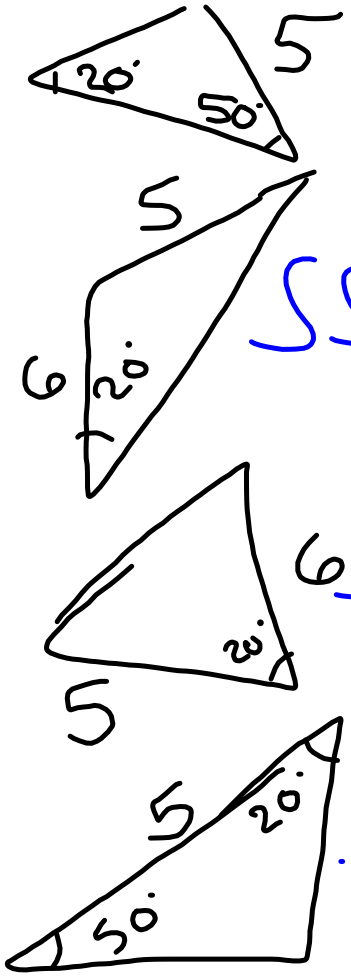
Warm-up

April 28, 2017

Match each triangle with its sides and angles.



AAS

SSA

SAS

ASA

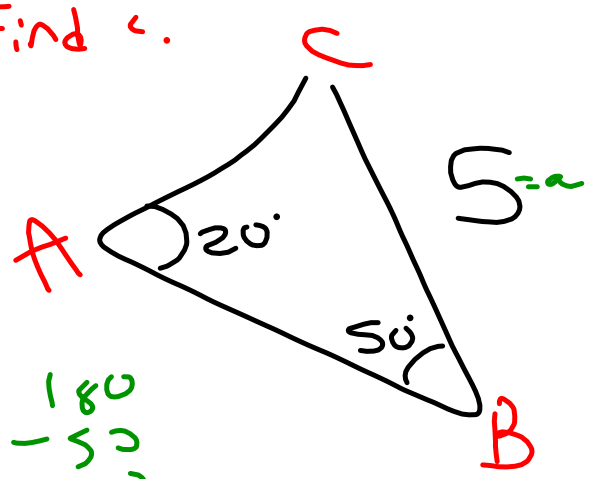
SAS

ASA

AAS

SSA

Find c.



$180 - 50 - 20 = 110°$

AAS

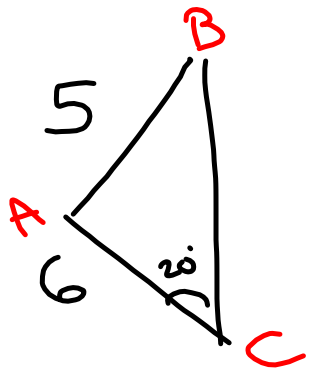
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin(20°)}{5} = \frac{\sin(110°)}{c}$

$c\sin(20) = 5\sin(110)$

$\frac{c\sin(20)}{\sin(20)} = \frac{5\sin(110)}{\sin(20)}$

$c = 13.7$



Find $m\angle B$.

SSA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

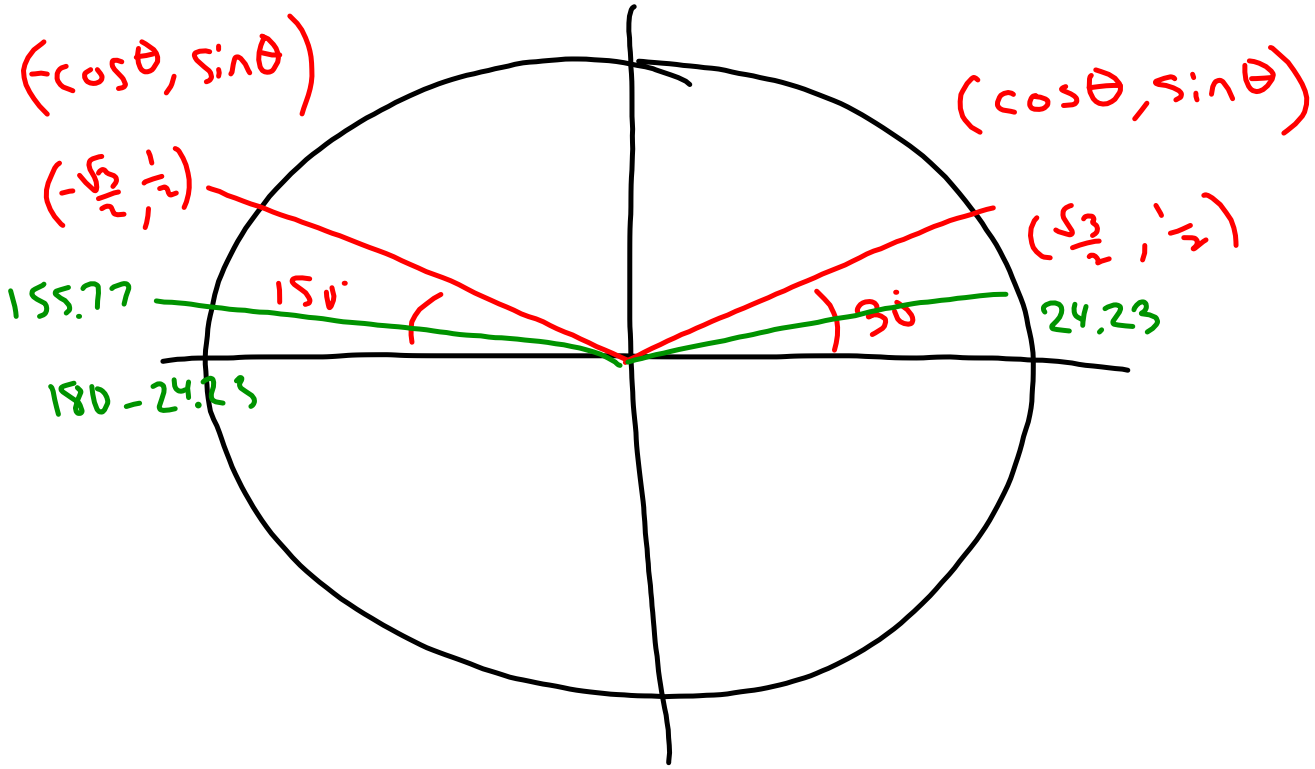
$$\frac{\sin B}{6} = \frac{\sin(20^\circ)}{5}$$

