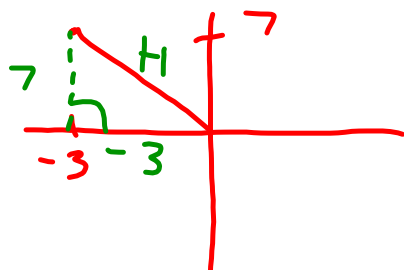


Warm-up

March 27, 2017

Find $\sec \theta$ for the angle whose terminal side passes through the point $(-3, 7)$.



$$\cos \theta = \frac{A}{H}$$

$$\sec \theta = \frac{H}{A} = \frac{\sqrt{58}}{-3}$$

$$7^2 + (-3)^2 = c^2$$

$$49 + 9 = c^2$$

$$\sqrt{58} = \sqrt{c^2}$$

$$\sqrt{58}$$

$$\begin{array}{c} \sqrt{58} \\ \swarrow \quad \searrow \\ 2 \quad 29 \end{array}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Which of the following angles is coterminal to $\theta = -\frac{20\pi}{12}$?

$$\pm 2\pi$$

Positive

$$-\frac{20\pi}{12}$$

$$+\frac{24\pi}{12}$$

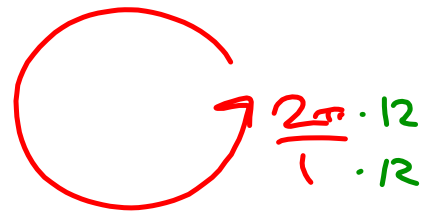
$$\frac{4\pi}{12} = \frac{\pi}{3}$$

Negative

$$-\frac{20\pi}{12}$$

$$-\frac{24\pi}{12}$$

$$\frac{-44\pi}{12} = \frac{-11\pi}{3}$$



$$\frac{24\pi}{12}$$

$$\frac{-20\pi}{12} \left(\frac{180i}{\pi} \right) = -300i$$

Pos

$$-300i + 360i$$

$$60i \left(\frac{\pi}{180i} \right)$$

$$\frac{\pi}{3}$$

Neg

$$-300 - 360$$

$$-660i$$

$$-660 \left(\frac{\pi}{180} \right)$$

$$\frac{11\pi}{3}$$

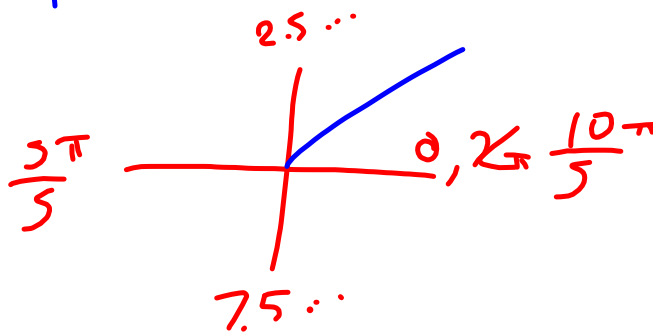
2. Find the reference angle for $\theta = \frac{21\pi}{5}$?

① $0 \leq \theta < 2\pi \cdot 5 = \frac{10\pi}{5}$

②

$\pi - A$	A
$A - \pi$	$2\pi - A$

$$\begin{array}{r} 21 \\ -10 \\ \hline 11 \\ -10 \\ \hline 1\pi \end{array}$$



$\frac{11\pi}{5}$

R → D

$$\frac{21\pi}{5} \left(\frac{180}{\pi} \right) = 756$$

$$\frac{-360}{-}$$

$$396$$

$$\frac{-360}{-}$$

$$36$$

① $0^\circ \leq \theta < 360^\circ$

②
$$\begin{array}{c|c} 180 - A & A \\ \hline A - 180 & 360 - A \end{array}$$

36

D → R

$$36 \left(\frac{\pi}{180} \right) = \left(\frac{\pi}{5} \right)$$

$$R \rightarrow D$$

$$\frac{2\pi}{5} \left(\frac{180}{\pi} \right) = 756^\circ$$

$$\textcircled{1} \quad 0^\circ \leq \theta < 360^\circ$$

36°

$$\begin{array}{r} 756 \\ - 360 \\ \hline 396 \\ - 360 \\ \hline 36 \end{array}$$

