

## Warm-up

January 20, 2017

Simplify.

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} - 4 \begin{bmatrix} 5 & 6 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 20 & 24 \\ 0 & -4 \end{bmatrix}$$

Solve.

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 20 & 24 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 20 & -23 \\ 3 & 0 \end{bmatrix}$$

$$4X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$4X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\left(\frac{1}{4}\right) 4X = \frac{1}{4} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{5}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

$$4X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

$\rightarrow$   $3 \times 3$   $\rightarrow$   $3 \times 3$   $\rightarrow$   $3 \times 3$

$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} 4X = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

No Division Allowed.

$$X = \begin{bmatrix} .5 & -.25 & .125 \\ 0 & .75 & .75 \end{bmatrix}$$

$$= \left[ \begin{array}{ccc|c} 1 & & & 5 \\ \hline 2 & & & 5 \\ \hline 0 & 4 & 3 & 3 \end{array} \right]$$

$$4X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

↑  
R1+R2  
R3+R2

$$\frac{1}{4}4X = \frac{1}{4} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

invert

Cannot  
DIVIDE  
MATRICES!

$$X = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{5}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

$$\begin{aligned}
 & \cancel{4}X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \\
 & + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 & \cancel{\frac{1}{4}}4X = \frac{1}{4} \begin{bmatrix} 2 & -1 & 5 \\ 0 & 3 & 3 \end{bmatrix}
 \end{aligned}$$

$$X = \begin{bmatrix} \frac{2}{4} = \frac{1}{2} & \frac{-1}{4} & \frac{5}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

$$4X - \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$+ \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$


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$$\cancel{4}X = \begin{bmatrix} 2 & -1 & 5 \\ 0 & 3 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{5}{4} \\ 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} + (-4) \begin{bmatrix} 5 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} -20 & -24 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -20 & -23 \\ 3 & 0 \end{bmatrix}$$

✓

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} - 4 \begin{bmatrix} 5 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 20 & 24 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -20 & -23 \\ 3 & 0 \end{bmatrix}$$

✓



2x3

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

# Scalar Multiplication

$$4 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$



Simplify.

$$-3 \begin{bmatrix} 1 & 2 & -2 \\ 5 & 7 & 0 \end{bmatrix} + \begin{bmatrix} 9 & 12 & 0 \\ -5 & -7 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 6 \\ -20 & -28 & 0 \end{bmatrix}$$

$$\begin{array}{c} \xrightarrow{\hspace{2cm}} \\ [1 \quad 2] \cdot \begin{array}{c} \downarrow \\ \left[ \begin{array}{c} 3 \\ 6 \end{array} \right] \end{array} \\ \begin{array}{c} \textcircled{1} \times \textcircled{2} \rightarrow \textcircled{2} \times \textcircled{1} \\ \text{R} \quad \text{C} \quad \text{R} \quad \text{C} \end{array} \end{array}$$

### Multiply Matrices

① Lock & Key

②  $\rightarrow$   $\downarrow$   
Add the Products

$$= \begin{array}{c} \left[ \begin{array}{c} 1(3) + 2(6) \\ 3 + 12 \\ 15 \end{array} \right] = [15] \\ \begin{array}{c} 1 \times 1 \\ \text{R} \quad \text{C} \end{array} \end{array}$$

9.  $\begin{bmatrix} -4 \\ -3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & -1 \end{bmatrix}^{TL}$

$3 \times \begin{bmatrix} 1 & 1 \end{bmatrix} \times 2$

$3 \times 2$        $ML$        $MR$

$BL$        $BR$

$$\begin{bmatrix} -4(-2) & -4(-1) \\ -3(-2) & -3(-1) \\ 0(-2) & 0(-1) \end{bmatrix}$$

$\begin{bmatrix} \underline{8} & \underline{4} \\ \underline{6} & \underline{3} \\ \underline{0} & \underline{0} \end{bmatrix}$