$$\frac{5\sin(B)}{5} = \frac{6\sin(20)}{5}$$

$$\sin(B) = \frac{6\sin(20)}{5}$$

$$B = \sin^{-1}\left(\frac{6\sin(20)}{5}\right)$$

$$B = 24.23^\circ$$

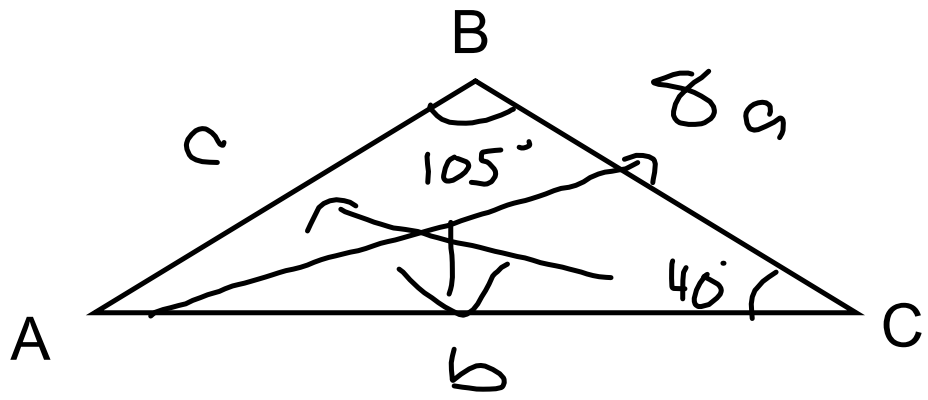

(-cosθ, sinθ)
(cosθ, sinθ)
(-√3/2, 1/2)
(√3/2, 1/2)
155.77
15°
30°
24.23
180 - 24.23

Warm-up

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

May 1, 2017

Solve the triangle. (Find ev'thing.)



$$\frac{\sin 35}{8} = \frac{\sin 105}{13.47} = \frac{\sin 40}{C?}$$

$$\frac{\sin 105}{13.47} \neq \frac{\sin 40}{C}$$

$$\frac{C \sin 105}{\sin 105} = \frac{13.47 \sin 40}{\sin 105}$$

$C = 8.96$ or 9

$$\frac{\sin A^{35}}{8} = \frac{\sin B^{105}}{b\,13.47} = \frac{\sin C^{40}}{c}$$

$$\sin(35) \neq \sin 105$$

$$\frac{b\sin(35)}{\sin 35} = \frac{8\sin 105}{\sin 35}$$

$$b = 13.47.$$

$$\frac{\sin 35}{8} = \frac{\sin 40}{c}$$

$$\frac{c \sin 35}{\sin 35} = \frac{8 \sin(40)}{\sin 35}$$

$$c = 8.97$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(35)}{8} = \frac{\sin(105)}{b}$$

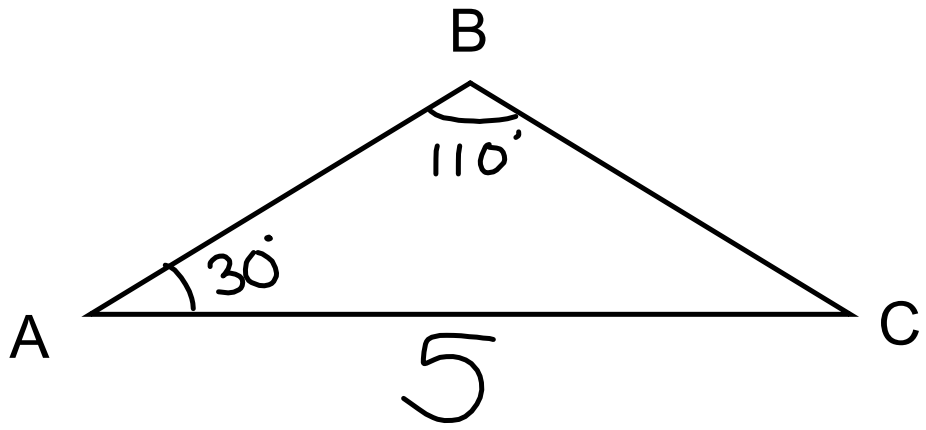
$$\frac{b \cdot \sin(35)}{\sin(35)} = \frac{8 \cdot \sin(105)}{\sin(35)}$$

$$b = 13.4$$

$$\frac{\sin(35)}{8} = \frac{\sin(40)}{c}$$

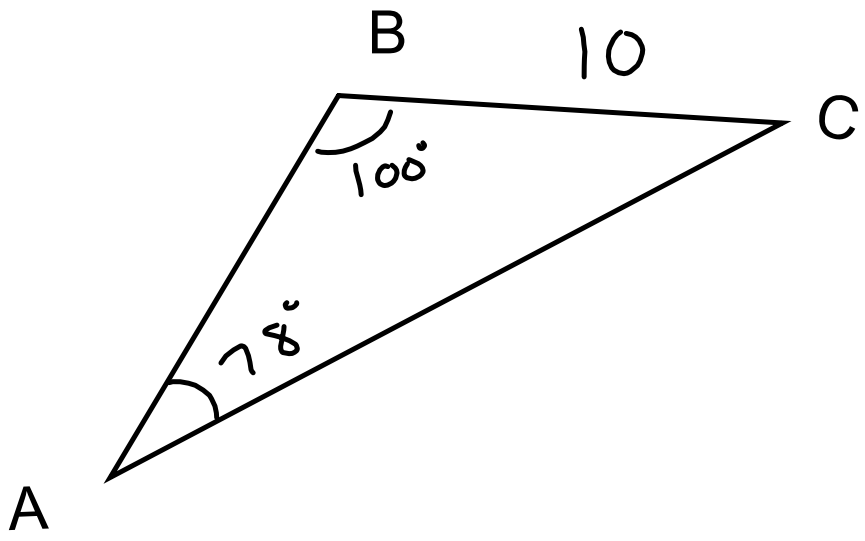
$$\frac{c \cdot \sin(35)}{\sin(35)} = \frac{8 \cdot \sin(40)}{\sin(35)}$$

$$c = 8.96$$


B
110°
30°
A
C
5

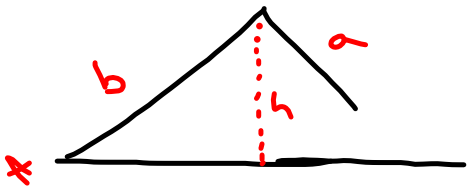

I'm not angry anymore because it's time to celebrate learning.

Solve the triangle.