②

$180 - A$	A
$A - 180$	$360 - A$

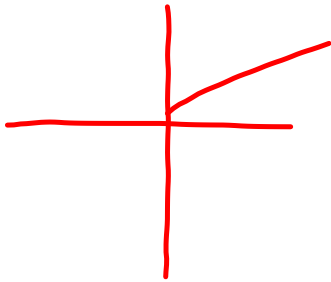
36°

$$D \rightarrow R$$

$$36 \left(\frac{2\pi}{180} \right) = \left(\frac{1\pi}{5} \right)$$

$$\frac{2\pi}{\pi} \left(\frac{180}{\pi} \right) = 756^\circ$$

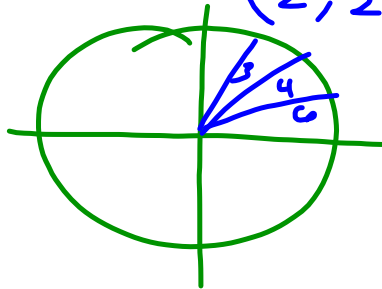
$$\begin{array}{r}
 0 < 0 < 360^\circ - \frac{360}{\pi} \\
 396 \\
 \underline{- 360} \\
 36^\circ
 \end{array}$$



$$36^\circ \left(\frac{\pi}{180} \right) = \frac{\pi}{5}$$

4. Give the exact value of $\tan \frac{13\pi}{3}$. $\textcircled{1} 0 \leq A < 2\pi$
 $\frac{6\pi}{3}$

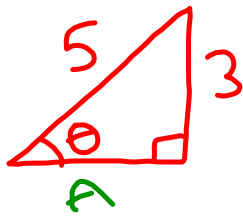
$$\tan\left(\frac{\pi}{3}\right) = \frac{y}{x} = \frac{\sqrt{3}}{1} = \sqrt{3} \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \frac{1}{\frac{1}{3}}$$



$$\frac{-6}{7} = \frac{-6}{1} \frac{1}{\frac{1}{3}}$$

$\sqrt{3}$

5. $\sin \theta = \frac{3}{5}$ find $\tan \theta$



$$\tan \theta = \frac{O}{A} = \frac{3}{4}$$

$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$b^2 = 16$$

$$b = 4$$

6. Determine the quadrant in which the terminal side of the angle $-\frac{11\pi}{9}$ lies.

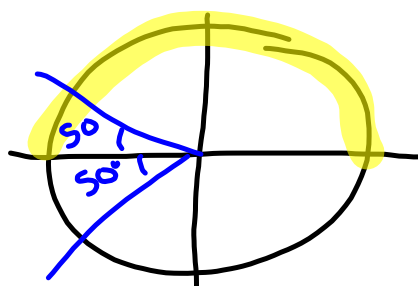
7. Find the two values of θ ($0 \leq \theta < 360^\circ$) that satisfy $\sec \theta = -1.5557$.

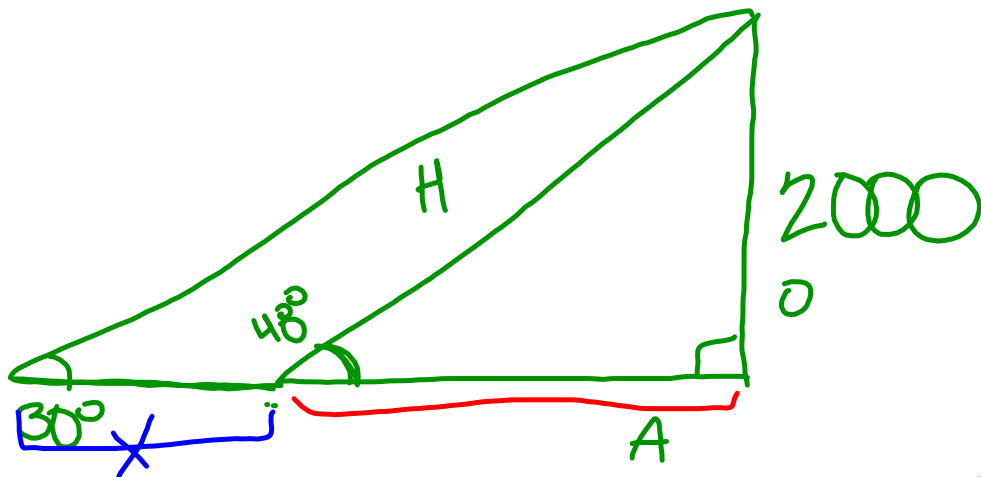
$$\cos^{-1}(\cos \theta) = \sec^{-1} \left(\frac{1}{-1.5557} \right)$$