# Try #2

$$\begin{bmatrix} 2 & 6 \\ -15 & -4 \end{bmatrix}$$

$$\begin{matrix} 2 \times & \boxed{3} & \times 3 \\ \begin{bmatrix} 3 & 0 & 3 \\ -3 & 1 & -3 \end{bmatrix} & & \begin{bmatrix} 5 & -2 \\ 6 & 2 \\ 2 & 3 \end{bmatrix} \end{matrix}$$

$$3(5) + 0(6) + 3(2)$$

$$15 + 0 + 6$$

$$\underline{21}$$

$$-3(5) + 1(6) - 3(2)$$

$$-15 + 6 - 6$$

$$15$$

$$3(-1) + 0(2) + 3(3)$$

$$-3 + 0 + 9$$

$$6$$

$$-3(-1) + 1(2) - 3(3)$$

$$3 + 2 - 9$$

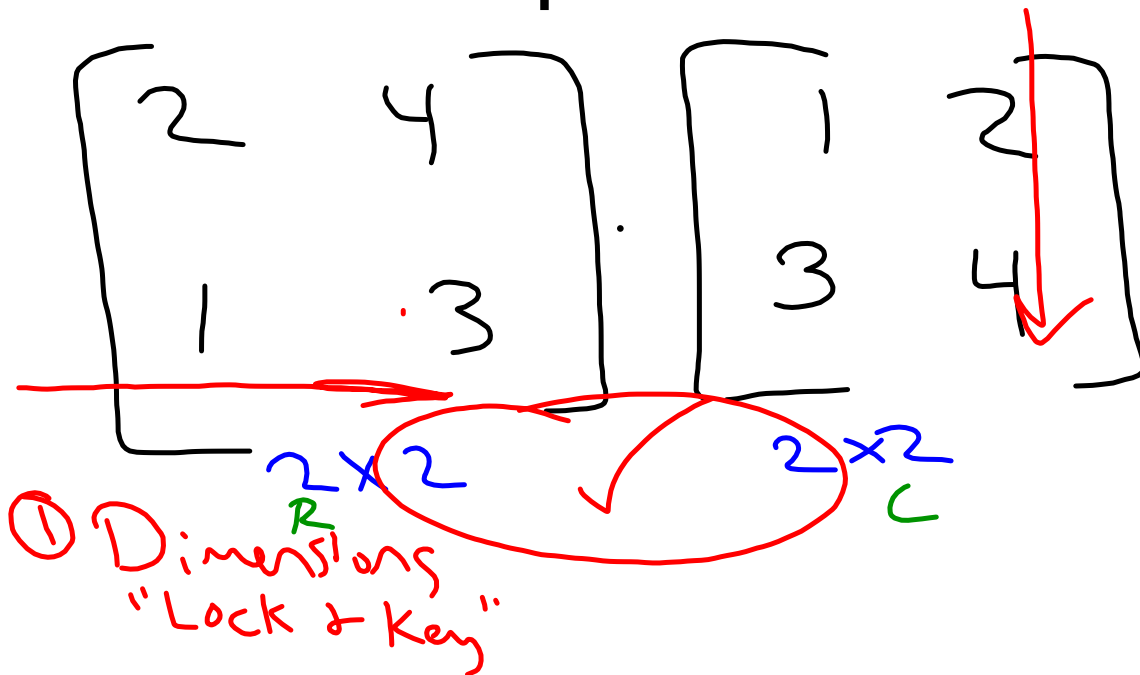
$$-4$$

$$\textcircled{3} \quad -3 \begin{bmatrix} 4 & 2 \\ x & -2 \end{bmatrix} + 2 \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 17 & a \end{bmatrix}$$
$$\begin{aligned} -3(4) + 2(5) &= 2 & -3(2) + 2(0) &= 6 \\ -3x + 2(1) &= 17 & -3(-2) + 2(4) &= a \end{aligned}$$



