Warm-up

January 25, 2017

Solve.

$$\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

1. Find inverse.
   ① Determinant
   ② $\frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
2. Multiply it to other side.

# Warm Up

Solve.

$$\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$(-1 \cdot 3) - (2 \cdot 0)$

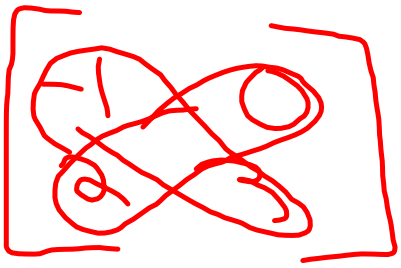
$-3 - 0 = -3$

$$-\frac{1}{3}\begin{bmatrix} 3 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} X = -\frac{1}{3}\begin{bmatrix} 3 & 0 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} (3 \cdot 5) + (0 \cdot 2) \\ (-2 \cdot 5) + (-1 \cdot -2) \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 15 \\ -8 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 \\ \frac{8}{3} \end{bmatrix}$$




$-1(3) - 2(0)$

$-3 - 0$

$-3$

Determinant

$-\frac{1}{3}\begin{bmatrix} 3 & 0 \\ -2 & -1 \end{bmatrix}$

Inverse

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} B + \begin{bmatrix} 7 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$-\begin{bmatrix} 7 \\ -2 \end{bmatrix} \qquad -\begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

$B = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$ $(1\cdot5)-(2\cdot3)$ $5-6=-1$

$-1\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} B = -1\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -6 \\ 1 \end{bmatrix}$

$B = -1\begin{bmatrix} (5\cdot-6)+(-3\cdot1) \\ (-2\cdot-6)+(1\cdot1) \end{bmatrix}$ $B = -1\begin{bmatrix} -33 \\ 13 \end{bmatrix}$ $B = \begin{bmatrix} 33 \\ -13 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} B = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

(X, Y)

$$D = 1(5) - 3(2) = -1$$

$$D_x = \begin{vmatrix} -6 & 3 \\ 1 & 5 \end{vmatrix} = -30 - 3 = -33$$

$$D_y = \begin{vmatrix} 1 & -6 \\ 2 & 1 \end{vmatrix} = 1 - (-12) = 13$$

$$x = \frac{D_x}{D} = \frac{-33}{-1} = 33 \qquad y = \frac{D_y}{D} = \frac{13}{-1} = -13$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} B = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$D = 5(1) - 2(3) = -1$$

$$D_x = \begin{vmatrix} -6 & 3 \\ 1 & 5 \end{vmatrix} = -6(5) - 3(1) = -30 - 3 = -33$$

$$D_y = \begin{vmatrix} 1 & -6 \\ 2 & 1 \end{vmatrix} = 1(1) - (-6)(2) = 1 + 12 = 13$$

$$x = \frac{D_x}{D} = \frac{-33}{-1} = 33$$

$$y = \frac{D_y}{D} = \frac{13}{-1} = -13$$

Cramer's Rule only works when there is a column matrix on right side.

1. $-5x-4y=-8$
$-3x-y=-2$

$x=\frac{D_x}{D}=\frac{0}{-7}=0$

$y=\frac{D_y}{D}=\frac{-14}{-7}=2$

$$\begin{bmatrix}-5 & -4\\ -3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-8\\ -2\end{bmatrix}$$

$x=0$
$y=2$

$D=5-12=-7$

$D_x=\begin{vmatrix}-8 & -4\\ -2 & -1\end{vmatrix}=8-8=0$

$D_y=\begin{vmatrix}-5 & -8\\ -3 & -2\end{vmatrix}=10-24=-14$

3) $2x-5y=19$

$-3x-y=-3$

$-2-15=-17$ det

$$\begin{bmatrix}2 & -5\\ -3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}19\\ -3\end{bmatrix}$$

$$D_x=\begin{bmatrix}19 & -5\\ -3 & -1\end{bmatrix}=-19-15=\frac{-34}{-17}=2$$

$$D_y=\begin{bmatrix}2 & 19\\ -3 & -3\end{bmatrix}=-6+57=51/-17=-3$$

$X=2\ y=-3$

16. $6x + 6y + z = -23$

$3x - 6z = 27$

$6x + 3y - 6z = 18$

$$\begin{bmatrix} 6 & 6 & 1 \\ 3 & 0 & -6 \\ 6 & 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -23 \\ 27 \\ 18 \end{bmatrix}$$

$$D = \begin{vmatrix} 6 & 6 & 1 \\ 3 & 0 & -6 \\ 6 & 3 & -6 \end{vmatrix} \begin{matrix} 6 & 6 \\ 3 & 0 \\ 6 & 3 \end{matrix} = 9$$