II $\theta = 130^\circ$

$\theta' = 50^\circ$

III $\theta = 180^\circ + 50^\circ$
 $= 230^\circ$





$$A \cdot \frac{\tan(48)}{\tan(48)} = \frac{2000}{A} \quad A \div \tan(48)$$

$$A = \frac{2000}{\tan(48)} = 1800.8$$

$$"30" = 3464.1$$

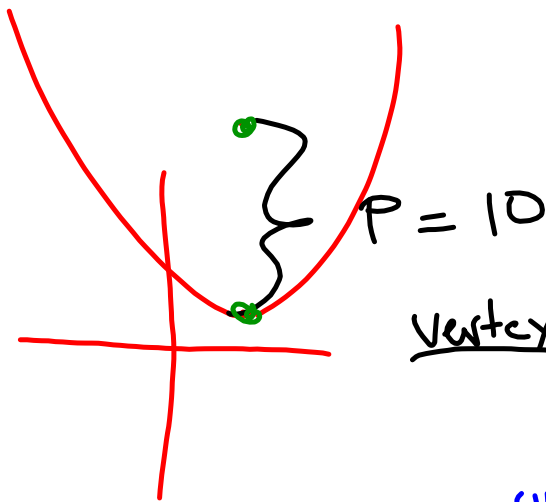
$$1663.3$$

18. Find the vertex of the parabola: $(x - 5)^2 - 7(y + 8) = 0$

(h, k)

$(5, -8)$

19. Find the equation of the parabola with vertex at $(3, 2)$ and focus at $(3, 12)$.



Vertex form

$$y = \frac{1}{4p} (x-h)^2 + k$$

$$y = \frac{1}{4(10)} (x-3)^2 + 2$$

$$y = \frac{1}{40} (x-3)^2 + 2$$

$$40 \cdot (y-2) = \frac{1}{40} (x-3)^2$$

Transformation $\rightarrow 40(y-2) = (x-3)^2$

center (6, 5)

20. Find the center of the ellipse: $x^2 + 2y^2 - 12x - 20y + 22 = 0$

constants on right. -22 -22

Group x 's $\rightarrow y$'s

$$x^2 - 12x + 2y^2 - 20y = -22$$

Factor out coeff. 1 $(x^2 - 12x + 36) + 2(y^2 - 10y + 25) = -22$

on squared term.

$$\frac{(x-6)^2}{64} + \frac{2(y-5)^2}{64} = \frac{64}{64} + \frac{36}{32}$$

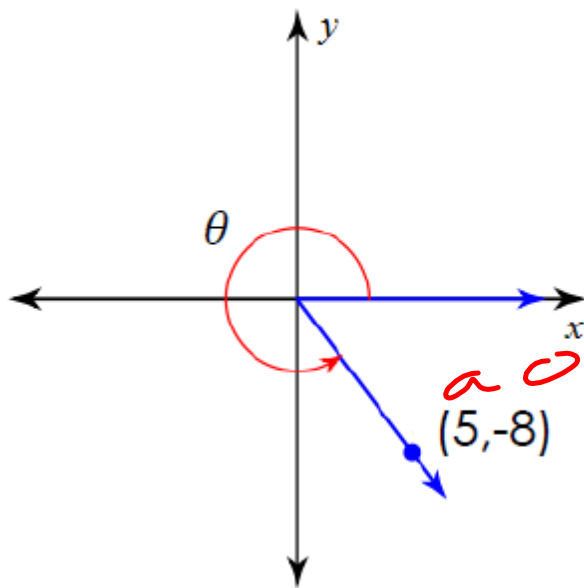
Half middle term.

Square it.

Divide both sides
by # on right.

$$\frac{(x-6)^2}{64} + \frac{(y-5)^2}{32} = 1$$

23. Find $\tan \theta$, for the angle θ shown below.




$$\tan \theta = \frac{-8}{5}$$

21. Find the standard form of the equation of the ellipse with the foci $(-\sqrt{13}, 6)$ and $(\sqrt{13}, 6)$ and a vertex $(5, 6)$.

$$\frac{x^2}{25} + \frac{(y-6)^2}{12} = 1 \quad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 5^2 = 25$ $b^2 = 12$



$a =$ Distance = 5
Vertex to center

$c =$ Distance = $\sqrt{13}$
focus to center

$$a^2 - b^2 = c^2$$

$$25 - b^2 = 13$$

$$b^2 = 25 - 13$$

$$b^2 = 12$$

center $\left(\frac{-\sqrt{13} + \sqrt{13}}{2}, \frac{6+6}{2} \right)$
 $\left(\frac{0}{2}, \frac{12}{2} \right)$
 center $(0, 6)$

12. Determine the order of the matrix: $\begin{bmatrix} 1 & 2 & 3 \\ -5 & -8 & 2 \end{bmatrix}$ Rows

Rows x Columns
2 x 3

13. Evaluate: $3 \begin{bmatrix} 2 & 7 \\ 3 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 4 \\ -1 & 5 \end{bmatrix}$

$$\begin{bmatrix} 6 & 21 \\ 9 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 8 \\ -2 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 13 \\ 11 & -10 \end{bmatrix}$$

14. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 0 \\ -1 & -1 \end{bmatrix}$, find $2B - A$.