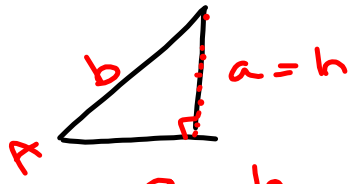
$b \cdot \frac{height}{b} = b\sin A$

$height = b\sin A$



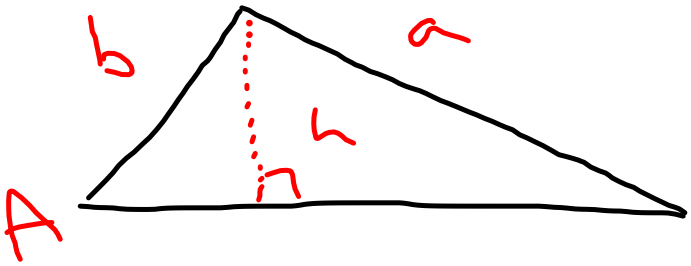
$a < h$

none



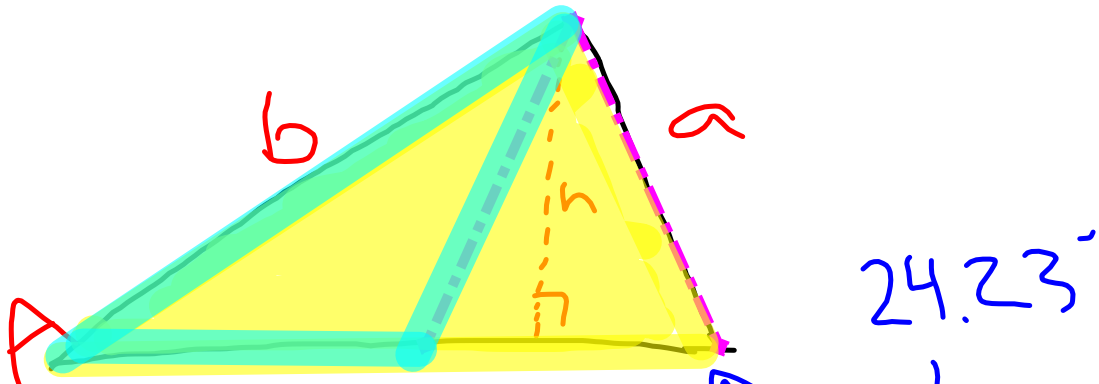
$a = h$

one



$h < b < a$

one



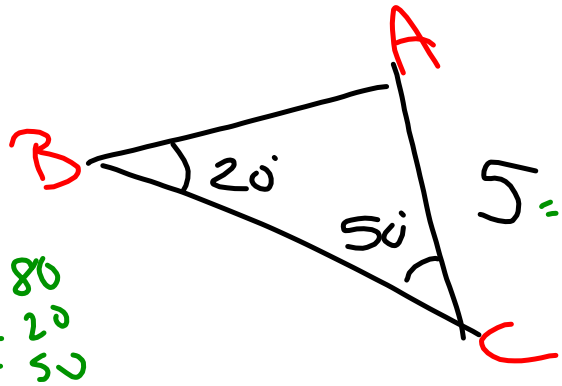
$h < a < b$

two

Find a.

AAS

LOS



$180 - 20 - 50 = 110°$

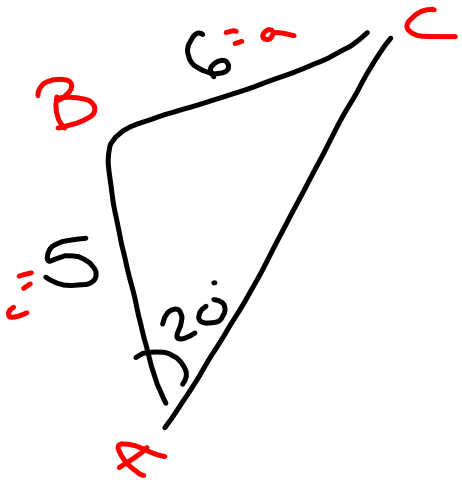
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin(110)}{a} = \frac{\sin(20)}{5}$

$\frac{5\sin(110)}{\sin(20)} = \frac{a \cdot \sin(20)}{\sin(20)}$

$a = 13.7$

Find m∠C.