$= (0 + -216 + 9)$

$- (0 + -108 + -108)$

$= -207 - -216$

$= 9$

Solve for X

$$D_x = \begin{vmatrix} -23 & 6 & -23 & 6 \\ 27 & 0 & -6 & 27 & 0 \\ 18 & 3 & -6 & 18 & 3 \end{vmatrix}$$

$$= (0 + 648 + 81) - (0 + 414 + -972)$$

$$= -567 - -558 = -9$$

$$X = \frac{D_x}{D} = \frac{-9}{9} = -1$$

$$\begin{matrix} x & y \\ \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \end{matrix} B = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$x = \frac{Dx}{D} = \frac{-33}{-1} = 33$$

$$y = \frac{Dy}{D} = \frac{13}{-1} = -13$$

$$D = 1(5) - 2(3)$$
$$5 - 6$$
$$-1$$

$$D_x = \begin{vmatrix} -6 & 3 \\ 1 & 5 \end{vmatrix} = -6(5) - 1(3)$$
$$-30 - 3$$
$$= -33$$

$$D_y = \begin{vmatrix} 1 & -6 \\ 2 & 1 \end{vmatrix} = 1(1) - 2(-6)$$
$$1 + 12$$
$$13$$

$$-5x - 4y = -8$$
$$-3x - y = -2$$

$$\begin{bmatrix} -5 & -4 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$$

$x = \frac{Dx}{D} = \frac{0}{-7} = 0$

$y = \frac{Dy}{D} = \frac{-14}{-7} = 2$

$x: 0$

$y: 2$

$D = -5(-1) - -3(-4) = 5 - 12$

$= -7$

$Dx = \begin{vmatrix} -8 & -4 \\ -2 & -1 \end{vmatrix} = -8(-1) - -4(-2)$

$8 - 8 = 0$

$Dy = \begin{vmatrix} -5 & -8 \\ -3 & -2 \end{vmatrix} = -5(-2) - -3(-8)$

$10 - 24$

$-14$

$$2x - 5y = 19$$
$$-3x - y = -3$$

$$\begin{bmatrix} 2 & -5 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ -3 \end{bmatrix}$$

$D = 2(-1) - -3(-5) = -17$

$D_X \begin{vmatrix} 19 & -5 \\ -3 & -1 \end{vmatrix}$ $19(-1) - -3(-5) = -34$

$D_Y \begin{vmatrix} 2 & 19 \\ -3 & -3 \end{vmatrix}$ $2(-3) - -3(19) = 51$

$\frac{D_X}{D} \frac{-34}{-17} = 2$

$\frac{D_Y}{D} \frac{51}{-17} = -3$

$X = 2$

$Y = -3$

solve x

16. $6x + 6y + z = -23$

$3x - 6z = 27$

$6x + 3y - 6z = 18$

$$\begin{bmatrix} 6 & 6 & 1 \\ 3 & 0 & -6 \\ 6 & 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -23 \\ 27 \\ 18 \end{bmatrix}$$

$$D = \begin{vmatrix} 6 & 6 & 1 \\ 3 & 0 & -6 \\ 6 & 3 & -6 \end{vmatrix} \begin{matrix} 6 & 6 \\ 3 & 0 \\ 6 & 3 \end{matrix} = 9$$

$= (0 + -216 + 9)$

$- (0 + -108 - 108)$

$= -207 - -216$

$= 9$

$$Dx = \begin{vmatrix} -23 & 6 & 1 \\ 27 & 0 & -6 \\ 18 & 3 & -6 \end{vmatrix} \begin{matrix} -23 & 6 \\ 27 & 0 \\ 18 & 3 \end{matrix}$$

$$= -(0 + -648 + 81) - (0 + 414 - 972)$$

$$= 567 - -558 = -9$$

$$x = \frac{Dx}{D} = \frac{-9}{9} = -1$$

(16) $6x + 6y + z = -23$

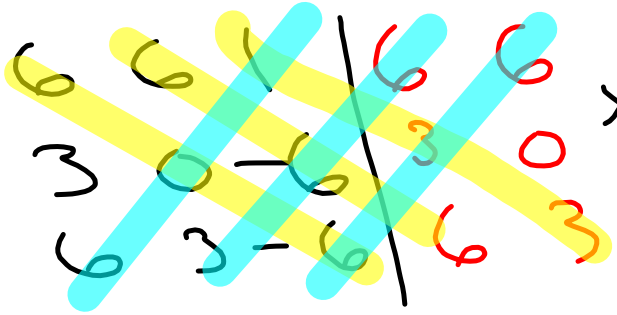
$3x - 6z = 27$

$6x + 3y - 6z = 18$

$$\begin{bmatrix} 6 & 6 & 1 \\ 3 & 0 & -6 \\ 6 & 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -23 \\ 27 \\ 18 \end{bmatrix}$$

Solve for x.

