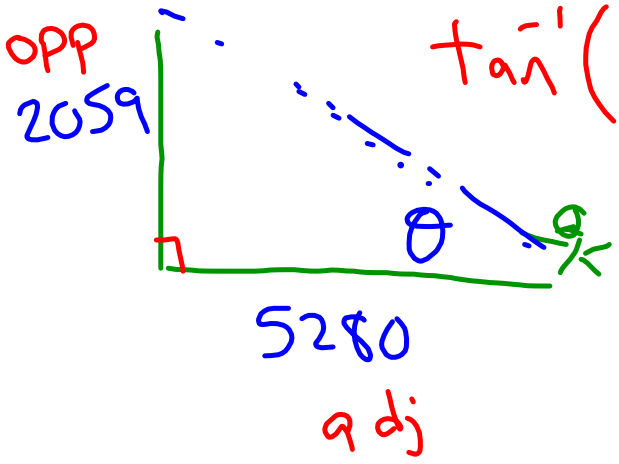
Warm-up

March 23, 2017

The tallest television transmitting tower in the world is in North Dakota, and it is 2059 feet tall. If you are on level ground exactly 5280 feet (one mile) from the base of the tower, what is your angle of elevation looking up at the top of the tower?

$\theta = 21.30°$



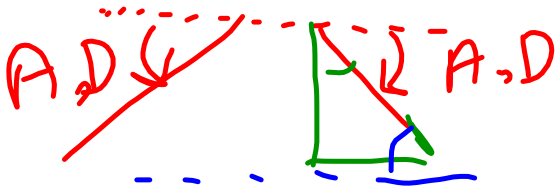
$\tan^{-1}\left(\frac{2059}{5280}\right) = 21.30°$

Angle of Elevation

Angle of Depression

$90 - A_oD$

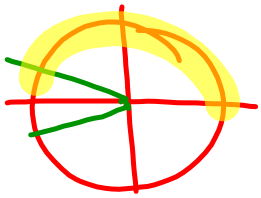
A_oE for bottom



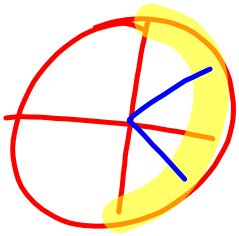
34 $\cos(\arccos(-\frac{\sqrt{3}}{2})) = -\frac{\sqrt{3}}{2}$

What angle

arccos
[0, 180]



arcsin
[-90, 90]
arctan
(-90, 90)



(9) $y = \cot\left(x + \frac{\pi}{2}\right)$ ← Radians

$a = 1 \quad b = 1 \quad c = -\frac{\pi}{2} \quad d = 0$

"amp" = 1

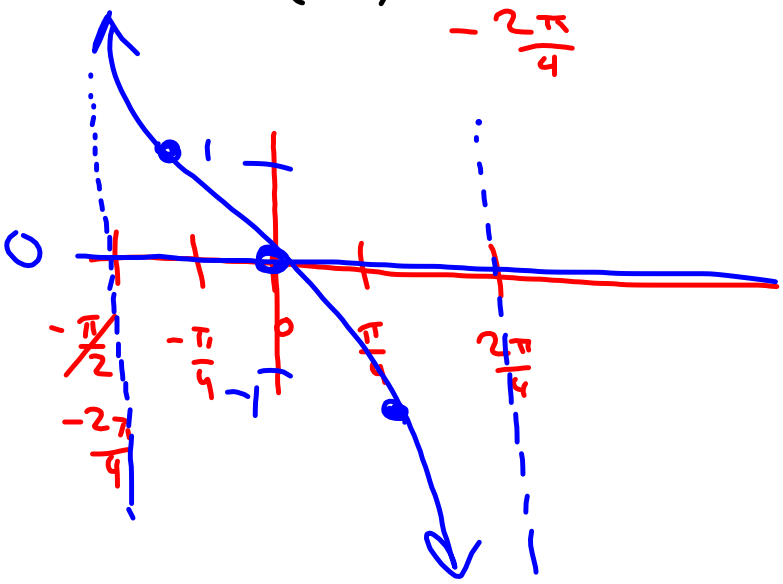
$\text{per} = \frac{\pi}{b} = \frac{\pi}{1} = \pi$

$\text{Interval} = \frac{\text{per}}{4} = \frac{\pi}{4}$

$PS = \frac{c}{b} = \frac{\left(-\frac{\pi}{2}\right)}{(1)} = -\frac{\pi}{2} \cdot \frac{2}{2}$ left

$-\frac{2\pi}{4}$

$VS = d = 0$ nuvs



(16) $y = \cot\left(\frac{1}{4}x\right)$ Find Dom, Ran, asy, per,

↕ Range: "y" $(-\infty, \infty)$

↔ Domain: "x" $x \neq$ asy $x \neq 0 + n(4\pi)$

asy: $x = (\text{first}) + n(\text{per})$

$bx - c = 0$

$4 \cdot \frac{x}{4} = 0 \cdot 4$

$x = 0$

$\text{per} = \frac{\pi}{b} = \frac{\pi}{\left(\frac{1}{4}\right)} = \pi \cdot \frac{4}{1} = 4\pi$

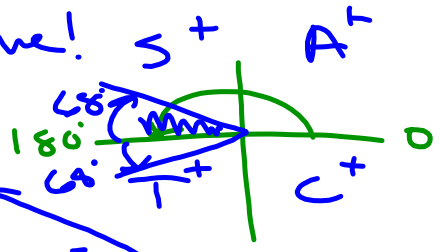
(28) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-2.6571}$ $[0, 360)$

$\cos^{-1}$ $\cos^{-1}$

↳ negative!

$\theta = 112°$

$\theta' = 180 - 112 = 68°$



II : 112°

III : 248°

Reciprocal

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$y = a \cdot trig(bx - c) + d$

$\frac{1 \cdot x}{4}$ $b = \frac{1}{4}$

$Per = \frac{2\pi}{b} = \frac{2\pi}{\left(\frac{1}{4}\right)} = \frac{\left(\frac{2\pi}{1}\right)}{\left(\frac{1}{4}\right)}$

$Per = 8\pi$

Asymptote

tan, cot Per: $\frac{\pi}{b}$

sec, csc Per: $\frac{2\pi}{b}$

cos, sin

sec, csc

$$x = (first) + n\left(\frac{Per}{2}\right)$$

cot, tan

$$x = (first) + n(Per)$$

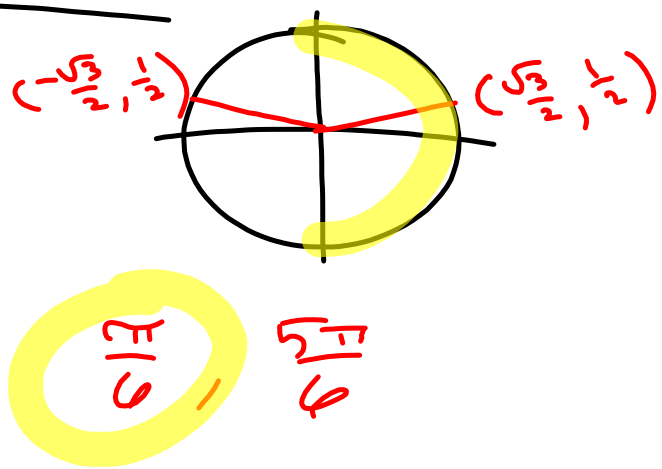
(first)