$$\begin{aligned} & 2B - A \\ & 2 \begin{bmatrix} -5 & 0 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} \\ & \begin{bmatrix} -10 & 0 \\ -2 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} \\ & \begin{bmatrix} -12 & 1 \\ -5 & -2 \end{bmatrix} \end{aligned}$$

15. Use a graphing utility to multiply: $\begin{bmatrix} 4 & 7 & -3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ -6 & 2 \end{bmatrix}$ ↓

$$= \begin{bmatrix} 14 & 17 \\ -11 & 0 \end{bmatrix}$$

2 × 3 ✓ 3 × 2

TL	$4(-1) + 7(0) + 3(-6)$ $-4 + 0 + 18$ <hr style="width: 50%; margin: 0 auto;"/> 14	TR
BL	$-1(-1) + 0(0) + 2(-6)$ $1 + 0 - 12$ <hr style="width: 50%; margin: 0 auto;"/> -11	BR

$4(4) + 7(1) + 3(2)$	$16 + 7 + 6$	
$-1(4) + 0(1) + 2(2)$	$-4 + 0 + 4$	
	<hr style="width: 50%; margin: 0 auto;"/>	
	17	
	<hr style="width: 50%; margin: 0 auto;"/>	
	0	

16. Use matrix multiplication to determine which of the following is the inverse of $A = \begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 0 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & 2 \end{bmatrix}$$

$$\textcircled{1} \begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix} = 4(0) - (-2(1)) \\ \phantom{\textcircled{1} \begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix}} = 0 + 2$$

17. Use a determinant to find the area of a triangle with the vertices $(3, -1)$, $(4, 2)$, and $(-2, 0)$.

$$\text{Area} = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ d & e & 1 \\ g & h & 1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 3 & -1 & 1 \\ 4 & 2 & 1 \\ -2 & 0 & 1 \end{vmatrix}$$

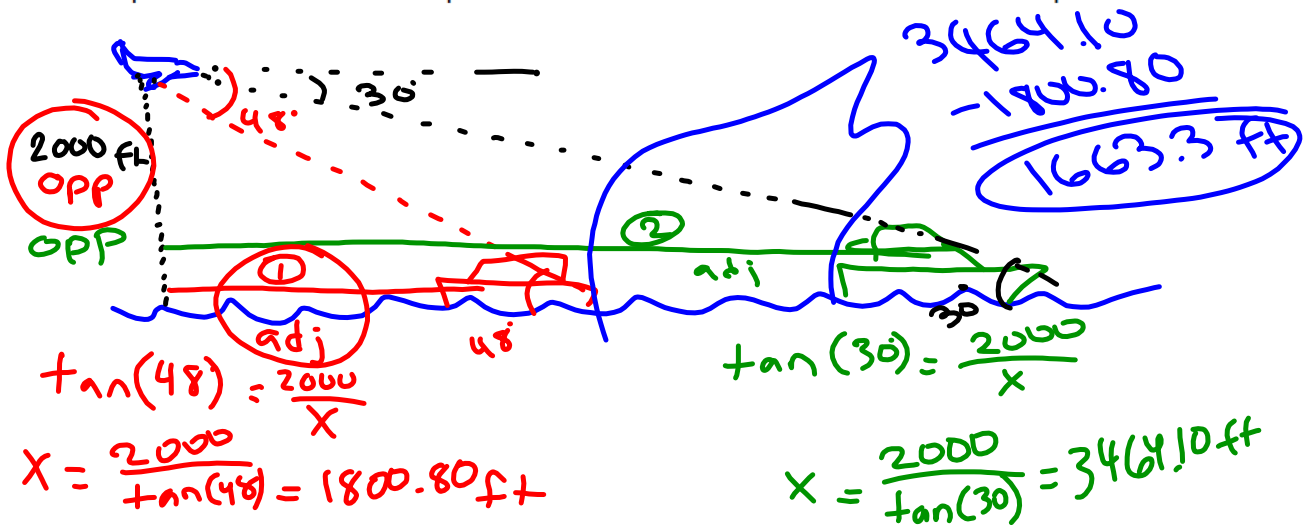
$$(\underline{6} + \underline{2} + \underline{0}) - (\underline{-4} + \underline{0} + \underline{-4})$$

$$(8) - (-8)$$

$$\frac{1}{2}(16) = \textcircled{8}$$

S O C A T A
H H T A

11. The pilot of an airplane flying at an altitude of 2000 feet sights two ships traveling in the same direction as the plane. The angle of depression of the farther ship is 30° and the angle of depression of the other ship is 48° . Find the distance between the two ships.



(22)

$$Ax^2 + Cy^2$$

$A = C$ circle

$AC > 0$ ellipse

$AC < 0$ hyperbola

$AC = 0$ parabola