SSA

AS S.A.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

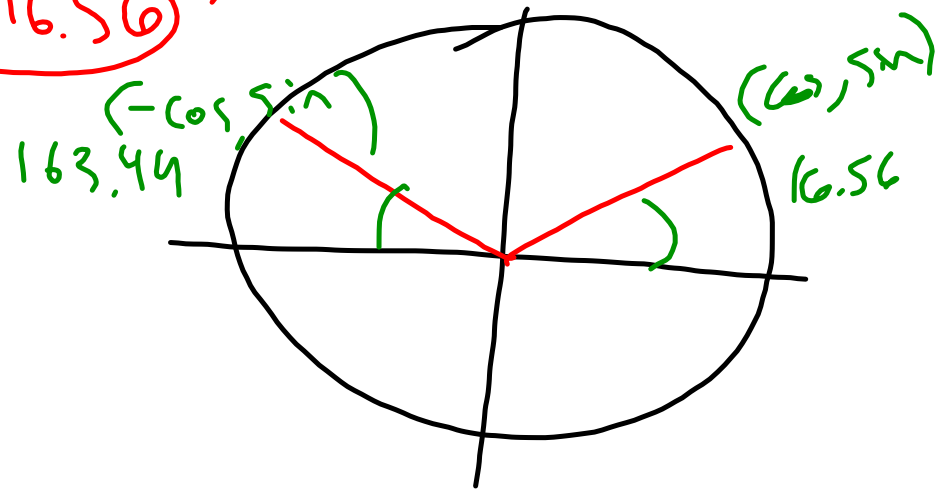
$\frac{\sin(20)}{6} = \frac{\sin C}{5}$

$5 \cdot \sin(20) = 6 \sin(C)$

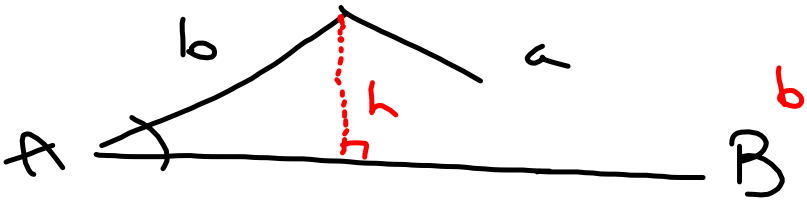
$\frac{5 \cdot \sin(20)}{6} = \sin(C)$

$\sin^{-1}\left(\frac{5\sin(20)}{6}\right) = C$

$C = 16.56$



A is $< 90°$



$b \cdot \sin A = \frac{h}{b} \cdot b$

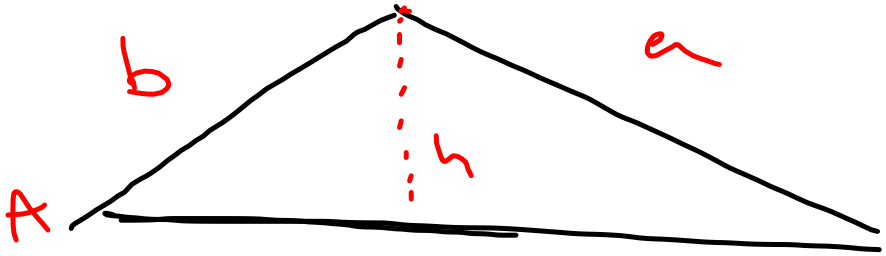
$h = b \sin A$

$a < h$

no solution

$a = h$

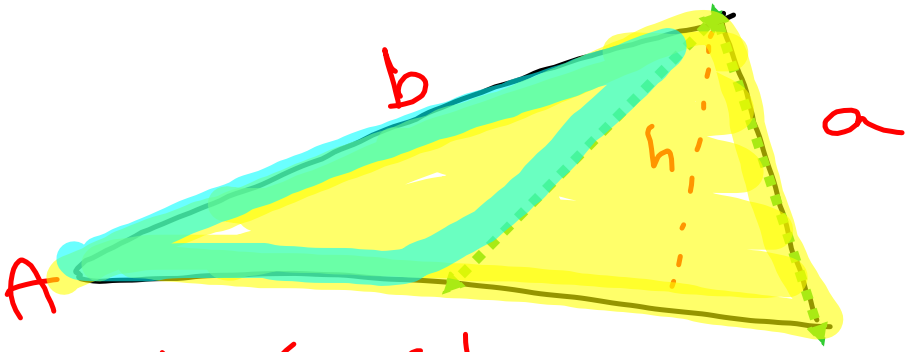
one solution



$a > b$

$h < b < a$

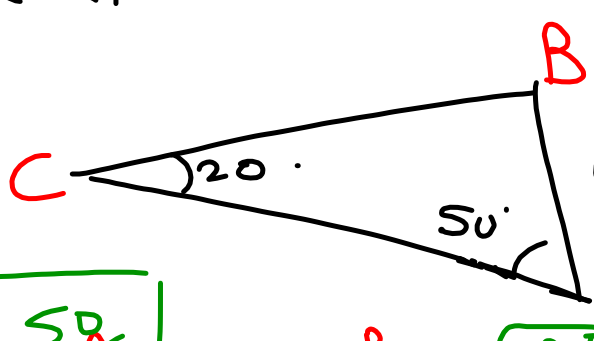
one solution



$h < a < b$

two solutions

Find a.



LS AAS SSA ASA

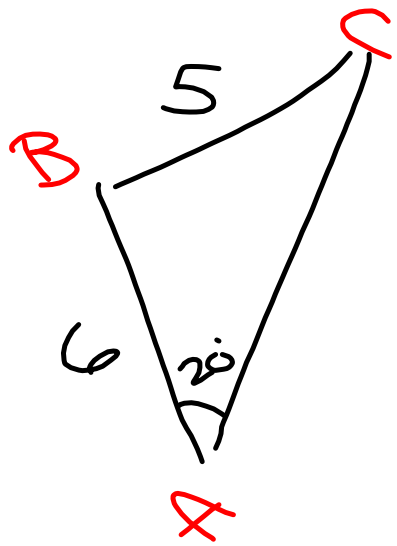
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin(50^\circ)}{a} = \frac{\sin(20^\circ)}{5}$

$a = 11.19$

$\frac{5 \cdot \sin(50)}{\sin(20)} = \frac{a \cdot \sin(20)}{\sin(20)}$

Find $m\angle C$.



$\frac{\sin 20}{5} = \frac{\sin B}{b} = \frac{\sin C}{6}$

$\frac{\sin(20)}{5} = \frac{\sin(C)}{6}$

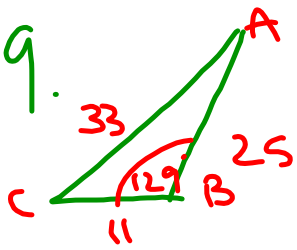
$\frac{5\sin(C)}{5} = \frac{6\sin(20)}{5}$

$\sin(C) = \left(\frac{6\sin(20)}{5}\right)$

$\sin^{-1}$ $\sin^{-1}$

$C = 24.23^\circ$

9.



$$\frac{\sin A}{11} = \frac{\sin 129}{33} = \frac{\sin C}{25}$$

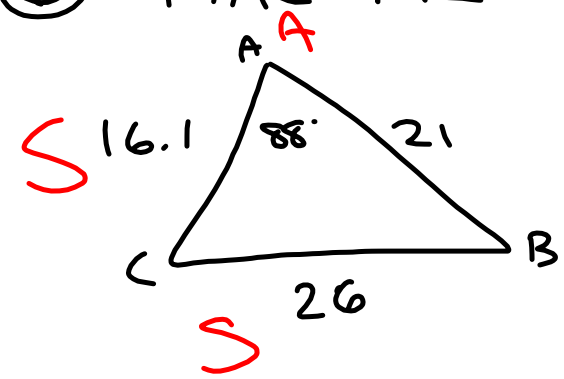
$$\frac{\sin A}{11} = \frac{\sin 129}{33} \qquad 33\sin A = 11\sin 129$$

$$\sin A = \frac{11\sin 129}{33} \qquad \sin^{-1}$$

$$A = 15^\circ$$

A

⑥ Find m∠C.