tan, sec: $bx - c = \frac{\pi}{2}$

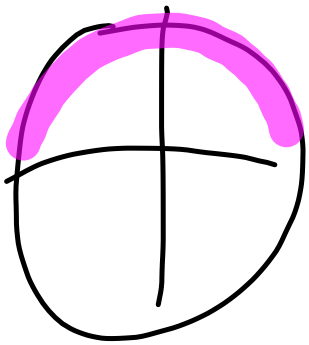
cot, csc: $bx - c = 0$

Exact values

① $\arcsin\left(\frac{1}{2}\right)$

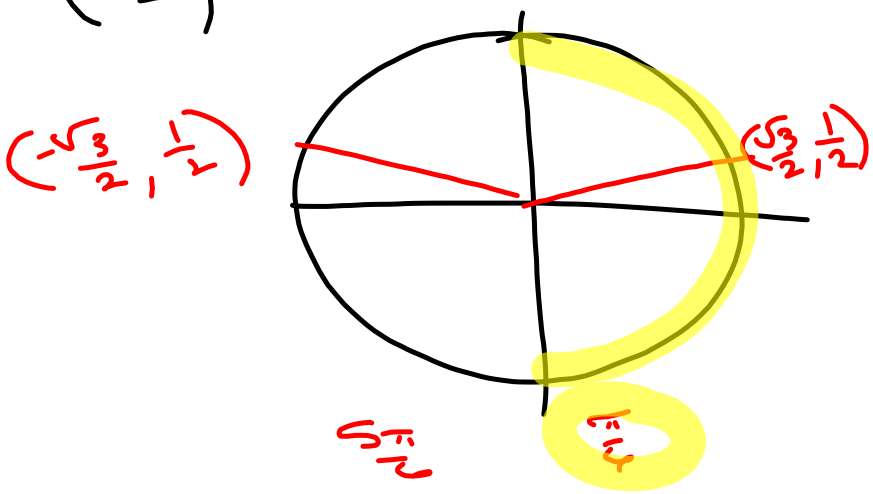


cos

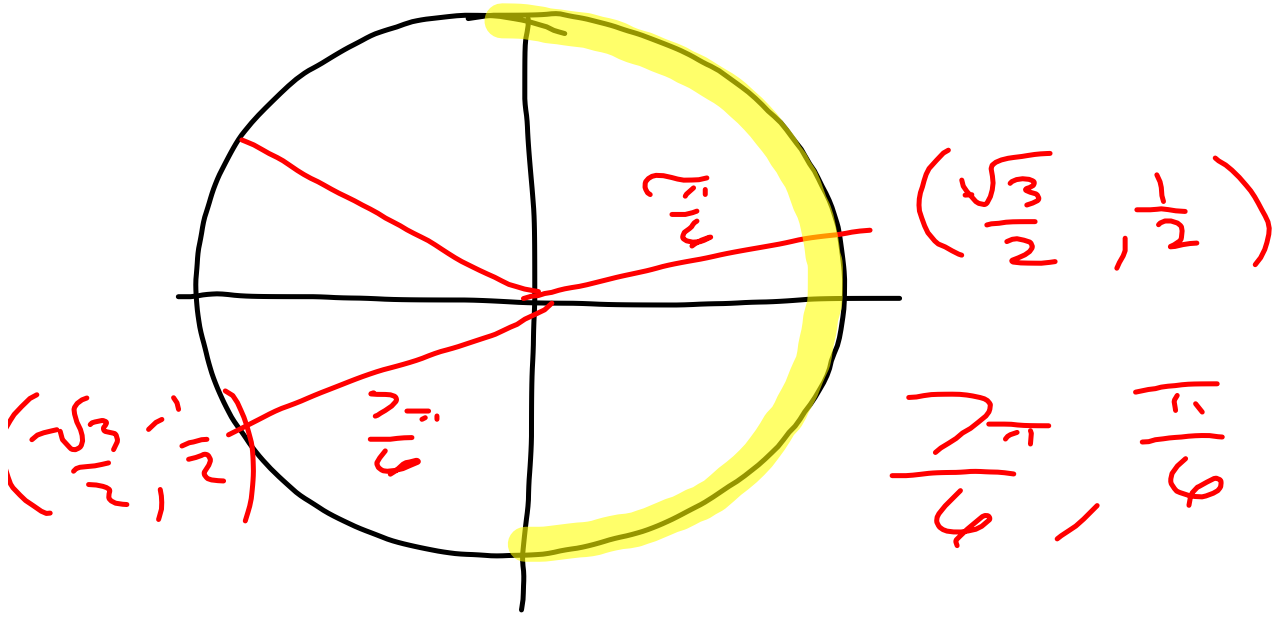


① $\arcsin\left(\frac{1}{2}\right)$

Exact value



② $\arctan\left(\frac{\sqrt{3}}{3}\right)$



$\tan\theta = \frac{y}{x}$

$y \quad \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

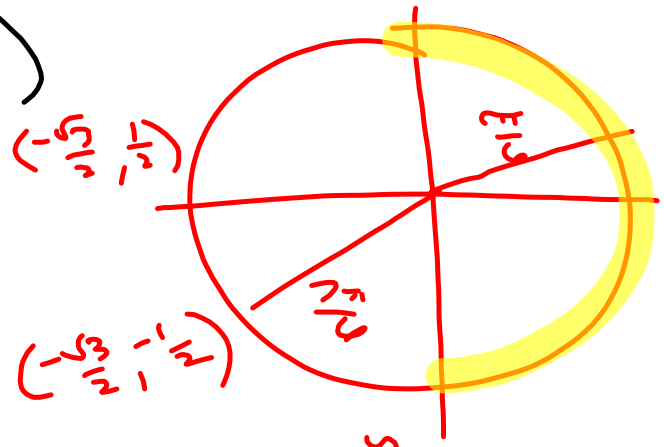
x

3

(2) $\arctan\left(\frac{\sqrt{3}}{3}\right)$

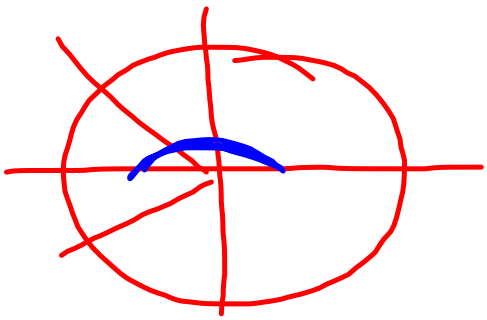
$\frac{y}{x}\ \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3



$\tan\theta = \frac{y}{x}$

3) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$



which θ has $x = -\frac{\sqrt{2}}{2}$?

$\frac{3\pi}{4}$, $\frac{5\pi}{4}$

$y = \cos(x)$

$R: [0, \pi]$

(4)

$y = \cos(x - \pi)$

$a = 1$

$b = 1$

$c = \pi$

$d = 0$

amp = 1

$per = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

midline = 0

$HS = \frac{c}{b} = \frac{\pi}{1} = \pi$ to the right

(5) $y = a\,trig(bx - c) + d$

$y = -5\cos\left(\frac{1}{4}x - \pi\right) + 4$

$a = -5 \quad b = \frac{1}{4} \quad c = \pi \quad d = 4$

amp = 5

vertical reflection

$Per = \frac{2\pi}{b} = \frac{2\pi}{\left(\frac{1}{4}\right)} \quad \frac{\left(\frac{2\pi}{1}\right)\cdot\frac{4}{1}}{\left(\frac{1}{4}\right)}$