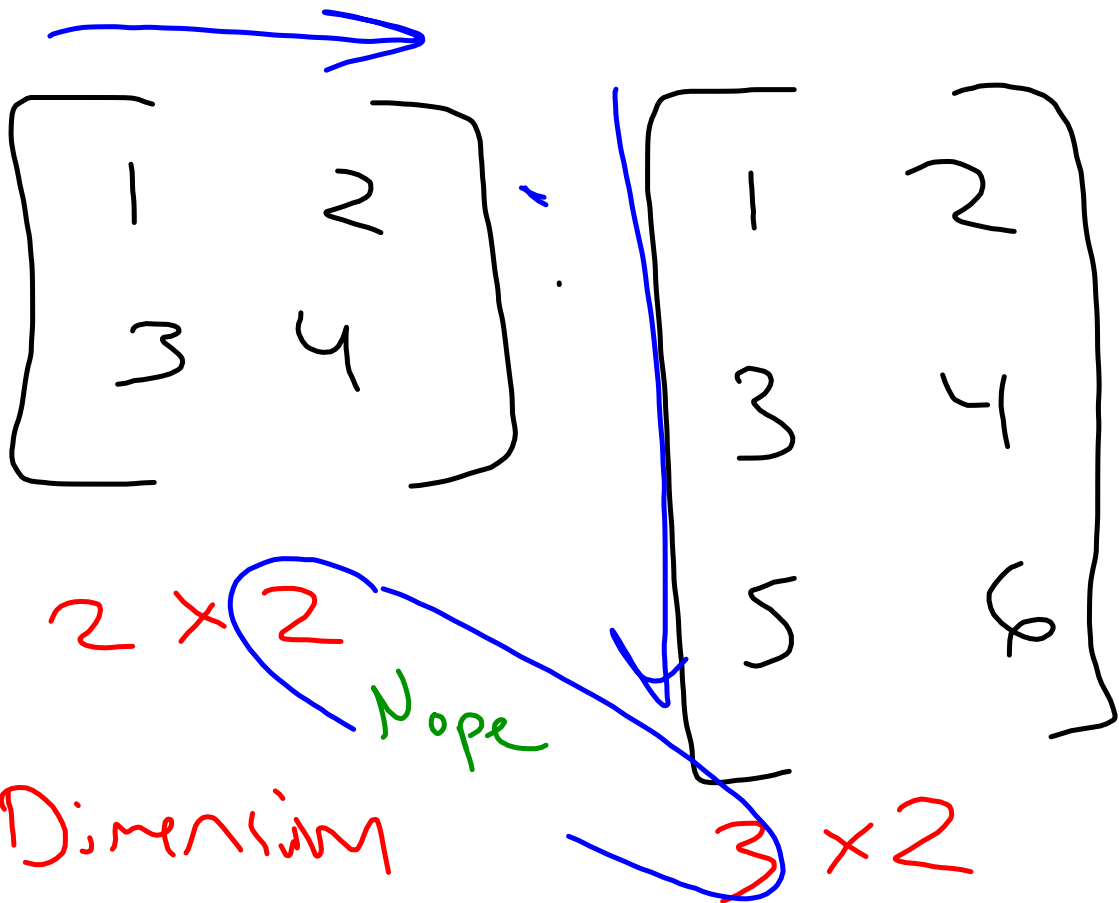
$$\begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

# Matrix Multiplication



TL	$2(1) + 4(3)$ $2 + 12$ $\underline{14}$	TR
BL	$1(1) + 3(3)$ $1 + 9$ $\underline{10}$	BR

$2 \times 2$



Undefined

2)

$$\begin{bmatrix} 3 & 0 & 3 \\ -3 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 6 & 2 \\ 2 & 3 \end{bmatrix}$$

① Dimensions  $2 \times 3$   $3 \times 2$

$$\begin{array}{l} \text{TL} \\ \begin{bmatrix} 3(5) + 0(6) + 3(2) \\ -3(-1) + 0(2) + 3(3) \\ -15 \\ 6 \\ -4 \end{bmatrix} \\ \text{BL} \end{array} \quad \begin{array}{l} \text{TR} \\ \begin{bmatrix} 15 + 0 + 6 \\ -3 + 0 + 9 \\ 6 \\ -4 \end{bmatrix} \\ \text{BR} \end{array}$$

$2 \times 2$

2)

$$\begin{bmatrix} 5 & -1 \\ 6 & 2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 & 3 \\ -3 & 1 & -3 \end{bmatrix}$$

① Dim.  $3 \times 2$   $\cdot$   $2 \times 3$

	T M	
<p>TL</p> $5(3) + 1(-3)$ $15 - 3$ $\underline{12}$	$6(0) + 2(1)$ $0 + 2$ $\underline{2}$	<p>TR</p> $6(3) + 2(-3)$ $18 - 6$ $\underline{12}$
<p>ML</p>	$2(0) + 3(1)$ $0 + 3$ $\underline{3}$	<p>MR</p>
<p>BL</p>	$3 \times 3$ BM	<p>BR</p>

Ex.?