$D = \begin{vmatrix} 6 & 1 \\ 3 & -6 \end{vmatrix}$



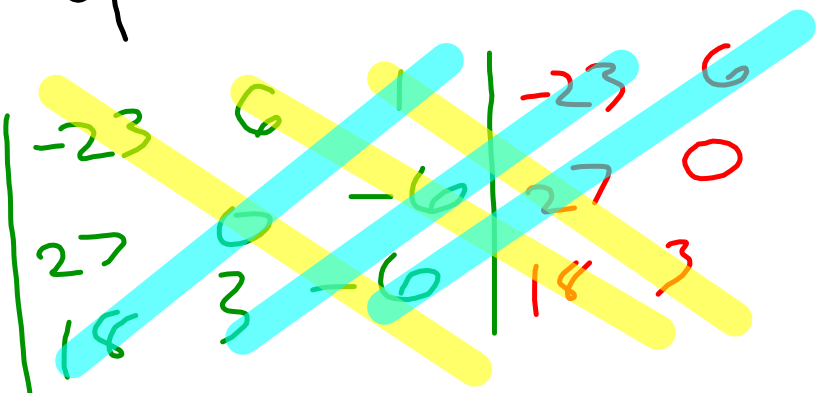
$x = \frac{D_x}{D} = \frac{-9}{9} = -1$

$(0 + 216 + 9) - (0 + -108 + -108)$

$-207 - (-216)$

$= 9$

$D_x = \begin{vmatrix} -23 & 1 \\ 27 & -6 \end{vmatrix}$



$(0 + 648 + 81) - (0 + 414 + 972)$

$-567 - (-558)$

$= -9$

① $-5x-4y=-8$

$-3x-y=-2$

$x=\frac{D_x}{D}=\frac{0}{-7}$

$x=0$

$y=\frac{D_y}{D}=\frac{-14}{-7}$

$y=2$

$$\begin{bmatrix}-5 & -4\\ -3 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-8\\ -2\end{bmatrix}$$

$$D=\begin{vmatrix}-5 & -4\\ -3 & -1\end{vmatrix}=-5(-1)-(-3)(-4)=5-12=-7$$

$$D_x=\begin{vmatrix}-8 & -4\\ -2 & -1\end{vmatrix}=8-8=0$$

$$D_y=\begin{vmatrix}-5 & -8\\ -3 & -2\end{vmatrix}=-5(-2)-(-3)(-8)=10-24=-14$$

$$3x + 5 = 13$$
$$-5 \quad -5$$
$$3x = 8$$

$-1 - -2$

$-1 + 2$

$1$

$1 - 7$

$= -6$

$$\begin{matrix} x & y \\ \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} \end{matrix} X = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$-1x + 0y = 5$

$2x + 3y = -2$

$$\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 \\ 8/3 \end{bmatrix}$$

$$D = \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} = -1(3) - 2(0) = -3 - 0 = -3$$

$$D_x = \begin{vmatrix} 5 & 0 \\ -2 & 3 \end{vmatrix} = 5(3) = 15$$

$$D_y = \begin{vmatrix} -1 & 5 \\ 2 & -2 \end{vmatrix} = -1(-2) - 5(2) = 2 - 10 = -8$$

$$x = \frac{D_x}{D} = \frac{15}{-3} = -5$$

$$y = \frac{D_y}{D} = \frac{-8}{-3} = \frac{8}{3}$$

* Answer matrix is ONE column.

* IF original Determin is zero, no solution.

$$\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$D = \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} = -3$$

$-x + 0y = 5$

$2x + 3y = -2$

$$\begin{bmatrix} x & y \\ -1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$D_x = \begin{vmatrix} 5 & 0 \\ -2 & 3 \end{vmatrix} = 5(3) - 0(-2) = 15 - 0 = 15$$

$$D_y = \begin{vmatrix} -1 & 5 \\ 2 & -2 \end{vmatrix} = -1(-2) - 2(5) = 2 - 10 = -8$$

$$x = \frac{D_x}{D} = \frac{15}{-3} = -5$$

$$y = \frac{D_y}{D} = \frac{-8}{-3} = \frac{8}{3}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ \frac{8}{3} \end{bmatrix}$$