$Per = 8\pi \qquad \frac{8\pi}{1} = 8\pi$

$PS = \frac{c}{b} = \frac{\pi}{\frac{1}{4}} = 4\pi$ right 4π

$VS = d = 4$ up 4

(5) $y = -5\cos\left(\frac{x}{4} - \pi\right) + 4$

$a = -5$

$b = \frac{1}{4}$

$c = \pi$

$d = 4$

amp = 5

vertical reflection

$per = \frac{2\pi}{b} = \frac{2\pi}{(1/4)} = 8\pi$

midline = 4

$HS = \frac{c}{b} = \frac{\pi}{(1/4)} = 4\pi$ to the right

(6) $y = \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)$

$a = \frac{1}{2}$

$b = 2$

$c = \frac{\pi}{3}$

$d = 0$

amp $= \frac{1}{2}$

Per $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

midline $= 0$

$HS = \frac{c}{b} = \frac{\left(\frac{\pi}{3}\right)}{\left(\frac{2}{1}\right)} = \frac{\pi}{6}$ to the right

(6) $y = \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)$

$a = \frac{1}{2}$ $b = 2$ $c = \frac{\pi}{3}$ $d = 0$

amp $= \frac{1}{2}$

Per $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

PS $= \frac{c}{b} = \frac{\left(\frac{\pi}{3}\right)}{\left(\frac{2}{1}\right)} \cdot \frac{1}{2} = \frac{\pi}{6}$ to the right

VS $= d = 0$ no VS

⑦ $y = -3 + \sin(3x + \frac{\pi}{2})$

↳ $y = \sin(3x + \frac{\pi}{2}) - 3$

$a = 1$

$b = 3$

$c = -\frac{\pi}{2}$

$d = -3$

amp $= 1$

per $= \frac{2\pi}{b} = \frac{2\pi}{3}$

midline $= -3$

$HS = \frac{c}{b} = \frac{(-\frac{\pi}{2})}{(\frac{3}{1})} = \frac{-\pi}{6}$ to the left

⑧ $y = \cos\left(2x - \frac{\pi}{2}\right)$

$a = 1$

$b = 2$

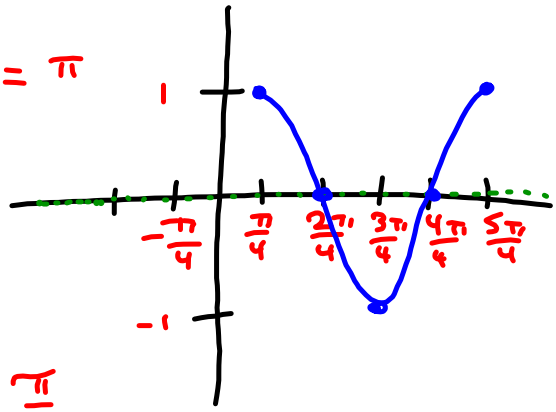
$c = \frac{\pi}{2}$

$d = 0$

amp = 1

$per = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$\frac{per}{4} = \frac{\pi}{4}$



- midline = 0
- $HS = \frac{c}{b} = \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{2}{1}\right)} = \frac{\pi}{4}$ to the right

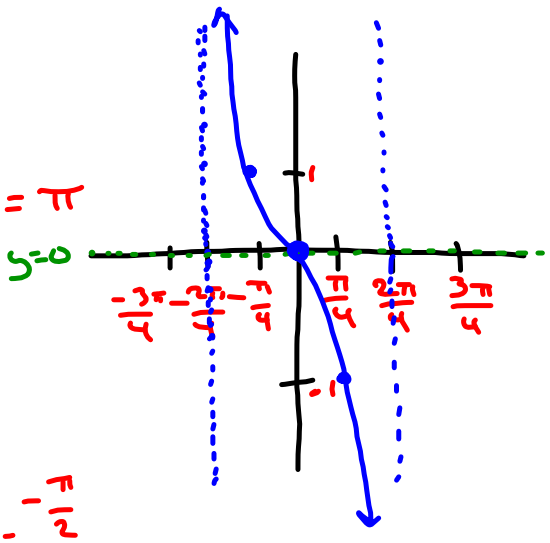
(9) $y = \cot\left(x + \frac{\pi}{2}\right)$

$a = 1$ "amp" $= 1$

$b = 1$ $\text{per} = \frac{\pi}{b} = \frac{\pi}{1} = \pi$

$c = -\frac{\pi}{2}$

$d = 0$ $\frac{\text{per}}{4} = \frac{\pi}{4}$



midline $= 0$

$HS = \frac{c}{b} = \frac{\left(-\frac{\pi}{2}\right)}{1} = -\frac{\pi}{2}$ to the left

asymptote: $x = (\text{first}) + k(\text{per})$

first: $bx - c = 0$

$x + \frac{\pi}{2} = 0$

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + k\pi$

⑨ $y = \cot\left(x + \frac{\pi}{2}\right)$

$y = a\,\text{trig}(bx - c) + d$

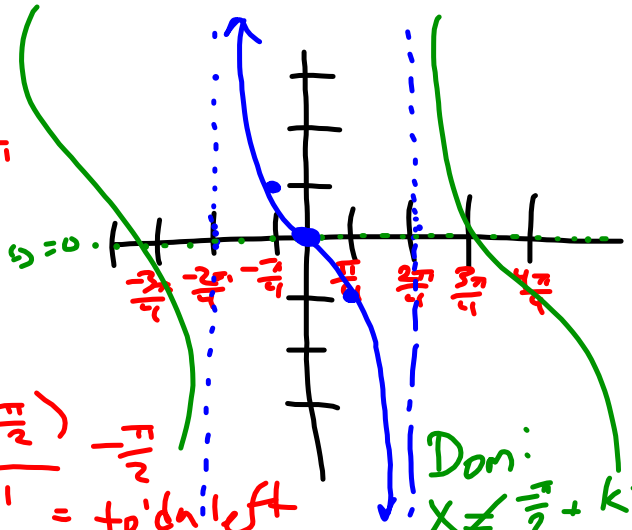
$a = 1$ "amp" $= 1$

$b = 1$ per $= \frac{\pi}{b} = \pi$

$c = -\frac{\pi}{2}$ per $\frac{}{4} = \frac{\pi}{4}$

$d = 0$

midline $= 0$



first: $bx - c = 0$, $x + \frac{\pi}{2} = 0$, $x = -\frac{\pi}{2}$

$HS = \frac{c}{b} = \frac{\left(-\frac{\pi}{2}\right)}{1} = -\frac{\pi}{2}$ to the left

Dom: $x \neq -\frac{\pi}{2} + k\pi$

Asymptote: $x = (\text{first}) + k(\text{per})$

$x = \frac{-\pi}{2} + k\frac{\pi}{1}$ .4 .4

Range: R

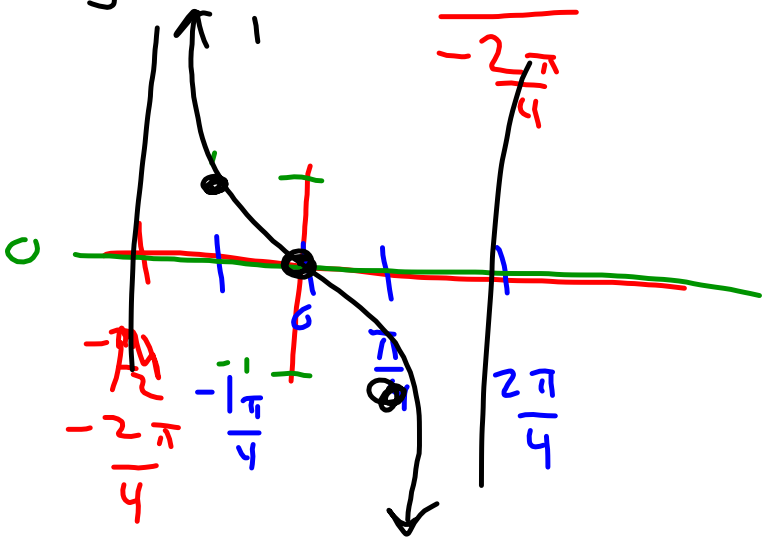
(9) $y = \cot\left(x + \frac{\pi}{2}\right)$ → Radians Graph.

