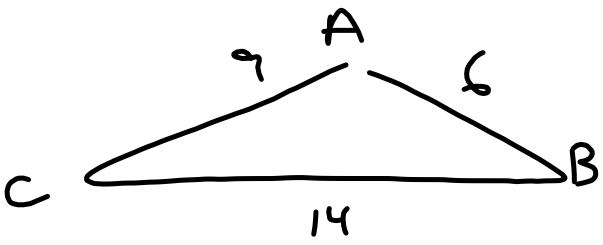



$a = \sqrt{(21^2 + 30^2 - 2 \cdot 21 \cdot 30 \cdot \cos 123°)}$

$a = 45.0$

SSS

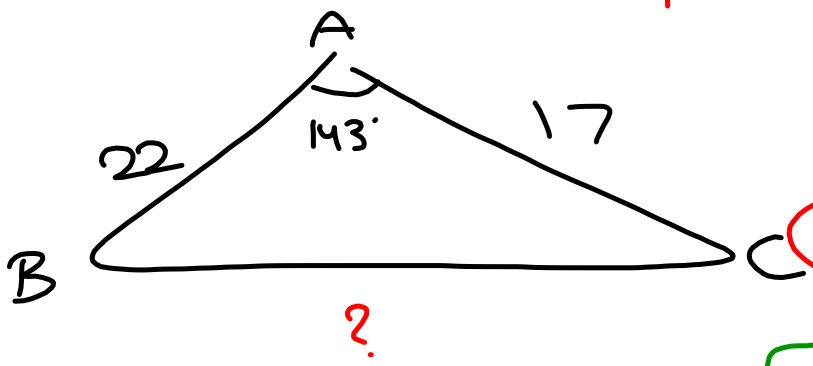
① $m\angle A$




$$A = \cos^{-1}\left(\frac{(9^2+6^2-14^2)}{(2\cdot 9\cdot 6)}\right)$$

$$A = 137.0^\circ$$

(8) Find $m\angle B$



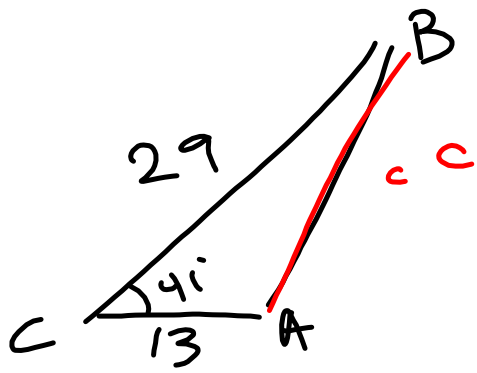

$B = \cos^{-1}\left(\frac{37^2 + 22^2 - 17^2}{2 \cdot 37 \cdot 22}\right)$

$B = 16.1°$

$a = \sqrt{17^2 + 22^2 - 2 \cdot 17 \cdot 22 \cdot \cos(143)}$

$a = 37.0$

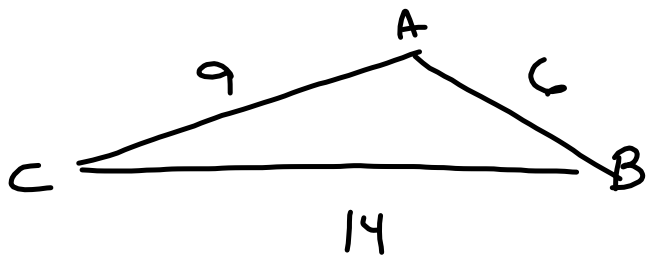
(1) Find AB := c



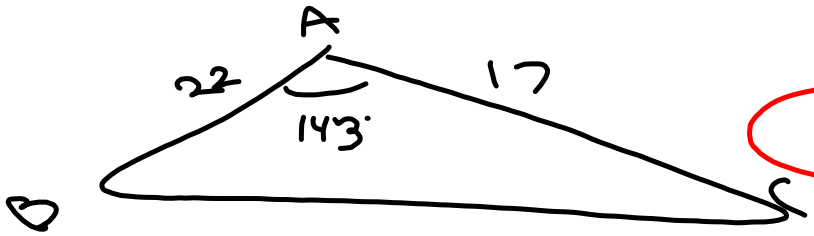

$c = \sqrt{(29^2 + 13^2 - 2 \cdot 29 \cdot 13 \cos(41))}$

$c = 21.0$

(7) Find $m\angle A$.



⑧ Find $\angle B$.



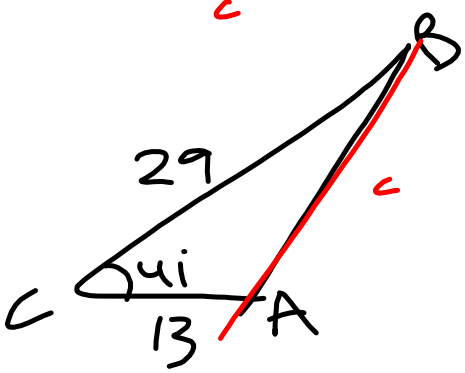

$B = \cos^{-1}\left(\frac{37^2 + 22^2 - 17^2}{2 \cdot 37 \cdot 22}\right)$

$B = 16.1°$

$a = \sqrt{17^2 + 22^2 - 2 \cdot 17 \cdot 22 \cdot \cos(143)}$

$a = 37.0$

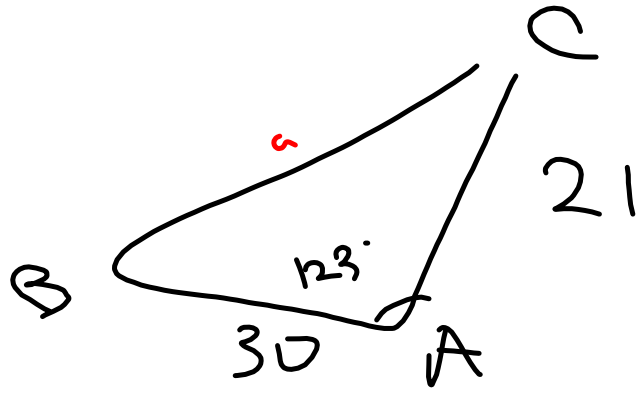
① Find ~~AB~~ c




$c = \sqrt{(29^2 + 13^2 - 2 \cdot 29 \cdot 13 \cos(41)}$

$c = 21.0$

② Find ~~BC~~ a.




$a = \sqrt{21^2 + 30^2 - 2(21)(30)\cdot\cos(123^\circ)}$

$a = 45.0$