S S

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

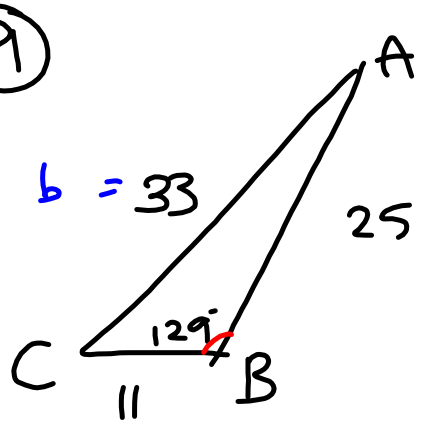
$\frac{\sin(88^\circ)}{26} = \frac{\sin C}{21}$

$\sin^{-1}\left(\frac{21 \cdot \sin(88)}{26}\right) = \frac{26 \cdot \sin(C)}{26}$

$\sin^{-1}(\sin C) = C$

$C = 53.8^\circ$

(9)

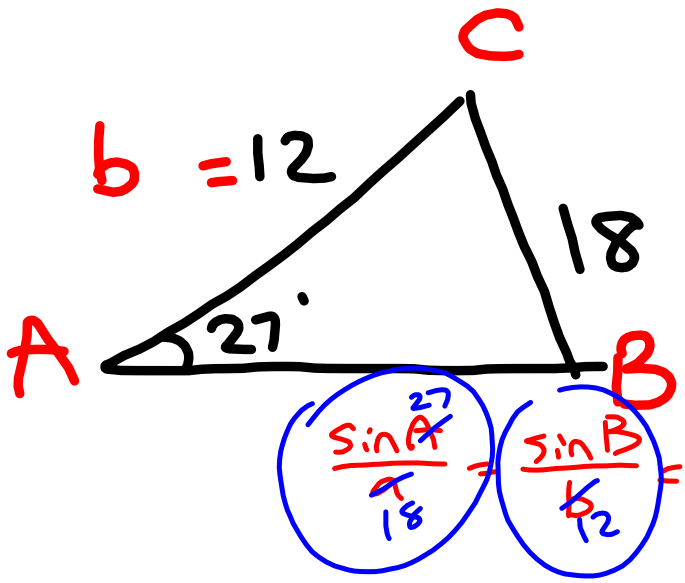


$$\frac{\sin A}{11} = \frac{\sin B}{33} = \frac{\sin C}{25}$$

$$\frac{33 \cdot \sin A}{11} = \frac{11 \sin(129)}{33}$$

$$\frac{33 \cdot \sin A}{33} = \frac{11 \cdot \sin(129)}{33}$$

$$A = 15.013$$



Find m∠B.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 27}{18} = \frac{\sin B}{12}$

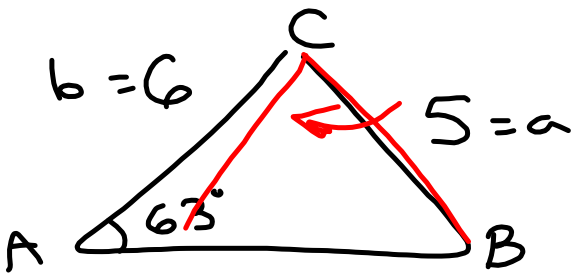
$\frac{12 \sin 27}{18} = \frac{18 \cdot \sin B}{18}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{12 \cdot \sin 27}{18}\right)$

$B = 17.6°$

m∠B = 17.6°

Find mLB.



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

SSA

Hinge Theorem

"The Donkey Case"

DOMAIN ERROR

NO solution

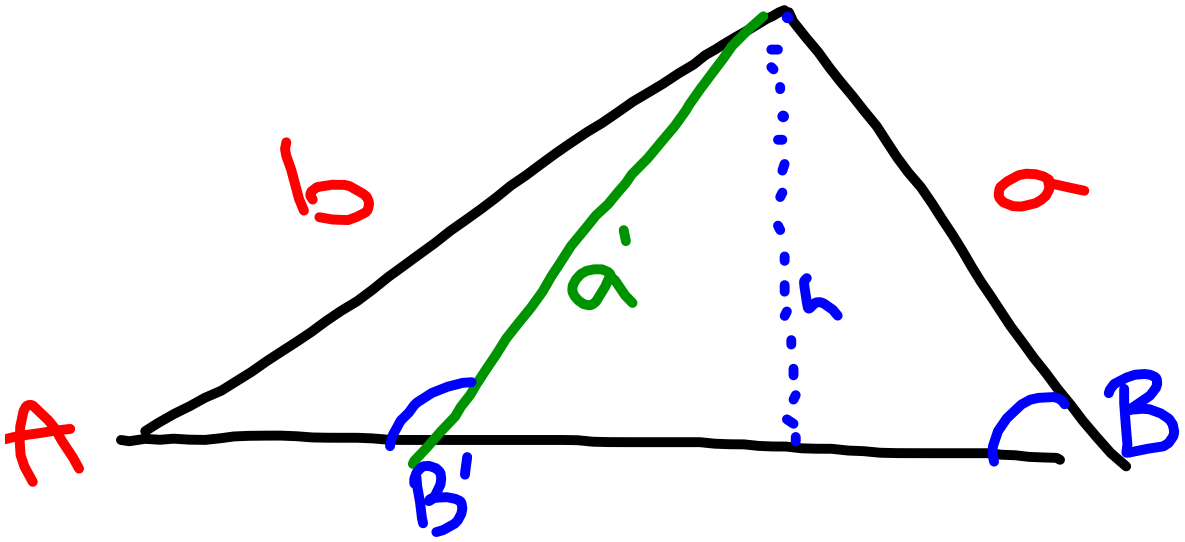
SSA

1. Find the height.

h = b*sinA

2. Order h, a, b from least to greatest.

3. Use the Law of Sines to solve.



(22) $A = 47^\circ, a = 25, b = 34$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(47)}{25} = \frac{\sin B}{34}$$

$$\frac{34\sin(47)}{25} = \sin B$$

$$B = 84.07^\circ$$

$$C = 180 - A - B$$
$$= 180 - 47 - 84.07$$
$$= 48.93^\circ$$

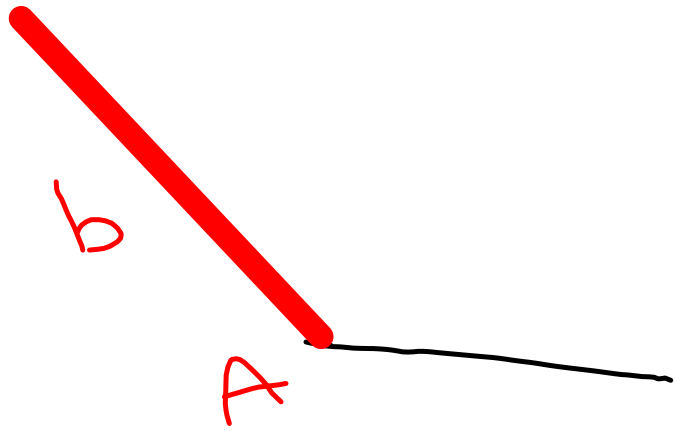
$$c = \frac{a \cdot \sin C}{\sin A} = \frac{25 \cdot \sin(48.93)}{\sin(47)} = 25.77$$

$$B' = 180 - B$$
$$= 180 - 84.07$$
$$= 95.93^\circ$$

$$C' = 180 - A - B'$$
$$= 180 - 47 - 95.93^\circ$$
$$= 37.07^\circ$$

$$c' = \frac{a \cdot \sin C'}{\sin A} = \frac{25 \cdot \sin(37.07)}{\sin(47)}$$

$$= 20.61$$


b
A

(21) $A = 61°$, $a = 14$, $b = 15$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{15\sin(61)}{14} = \frac{14\sin B}{14}$$