$a = 1 \quad b = 1 \quad c = -\frac{\pi}{2} \quad d = 0$

"amp" = 1 $\quad per = \frac{\pi}{b} = \frac{\pi}{1} = \pi$

Interval: $\frac{per}{4} = \frac{\pi}{4}$

$PS = \frac{c}{b} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2} \cdot \frac{2}{\pi} = -\frac{2\pi}{4}$ VS = d = 0 nos



(9) $y = \cot\left(x + \frac{\pi}{2}\right)$

$a = 1 \quad b = 1 \quad c = -\frac{\pi}{2} \quad d = 0$

"amp" = 1

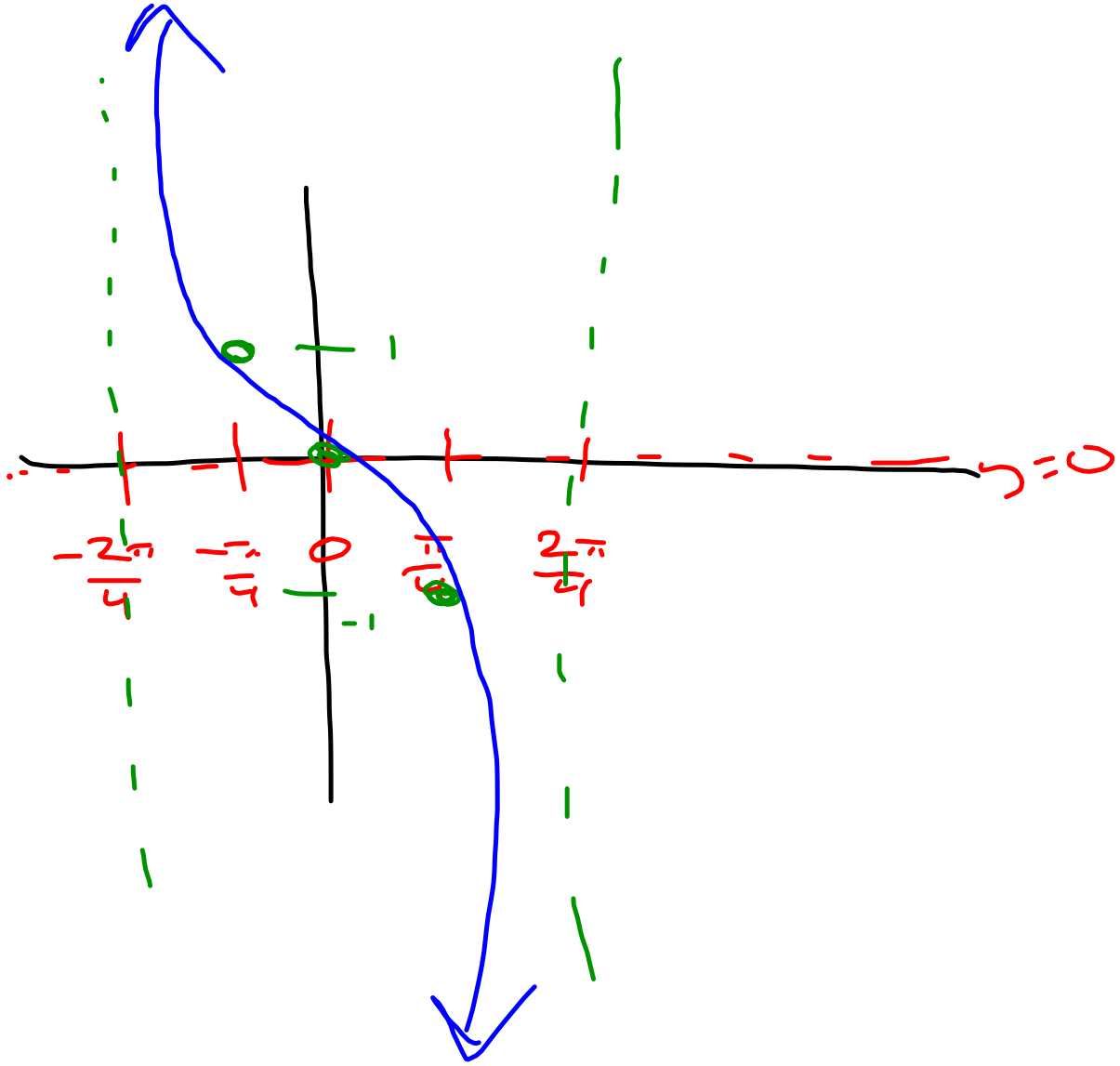
$per = \frac{1\pi}{b} = \frac{1\pi}{1} = \pi$

Int. $\frac{Per}{4} = \frac{\pi}{4}$

$\frac{\pi}{4}$ $\quad -\frac{2\pi}{4}$

$PS = \frac{c}{b} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$ to the left

$VS = d = 0$ midline


y=0
-2π/4
-π/4
0
π/4
2π/4

(10) $y = -2\csc(4x)$ "sin"

$a = -2$ "amp" $= 2$

$b = 4$ $\text{per} = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

$c = 0$ $\frac{\text{per}}{4} = \frac{\pi}{8}$

$d = 0$

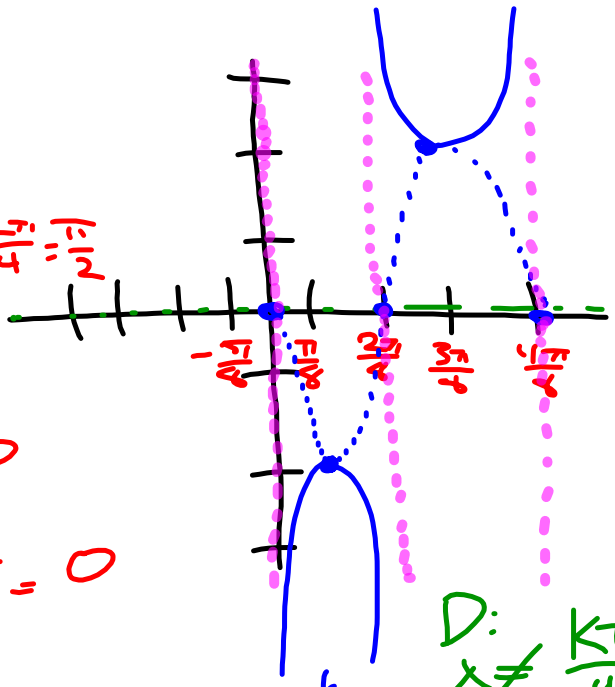
midline $= 0$

first: $bx - c = 0$ $HS = \frac{c}{b} = \frac{0}{4} = 0$

$\frac{4x}{4} = \frac{0}{4}$

$x = 0$ Asymptote:

$x = (\text{first}) + \frac{k}{1}(\text{per})$



$x = \frac{k\pi}{4}$

D: $x \neq \frac{k\pi}{4}$

R: $(-\infty, -2] \cup [2, \infty)$

(11) $y = -\sin\left(2x - \frac{\pi}{3}\right)$

$a = -1$ $b = 2$ $c = \frac{\pi}{3}$ $d = 0$

vertical reflect

amp = 1

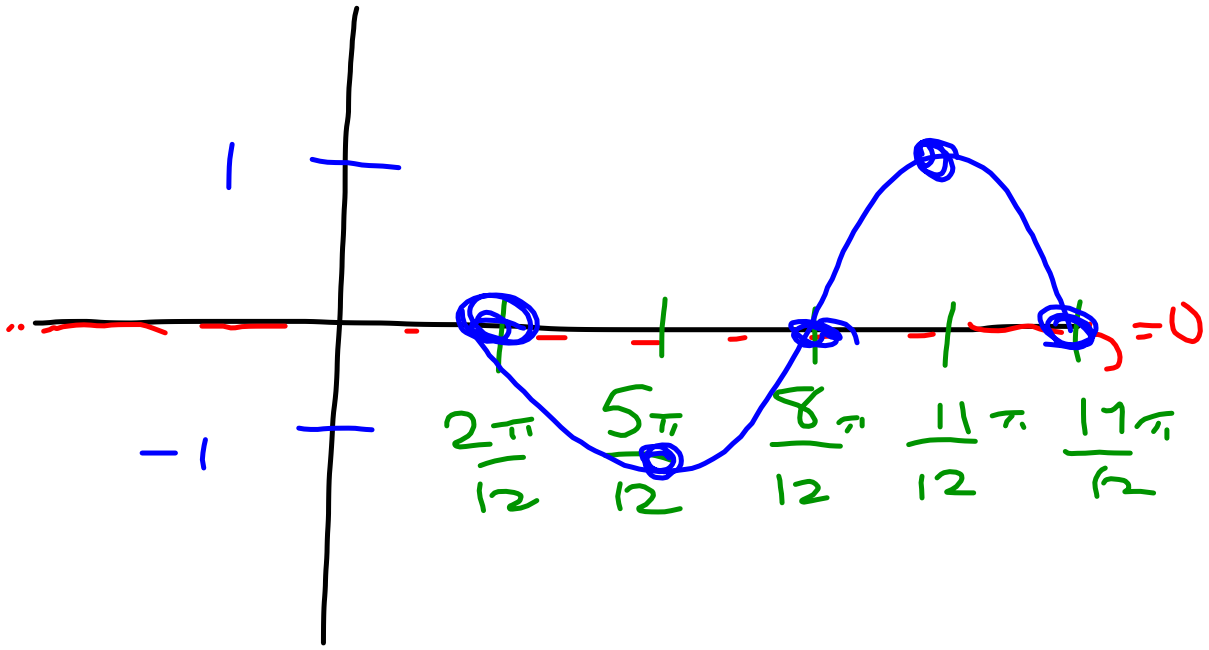
$\text{Per} = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Int: $\frac{\text{Per}}{4} = \frac{\pi}{4}$ · 3 $= \frac{3\pi}{12}$

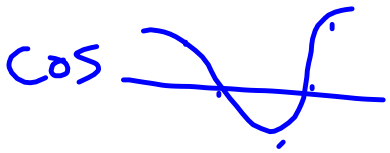
$\frac{\pi}{6}$ .2 $\frac{2\pi}{12}$ .2

$PS = \frac{c}{b} = \frac{\left(\frac{\pi}{3}\right)\cdot\frac{1}{2}}{\left(\frac{2}{1}\right)}$ to the right

VS: $d = 0$ no VS


1
-1
=0
2π/12
5π/12
8π/12
11π/12
17π/12

(12) $y = \sec 4x$



$a = 1 \quad b = 4 \quad c = 0 \quad d = 0$

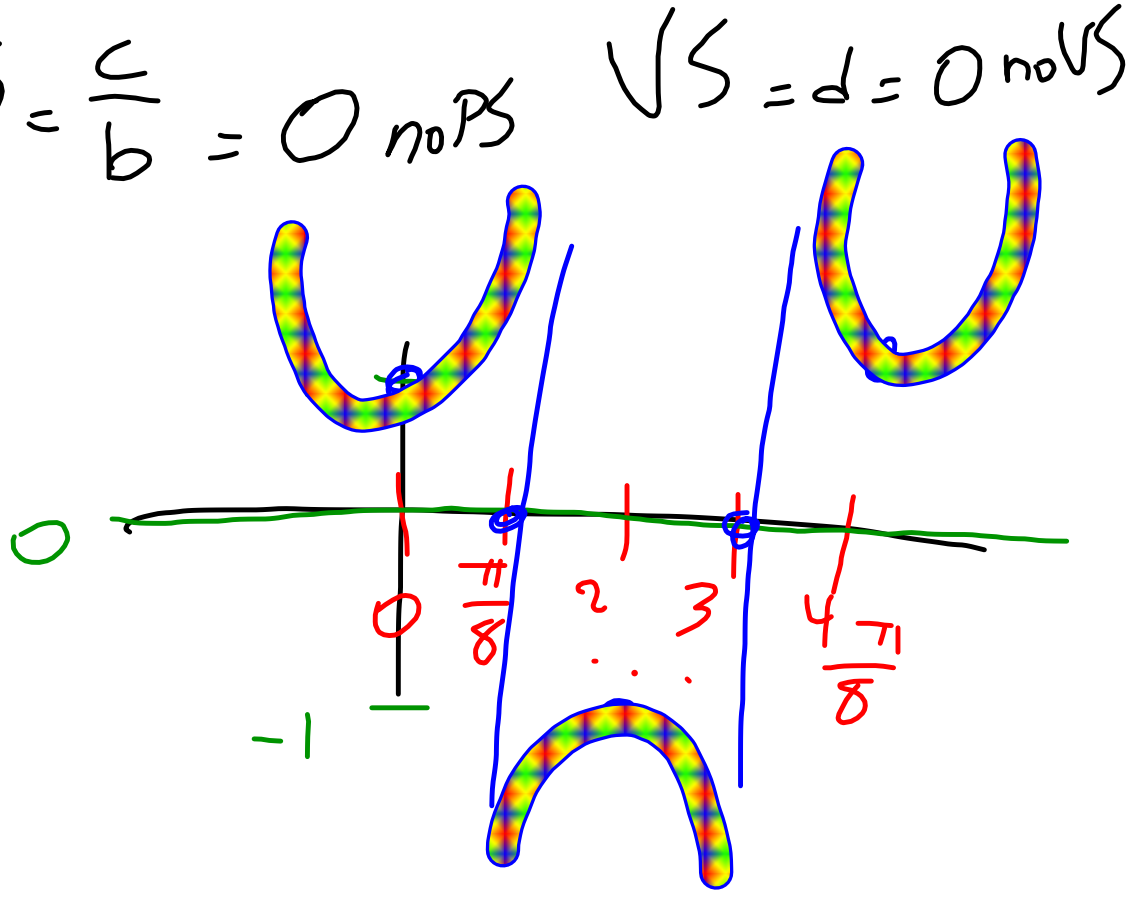
"amp" $= 1$ per $= \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

mid + amp = max

mid − amp = min

Int: $\frac{\text{per}}{4} = \left(\frac{\frac{\pi}{2}}{4}\right) = \left(\frac{\frac{\pi}{2}}{\frac{4}{1}}\right) \cdot \frac{1}{4} = \frac{\pi}{8}$