$$-5x - 4y = -8$$
$$-3x - y = -2$$

$x = 0$, $y = \frac{-14}{-7} = 2$

$$\begin{bmatrix} -5 & -4 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$$

$$D = \begin{vmatrix} -5 & -4 \\ -3 & -1 \end{vmatrix} = -5(-1) - -3(-4)$$
$$5 - 12$$
$$-7$$

$$D_x = \begin{vmatrix} -8 & -4 \\ -2 & -1 \end{vmatrix} = 8 - 8 = 0$$

$$D_y = \begin{vmatrix} -5 & -8 \\ -3 & -2 \end{vmatrix} = -5(-2) - -3(-8)$$
$$10 - 24$$
$$-14$$

$$\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$D = -1(3) - 2(0) = -3$

$D_x = 5(3) - -2(0) = 15 - 0 = 15$

$D_y = -1(-2) - 2(5) = 2 - 10 = -8$

$x = \frac{D_x}{D} = \frac{15}{-3} = -5$

$y = \frac{D_y}{D} = \frac{-8}{-3} = \frac{8}{3}$

$$\begin{bmatrix} -5 \\ 8/3 \end{bmatrix}$$

① $-5x-4y=-8$

$-3x-y=-2$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -5 & -4 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$

$D = -5(-1) - -3(-4)$

$5 - 12$

$-7$

$Dx = \begin{vmatrix} -8 & -4 \\ -2 & -1 \end{vmatrix} = 0$

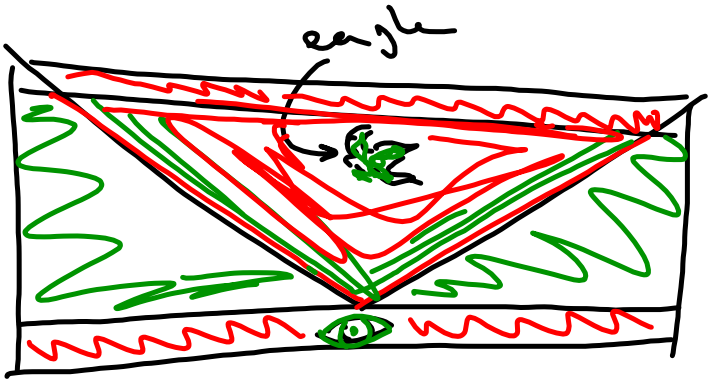
$x = \frac{Dx}{D} = \frac{0}{-7} = 0$

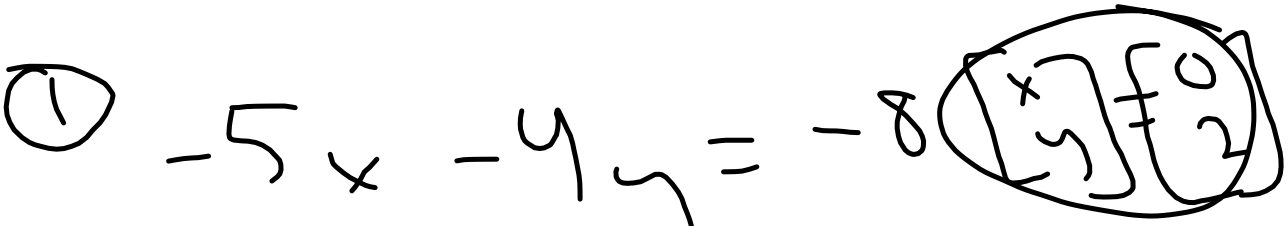
$Dy = \begin{vmatrix} -5 & -8 \\ -3 & -2 \end{vmatrix} = -5(-2) - -3(-8)$

$10 - 24$

$-14$

$y = \frac{Dy}{D} = \frac{-14}{-7} = 2$


eagle




$$\begin{bmatrix} -5 & -4 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$$

$$D = \begin{vmatrix} -5 & -4 \\ -3 & -1 \end{vmatrix} = -7$$

$$D_x = \begin{vmatrix} -8 & -4 \\ -2 & -1 \end{vmatrix} = 8 - 8 = 0$$

$$D_y = \begin{vmatrix} -5 & -8 \\ -3 & -2 \end{vmatrix} = -14$$

$$x = \frac{D_x}{D} = \frac{0}{-7} = 0 \qquad y = \frac{D_y}{D} = \frac{-14}{-7} = 2$$

$$-y - 3z = 4$$
$$-4x - 4y + z = -27$$
$$-6x - 3y - 2z = -15$$

$$\begin{bmatrix} 0 & -1 & -3 \\ -4 & -4 & 1 \\ -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -27 \\ -15 \end{bmatrix}$$