$2 \times 2$   $3 \times 2$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}$$

undefined

2)

$$\begin{bmatrix} 3 & 0 & 3 \\ -3 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$2 \times 3$        $3 \times 2$

✓

=

$$\begin{bmatrix} 3(5) + 0(6) + 3(2) & 3(-1) + 0(2) + 3(3) \\ -3(5) + 1(6) + 3(2) & -3(-1) + 1(2) + -3(3) \end{bmatrix}$$

$$\begin{bmatrix} 15 + 6 & -3 + 0 + 9 \\ -15 + 6 - 6 & 3 + 2 - 9 \end{bmatrix}$$

$$\begin{bmatrix} 21 & 6 \\ -15 & -4 \end{bmatrix}$$

3)  $\begin{bmatrix} 3 & -3 \\ 1 & 2 \\ -5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$

$\underline{3} \times 2 \quad \checkmark \quad 2 \times 2$

$3 \times 2$



TL	$3(-1) + -3(-1)$ $-3 + 3$ $\boxed{0}$	$3(-1) + -3(-2)$ $-3 + 6$ $\boxed{3}$	TR
ML	$-4(-1) + 2(-1)$ $4 - 2$ $\boxed{2}$	$-4(-1) + 2(-2)$ $4 + -4$ $\boxed{0}$	MR
BL	$-4(-1) + 0(-1)$ $\boxed{4}$	$-4(-1) + 0(-2)$ $\boxed{4}$	BR

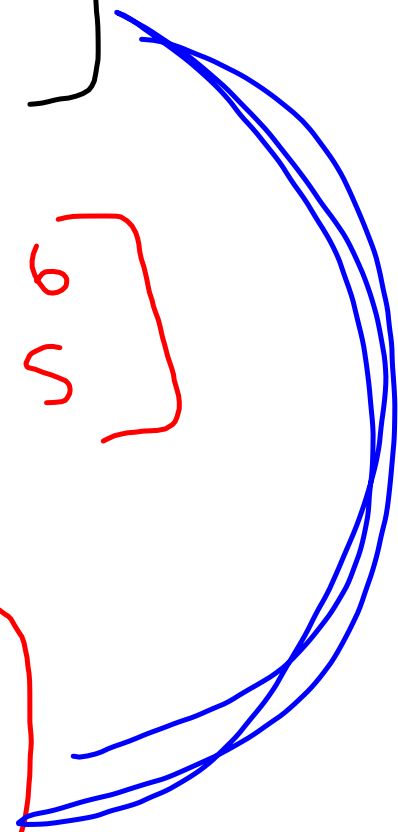
TL	$3(-1) + -3(-1)$ $-3 + 3$ $\boxed{0}$	$3(-1) + -3(-2)$ $-3 + 6$ $\boxed{3}$	TR
ML	$-4(-1) + 2(-1)$ $4 - 2$ $\boxed{2}$	$-4(-1) + 2(-2)$ $4 - 4$ $\boxed{0}$	MR
BL	$-4(-1) + 0(-1)$ $\boxed{4}$	$-4(-1) + 0(-2)$ $\boxed{4}$	BR

$$\begin{matrix} & A & & & + & & B \\ \left[ \begin{array}{cc} 3 & 6 \\ -3 & 5 \end{array} \right] & & & & & & \left[ \begin{array}{cc} 2 & -2 \\ -4 & -1 \end{array} \right] \end{matrix}$$

$$= \left[ \begin{array}{cc} 5 & 4 \\ -7 & 4 \end{array} \right]$$

$$\begin{matrix} & B & & + & & A \\ \left[ \begin{array}{cc} 2 & -2 \\ -4 & -1 \end{array} \right] & & & & & \left[ \begin{array}{cc} 3 & 6 \\ -3 & 5 \end{array} \right] \end{matrix}$$

$$= \left[ \begin{array}{cc} 5 & 4 \\ -7 & 4 \end{array} \right]$$



$$\begin{matrix} \text{A} \\ \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \end{matrix} + \begin{matrix} \text{B} \\ \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} 5 & 4 \\ -7 & 4 \end{bmatrix}$$

$$\begin{matrix} \text{B} \\ \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix} \end{matrix} + \begin{matrix} \text{A} \\ \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} 5 & 4 \\ -7 & 4 \end{bmatrix}$$

$$A + B = B + A$$

Matrix Addition  
is Commutative!

$$A \cdot B \neq B \cdot A$$

Matrix Multiplication  
is <sup>NOT</sup> Commutative!