$PS = \frac{c}{b} = 0$ no PS $\quad VS = d = 0$ no VS



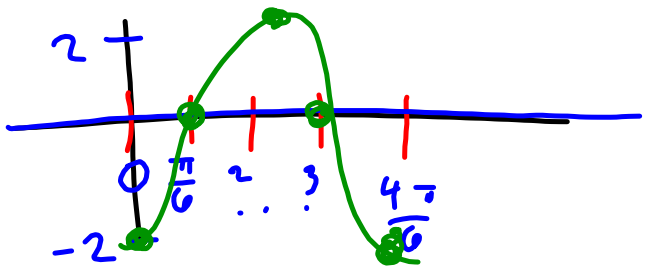
(13) $y = -2\cos(3x)$

$a = -2 \quad b = 3 \quad c = 0 \quad d = 0$

$\text{amp} = 2 \quad \text{per} = \frac{2\pi}{b} = \frac{2\pi}{3}$

$\text{Int.} = \frac{\text{per}}{4} = \frac{\frac{2\pi}{3}}{\left(\frac{4}{1}\right)} \cdot \frac{1}{4} = \frac{2\pi}{12} = \frac{\pi}{6}$

$PS = \frac{c}{b} = \frac{0}{3} = 0 \quad VS = d = 0$



(19) $y = 3\cos(3x + \pi)$

$a = 3$

$b = 3$

$c = -\pi$

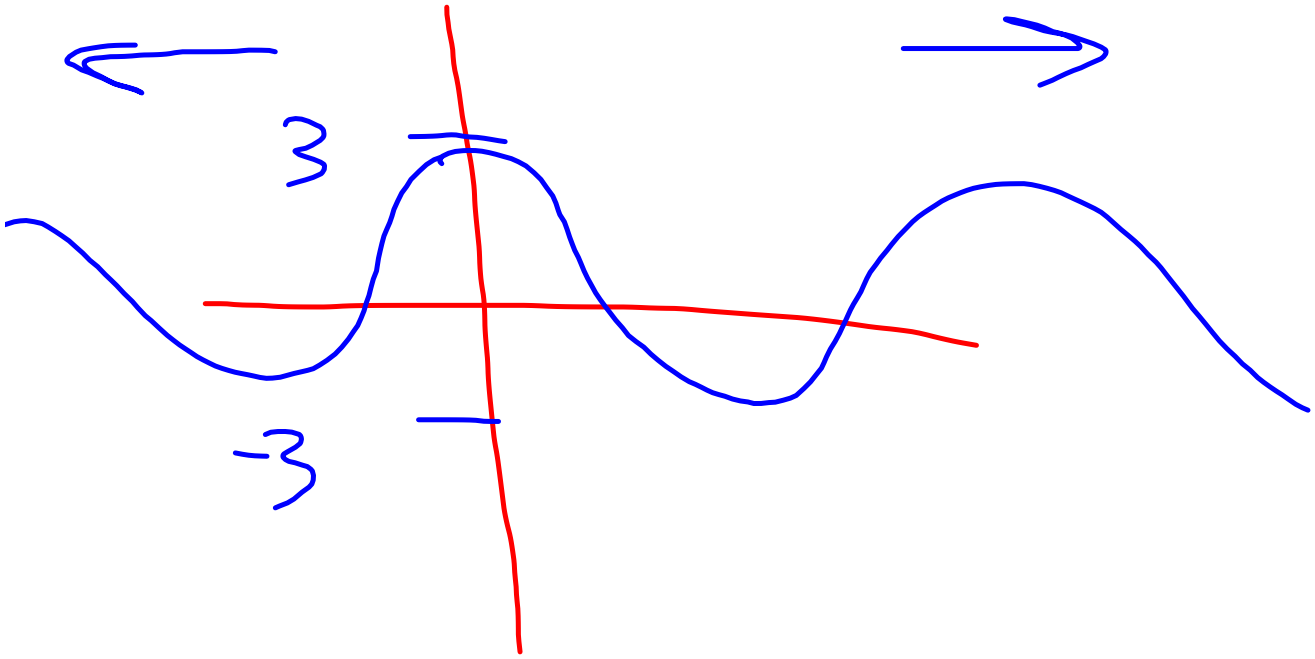
$d = 0$

$per = \frac{2\pi}{b} = \frac{2\pi}{3}$

no asymptotes for sine & cosine

x Domain: $(-\infty, \infty)$

y Range: $[-3, 3]$


3
-3

(15) $y = 4\cot(3x)$

Domain: $x \neq$ asy $\neq n\left(\frac{\pi}{3}\right)$

asy: $x = (\text{first}) + n(\text{per})$

$x = 0 + n\left(\frac{\pi}{3}\right) = n\left(\frac{\pi}{3}\right)$

first: $bx - c = 0$

$\frac{3x}{3} = \frac{0}{3}$

$x = 0$

Per: $\frac{\pi}{b} = \frac{\pi}{3}$

Range: $(-\infty, \infty)$

#14-16

(16) $y = \cot\left(\frac{1}{4}x\right)$

Find: domain, range, asymptote, period.

Range: "y" ↕ $(-\infty, \infty)$

Domain: "x" ↔ $x \neq$ asy $\quad x \neq 0 + n(4\pi)$

$\cot$ / $\tan$: $x = (\text{first}) + n(\text{per})$

first: $bx - c = 0$

cot / csc: $\frac{1}{4}x = 0$, $x = 0$

$\text{per} = \frac{\pi}{b} = \frac{\pi}{\frac{1}{4}} = \pi \cdot \frac{4}{1} = 4\pi$

(16) $y = \cot\left(\frac{1}{4}x\right)$

Range: $(-\infty, \infty)$

Domain: $x \neq 4n\pi$

asy: $x = (\text{First}) + n(\text{Per})$ → $x = 0 + n4\pi$

$x = 4n\pi$

First: $bx - c = 0$

$\frac{x}{4} = 0 \cdot 4$

$x = 0$

$\text{Per} = \frac{\pi}{b}$

$= \frac{(\pi)}{(\frac{1}{4})} \cdot \left(\frac{4}{1}\right) = 4\pi$

(23) $y = 2\cos(4x + \pi)$

amp $= 2 = a$

per $= \frac{\pi}{2} = \frac{2\pi}{b} \rightarrow \frac{b\pi}{\pi} = \frac{4\pi}{\pi}$

$b = 4$

PS $= -\frac{\pi}{4} = \frac{c}{b}$

$-\frac{\pi}{4} = \frac{c}{4}$

$c = \frac{-4\pi}{4} = -\pi$

23 amp = 2 per = $\frac{\pi}{2}$ PS = $-\frac{\pi}{4}$

$y = a\cos(bx - c) + d$

$y = 2\cos(4x + \pi)$

$b = \frac{2\pi}{per} = \frac{2\pi}{\frac{\pi}{2}} = \frac{\left(\frac{2\pi}{1}\right)}{\left(\frac{\pi}{2}\right)}$

$b = 4 \quad \frac{2\pi}{1} \cdot \frac{2}{\pi}$

$PS = -\frac{\pi}{4} = \frac{c}{b}$

$-\frac{\pi}{4} = \frac{c}{4}$ (4)