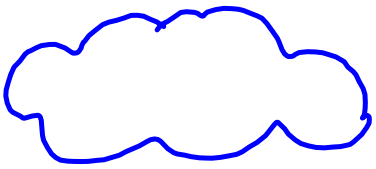
$$B = \sin^{-1}\left(\frac{15\cdot\sin(61)}{14}\right) = 69.56°$$

$$C = 180 - A - B$$
$$= 180 - 61 - 69.56$$
$$= 49.44°$$

$$c = \frac{a\cdot\sin C}{\sin A}$$
$$= \frac{14\sin(49.44)}{\sin(61)}$$
$$= 12.16$$

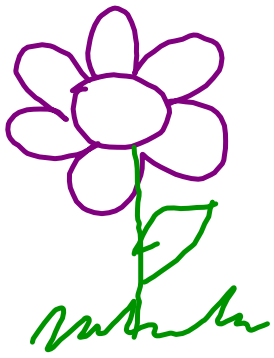
$B' = 180 - B = 180 - 69.56$
$= 110.44°$

$C' = 180 - A - B'$
$= 180 - 61 - 110.44$
$= 8.56°$



$c' = \frac{a \cdot \sin C'}{\sin A}$

$= \frac{14 \sin(8.56)}{\sin(61)}$



$= 2.36$

(17) $a = 10$, $b = \sqrt{200}$, $A = 45^\circ$

$$\sin^{-1}\frac{\sin 45}{10} = \frac{\sin B}{\sqrt{200}}$$

$$B = \sin^{-1}\left(\frac{\sqrt{200}\sin 45}{10}\right)$$

$B = 90$

(16) $a = 5, b = 6, A = 63°$

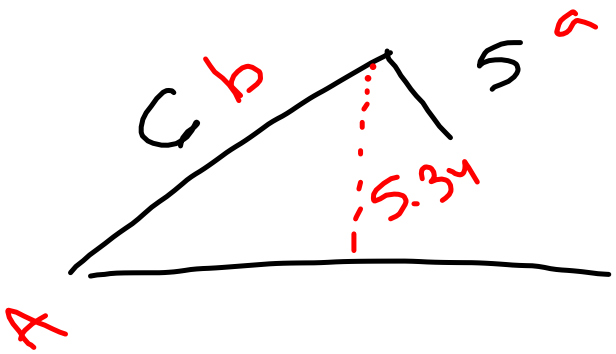
$h = b \cdot \sin A$

$h = 6 \sin(63) = 5.34$

$5 < 5.34 < 6$

$a < h < b$

none



(24) $A = 18^\circ$, $a = 8$, $b = 13$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(18)}{8} = \frac{\sin B}{13}$$

$$\left(\frac{13\sin(18)}{8}\right) = \frac{8(\sin(B))}{8}$$

$\sin^{-1}$

$B = 30.14^\circ$

$B' = 149.86^\circ$

$C = 180 - 18 - 30.14$

$C = 131.86$

$C' =$

(23) $A = 54°$, $a = 31$, $b = 36$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(54)}{31} = \frac{\sin B}{36}$$

$$\left(\frac{36\sin(54)}{31}\right) = \sin B$$

$$\sin^{-1}$$

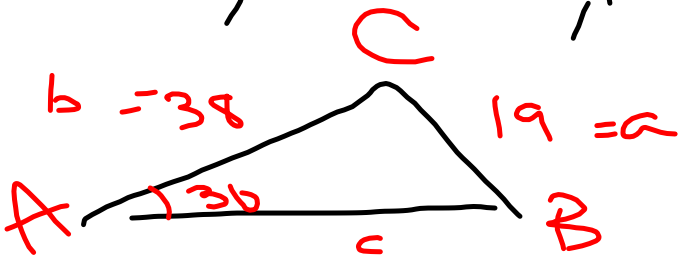
$$B = 69.97°$$

$$B' = 180 - 69.97 = 110.03°$$

$$C = 180 - A - B \qquad c = \frac{a\sin C}{\sin A}$$

$$C' = 180 - A - B' \qquad c' = \frac{a\sin C'}{\sin A}$$

(15) $a = 19, b = 38, A = 30°$



(1) $h = b \cdot \sin A$

$= 38 \sin 30$

$= 19$

(2) $19 = 19 < 38$ Right One Sol.

$a = h < b$

(3)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

30, 19, 38, 90°, 60

$$\frac{\sin(30)}{19} = \frac{\sin(B)}{38}$$

$$\frac{38 \cdot \sin(30)}{19} = \frac{19 \sin(B)}{19}$$

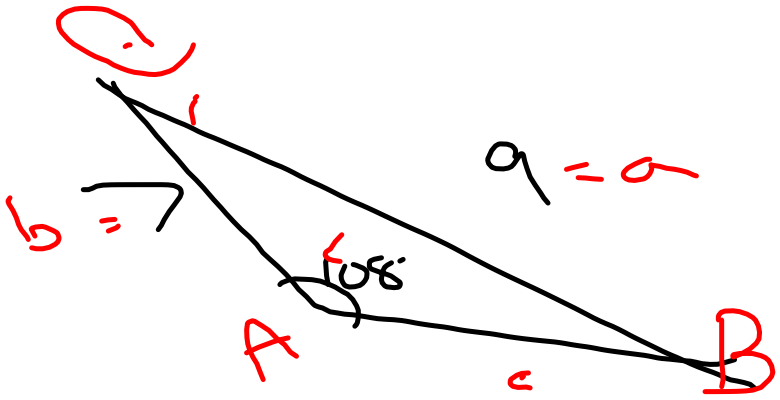
$$\sin^{-1}(\sin(B)) = \sin^{-1}\left(\frac{38\sin(30)}{19}\right)$$

$B = 90°$

$$\frac{\sin(30)}{19} = \frac{\sin(60)}{c}$$

$c = 19\sqrt{3}$

10 $a = 9, b = 7, A = 108°$



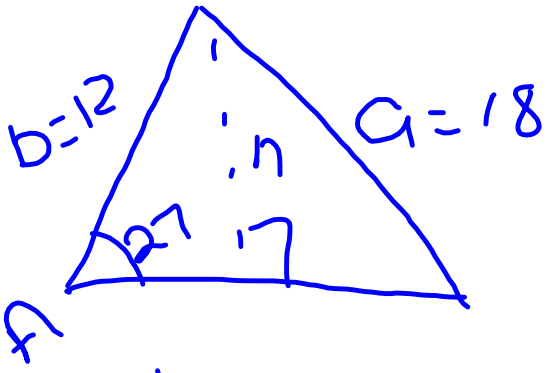
$7 < 9$ one solution

$b < a$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Find c.