Order Matters!

$$\begin{matrix} \text{A} \\ \rightarrow \end{matrix} \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \cdot \begin{matrix} \text{B} \\ \downarrow \end{matrix} \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix}$$

$$\begin{array}{l} 3(2) + 6(-4) \\ 6 + -24 \\ \boxed{-18} \end{array} \quad \begin{array}{l} 3(-2) + 6(-1) \\ -6 - 6 \\ \boxed{-12} \end{array}$$
$$\begin{array}{l} -3(2) + 5(-4) \\ -6 - 20 \\ \boxed{-26} \end{array} \quad \begin{array}{l} -3(-2) + 5(-1) \\ 6 - 5 \\ \boxed{1} \end{array}$$

$$\begin{matrix} B \\ \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix} \end{matrix} \cdot \begin{matrix} A \\ \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \end{matrix}$$

$$\begin{array}{l} 2(3) - 2(-3) \\ 6 + 6 \\ \boxed{12} \end{array} \quad \begin{array}{l} 2(6) + -2(5) \\ 12 - 10 \\ \boxed{2} \end{array}$$
$$\begin{array}{l} -4(3) + -1(-3) \\ -12 + 3 \\ \boxed{-9} \end{array} \quad \begin{array}{l} -4(6) + -1(5) \\ -24 - 5 \\ \boxed{-29} \end{array}$$

Matrix addition  
is Commutative!  
 $A + B = B + A$

Matrix multiplication  
is <sup>NOT</sup> Commutative!  
 $A \cdot B \neq B \cdot A$

Order matters!



$$\begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix}$$

$2 \times 2$        $2 \times 2$

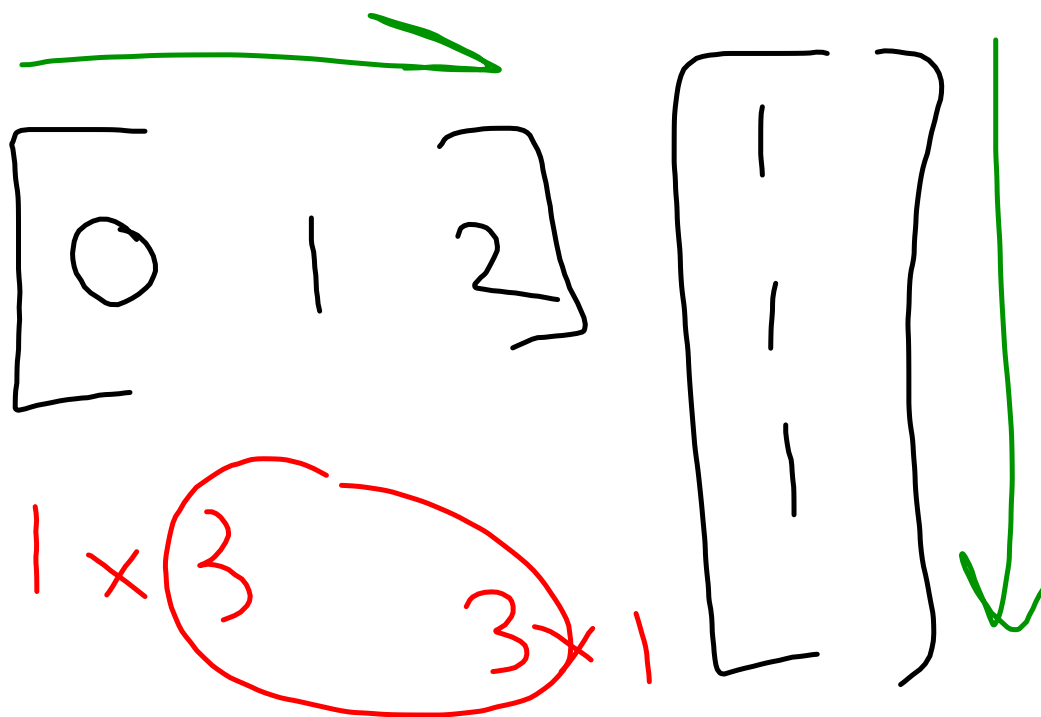
$$\begin{array}{l} 3(2) + 6(-4) \\ 6 - 24 \\ \boxed{-18} \end{array} \quad \begin{array}{l} 3(-2) + 6(-1) \\ -6 - 6 \\ \boxed{-12} \end{array}$$
$$\begin{array}{l} -3(2) + 5(-4) \\ -6 - 20 \\ \boxed{-26} \end{array} \quad \begin{array}{l} -3(-2) + 5(-1) \\ 6 - 5 \\ \boxed{1} \end{array}$$