$c = -\pi$

(24) cos

$Per = \frac{2\pi}{1} = \frac{2\pi}{b}$

$b = 1$

$\frac{2\pi}{1} = \frac{2\pi}{b}$

$\frac{b \cdot 2\pi}{2\pi} = \frac{2\pi}{2\pi}$

$b = 1$

$PS = -1 = \frac{c}{b} = \frac{c}{1} = c$

$c = -1$

$VS = -4 = d$

$y = a\cos(bx - c) + d$

$y = \cos(1x + 1) - 4$

25

$\frac{1}{\tan\theta} = \cot\theta = -1.456$

$\tan\theta = \frac{1}{-1.456}$

$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{1}{-1.456}\right)$

$\theta = -34.48^\circ$ $360-\theta$

$\theta' = 34.48^\circ$ $360-34.48$

IV: 325.52°

II: $180-\theta'$

$180-34.48$

145.52°

25

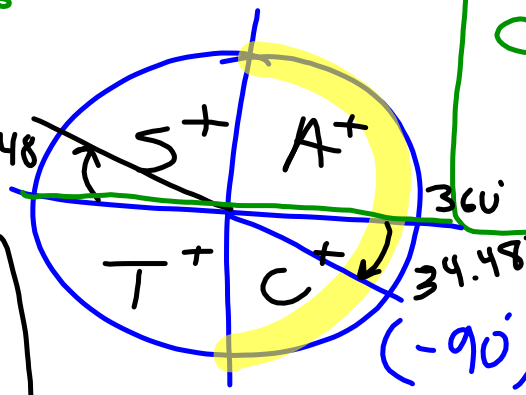
2 angles $[0°, 360°)$

$(\tan\theta) = \frac{1}{\cot\theta} = \left(\frac{1}{-1.456}\right)$

$\tan^{-1}$ $\tan^{-1}$

$\theta = -34.48°$

$\theta' = 34.48°$



$(-90°, 90°)$

Reciprocal
cot
sec
csc

II: $180 - 34.48 = 145.52°$

IV: $360 - 34.48 = 325.52°$

25

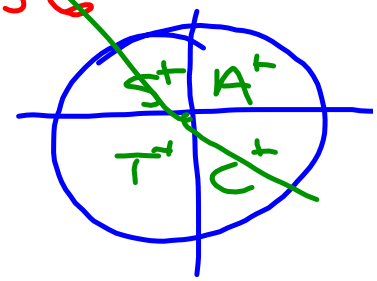
$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-1.456}$

$\tan\theta = \left(\frac{1}{-1.456}\right)$

$\tan^{-1}$ $\tan^{-1}$

$\theta = -34.48°$

$\theta' = 34.48°$



IV: $360 - \theta'$

$360 - 34.48°$

$= 325.52$

II: $180 - \theta'$

$180 - 34.48$

145.52

(26) $[0, 360)$

$\sin\theta = .9564$

sin sin positive

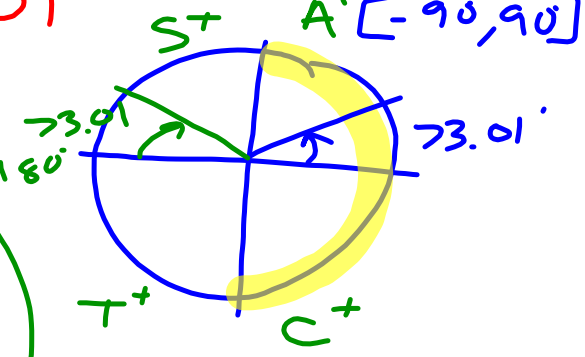
$\theta = 73.01°$

$\theta' = 73.01°$

I: $\theta_1 = 73.01°$

II: $\theta_2 = 180 - 73.01$

$= 106.99°$



27)

$\tan\theta = 0.3519$

tan −

tan

positive

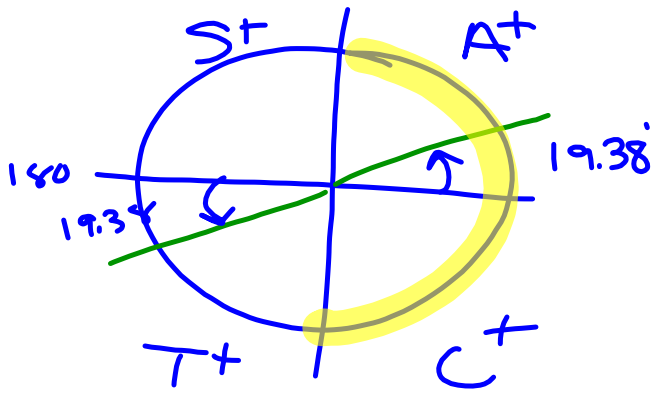
$\theta = 19.38°$

$\theta' = 19.38°$

I: $\theta_1 = 19.38°$

II: $\theta_2 = 180 + 19.38$

$= 199.38°$

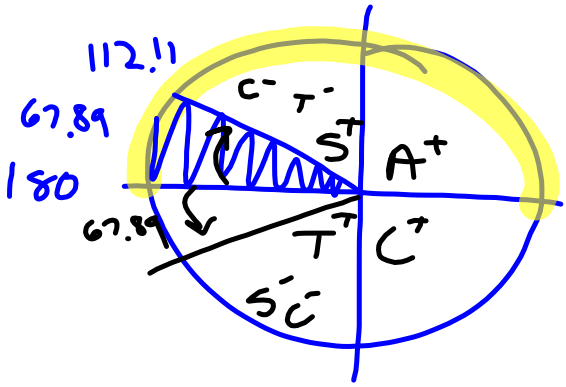


28) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-2.6571}$ → negative

cos⁻¹ cos⁻¹

$\theta = 112.11°$

$\theta' = 67.89°$



III: $180 + 67.89 = 247.89°$

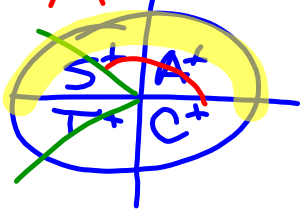
II: $180 - 67.89 = 112.11°$

(28)