$a = 18 \quad b = 12, A = 27°$



$12 < 18$

$b < a$

27 one solution

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(27)}{18} = \frac{\sin B}{12}$$

$$\sin^{-1}\left(\frac{12\sin(27)}{18}\right) = 17.62$$

$B = 17.62°$

10) $a = 9$, $b = 7$, $A = 108°$

ASS

obtuse

$7 < 9$

$b < a$

one solution

$\frac{\sin A}{9} = \frac{\sin B}{7} = \frac{\sin C}{c}$ 24.3

$\frac{7 \cdot \sin(108)}{9} = \frac{9 \cdot \sin B}{9}$

$$\sin^{-1}\left(\frac{7\sin(108)}{9}\right) = \sin B$$

$$B = 47.7^\circ$$

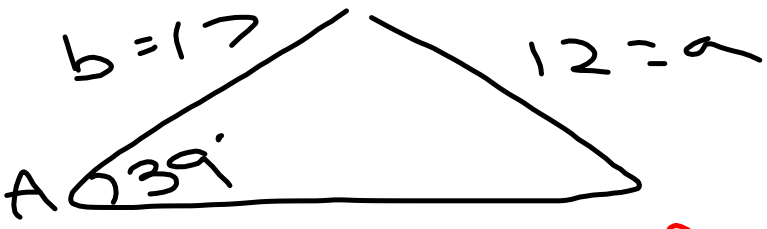
$$180 - 108 - 47.7 = 24.3^\circ$$

$$\frac{\sin(108)}{9} = \frac{\sin(24.3)}{c}$$

$$\frac{c\sin(108)}{\sin(108)} = \frac{9\sin(24.3)}{\sin(108)}$$

$$c = 3.89$$

19 $A = 39°$, $a = 12$, $b = 17$



① $h = b \cdot \sin A$
$= 17 \cdot \sin 39$
$= 10.69$

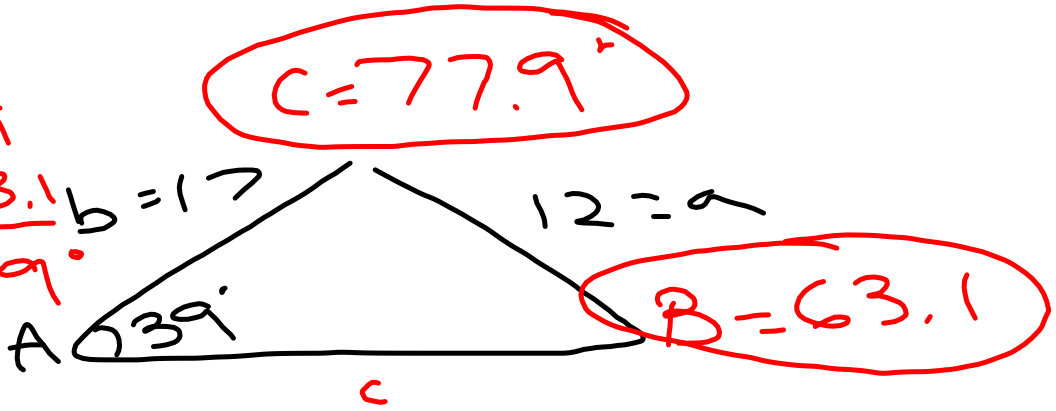
② $10.69 < 12 < 17$
$h < a < b$

2 solutions

③ $m\angle B = 63.1°$

$B' = 180 - B = 180 - 63.1 = 116.9°$

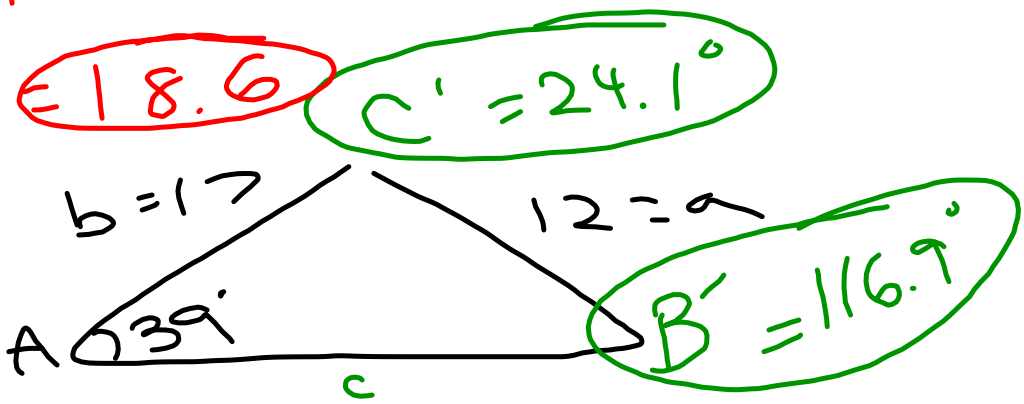
180
−39
−63.1
77.9°



$$\frac{\sin 39}{12} = \frac{\sin 77.9}{c}$$

$c = 18.6$

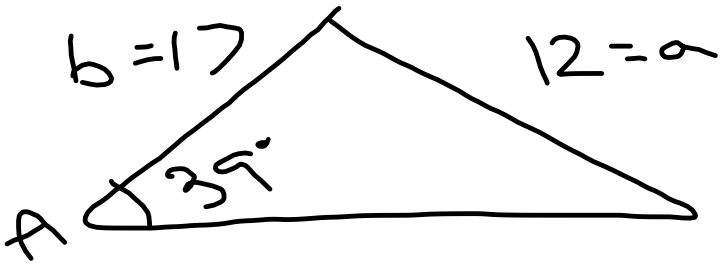
180
−116.9
−39
24.1



$$\frac{\sin 39}{12} = \frac{\sin 24.1}{c'}$$

$C' = 7.8$

19 $A = 39°$, $a = 12$, $b = 17$



1. $h = b \cdot \sin A$
$= 17 \cdot \sin(39°)$
$= 10.69$

2. $10.69 < 12 < 17$
$h < a < b$

2 solutions

3. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 39}{12} = \frac{\sin B}{17}$

$\sin B = \frac{17 \cdot \sin(39)}{12}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{17 \cdot \sin(39)}{12}\right)$