$2 \times 2$

$$\begin{matrix} B & + & A \\ \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix} & \cdot & \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \\ 2 \times 2 & & 2 \times 2 \end{matrix}$$

$$\begin{array}{ll} 2(3) + -2(-3) & 2(6) + -2(5) \\ 6 + 6 & 12 - 10 \\ \boxed{12} & \boxed{2} \\ \\ -4(3) + -1(-3) & -4(6) + -1(5) \\ -12 + 3 & -24 - 5 \\ \boxed{-9} & \boxed{-29} \end{array}$$

$$2 \times 2$$



$$[0(1) + 1(1) + 2(1)] = [3]$$

$$3X = [3 \ 6]$$

$$X = \frac{1}{3}[3 \ 6]$$

$$\left( \right) [0 \ 2] \times \neq \left( \right) [2 \ 0]$$

$\begin{bmatrix} 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

Row  $1 \times 2$   $\checkmark$   $2 \times 1$  Column =

Row  $\times$  Column

① Lock & key  
②  $\rightarrow$  Add the Product

$\begin{bmatrix} 1(5) + 3(-1) \\ s + -3 \end{bmatrix}$

$1 \times 1$

$= [2]$

③

$$\begin{bmatrix} 3 & -3 \\ -4 & 2 \\ -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$$

$3 \times 2$   
 $R \times C$

$2 \times 2$   
 $R \times C$

① Lock & key

$$= \begin{bmatrix} 3(-1) + (-3)(-1) & 3(-1) + (-3)(-2) \\ -4(-1) + 2(-1) & -4(-1) + 2(-2) \\ -4(-1) + 0(-1) & -4(-1) + 0(-2) \end{bmatrix}$$

$3 \times 2$

TL TR  
 ML MR  
 BL BR

③

$$\begin{bmatrix} 3 & -3 \\ -4 & 2 \\ -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3(-1) + -3(-1) & 3(-1) + -3(2) \\ -4(-1) + 2(-1) & -4(-1) + 2(-2) \\ -4(-1) + 0(-1) & -4(-1) + 0(-2) \end{bmatrix}$$

$3 \times 2$   $\checkmark$   $2 \times 2$   
 R C

TL TR  
 ML MR  
 BL BR

$3 \times 2$

$$\textcircled{1} \begin{bmatrix} 3 & 6 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 \\ -4 & -1 \end{bmatrix}$$

$\underline{\quad 2 \times 2 \quad \cdot \quad 2 \times 2 \quad \quad \underline{\quad}}$

$$= \begin{bmatrix}
 \begin{array}{l}
 \text{TL} \quad 3(2)+6(-4) \\
 6-24 \\
 -18
 \end{array} &
 \begin{array}{l}
 \text{TR} \quad 3(-2)+6(-1) \\
 -6-6 \\
 -12
 \end{array} \\
 \begin{array}{l}
 \text{BL} \quad -3(2)+5(-4) \\
 -6-20 \\
 -26
 \end{array} &
 \begin{array}{l}
 \text{BR} \quad -3(-2)+5(-1) \\
 6-5 \\
 1
 \end{array}
 \end{bmatrix}$$

$2 \times 2$