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-2.6571}$

$\cos\theta = \left(\frac{1}{-2.6571}\right)$

$\cos^{-1}$ $\cos^{-1}$



$\theta = 112.10°$

$\theta' = 180 - 112.10 = 67.90°$

II: $180 - \theta'$

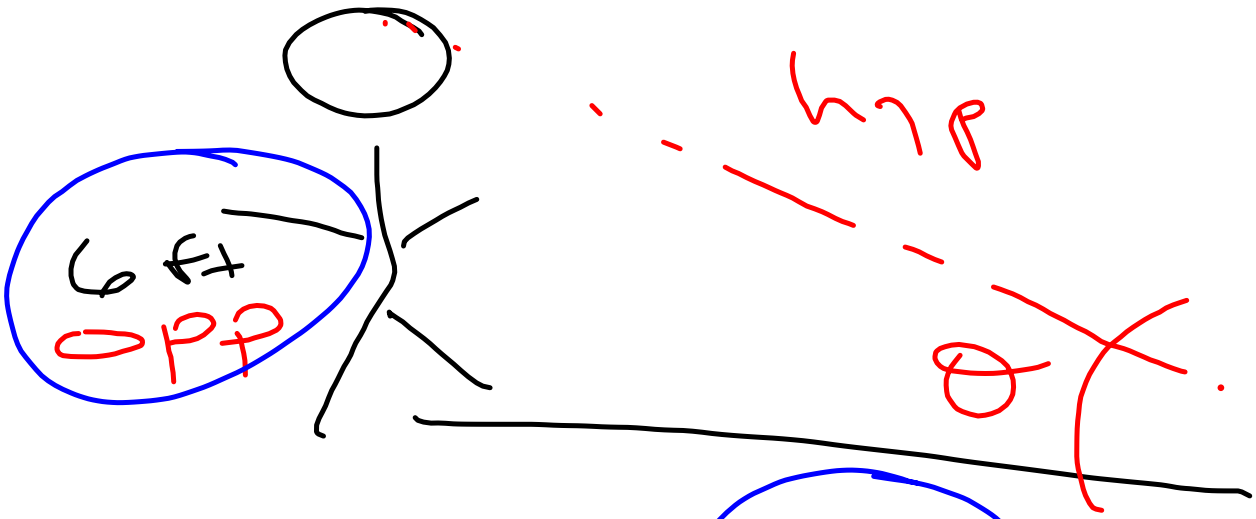
$180 - 67.9$

$112.10°$

III: $180 + \theta'$

$247.90°$

29

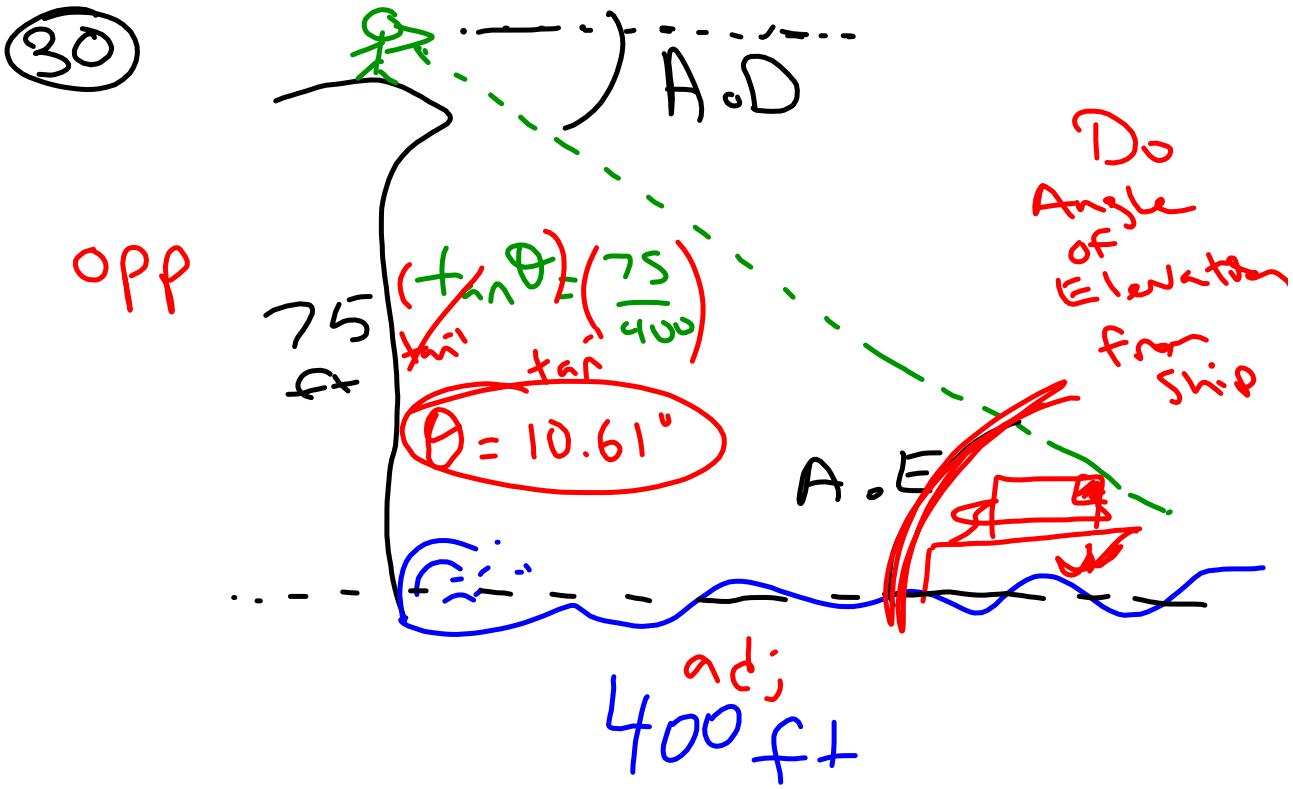


$\tan\theta = \frac{6}{14}$

$\tan^{-1}$

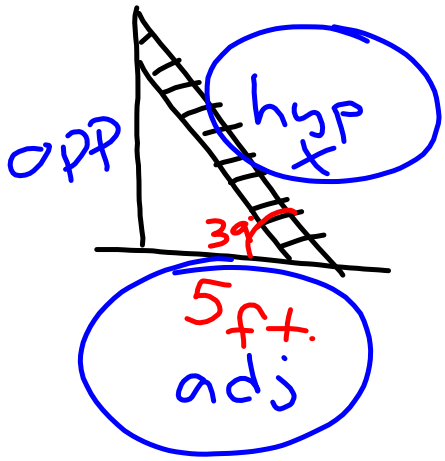
$\theta = 23.19^\circ$

30


A.D
Do
Angle
of
Elevation
from
Ship
opp
75 ft
$\tan\theta = \frac{75}{400}$
tan
$\theta = 10.61°$
A.E
adj
400 ft

31

SOH CAH TOA

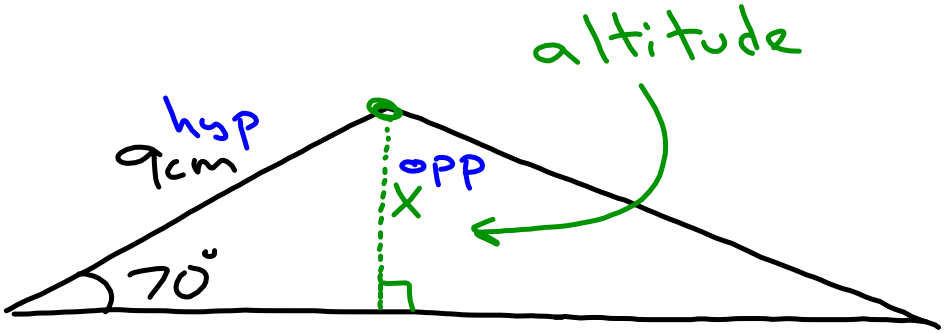


$\cos(39^\circ) = \frac{5}{x}$

Switch

$x = \frac{5}{\cos(39^\circ)} \approx 6.4 \text{ ft}$

32)



SOH CAH TOA

$\frac{9}{1} \cdot \sin(70^\circ) = \frac{x}{9} \cdot \frac{9}{1}$

$x = 8.4\text{cm}$

33

$\arcsin(\sin 3\pi)$

$= 3\pi$

$\arcsin(0)$

$= 0$

$\sin^{-1}(\sin(3\pi))$

$= 0$

34 $\cos\left(\arccos\left(-\frac{\sqrt{3}}{2}\right)\right) = -\frac{\sqrt{3}}{2}$

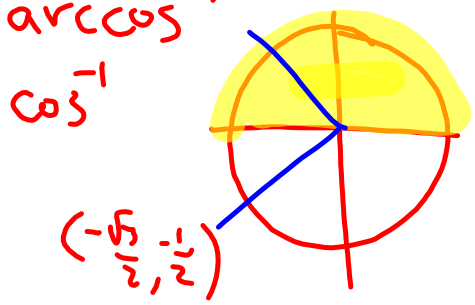
$\cos(150)$ $-.866$

$-\frac{\sqrt{3}}{2}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $[0, \pi]$ or $[0°, 180°]$

arccos

$\cos^{-1}$



$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$(-1, 0)$

$[-90°, 90°]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$90°$

arcsin

$\sin^{-1}$

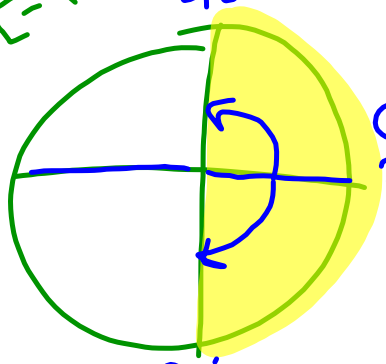
$(1, 0)$

2π

$(\ ,\ )$

arctan

$\tan^{-1}$



$-90°$