$B = 63.1°$

$B' = 180 - 63.1 = 116.9°$

(19) $A = 39°$, $a = 12$, $b = 17$

ASS

① height = $b \cdot \sin A$

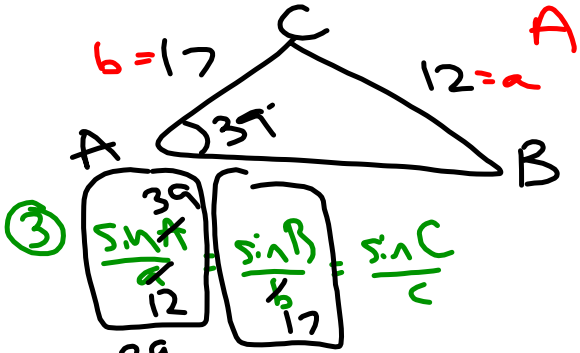
$17 \cdot \sin(39)$

$h = 10.69$

② $h < a < b$

$10.69 < 12 < 17$

2 solutions



③ $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 39}{12} = \frac{\sin B}{17}$

$\sin B = \frac{17 \cdot \sin(39)}{12}$

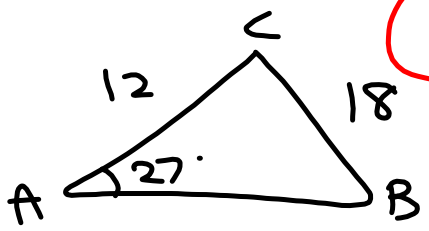
$B = 63.1°$

$B' = 180 - B$

$= 180 - 63.1$

$B' = 116.9°$

Find m∠B.



$A = 27°, a = 18, b = 12$

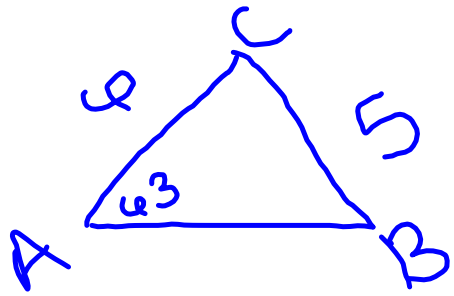
1) $\frac{\sin A 27°}{18} = \frac{\sin B}{12}$

$= \frac{\sin C}{C}$ 2) $\frac{\sin 27}{18} = \frac{\sin B}{12}$

3) $\sin^{-1}\left(\frac{12 \cdot \sin(27)}{18}\right)$

$B = 17.6$

(16) $a=5$, $b=6$, $A=63^\circ$ Find $m\angle B$.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 63}{5} = \frac{\sin B}{6}$$

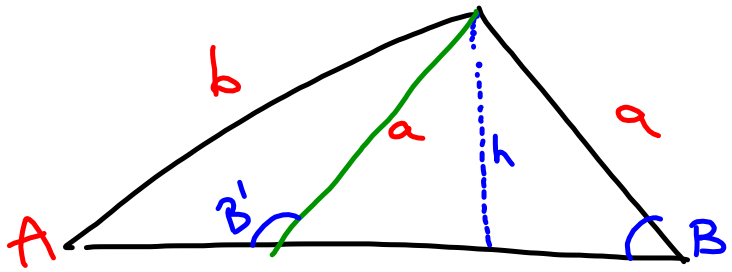
$$\sin^{-1}\left(\frac{\sin(63)\cdot 6}{5}\right) = \frac{\sin B \cdot 5}{5}$$

DOMAIN ERROR

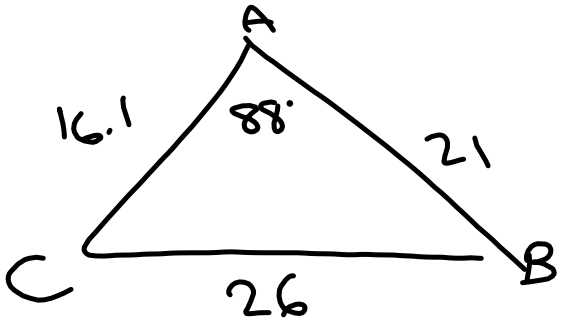
no solutions

ASA
AAS
AS

Hinge Theorem
"Donkey Case"


b
a
h
q
A
B'
B

⑥ Find m∠C.



$\frac{\sin A}{26} = \frac{\sin B}{16.1} = \frac{\sin C}{21}$

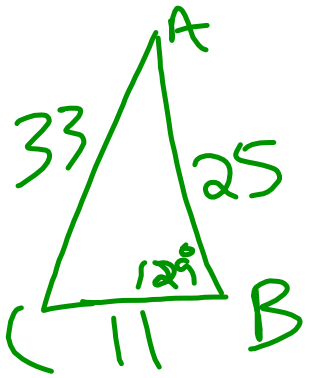
$\frac{\sin 88}{26} = \frac{\sin C}{21}$

$\frac{21\sin(88)}{26} = \frac{26\sin(C)}{26}$

$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{21\cdot\sin(88)}{26}\right)$

$C = 53.6°$

(9) Find m∠A

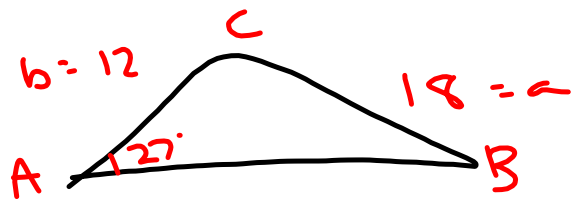


$$\frac{\sin A}{11} = \frac{\sin 129}{33} = \frac{\sin C}{25}$$

$$\frac{\sin A}{11} = \frac{\sin 129^\circ}{33}$$

$$= 15$$

(12) m∠B $a = 18, b = 12, A = 27°$



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

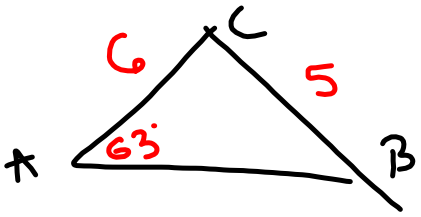
$$\frac{\sin(27)}{18} = \frac{\sin B}{12}$$

$$\frac{18\sin B}{18} = \frac{12\sin(27)}{18}$$

$$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{12\sin(27)}{18}\right)$$

$$B = 17.6$$

(16) $a=5$, $b=6$, $A=63^\circ$ Find $m\angle B$.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

DOMAIN ERROR

No Solution

ASS

Law of Sines

ASA

AAS

SSA → Hinge Theorem

"Donkey Case"