Hyperbola Notes

Objectives:
Find the center, vertices, and foci of a hyperbola.
Graph a hyperbola.
Write the equation of a hyperbola in standard form given the general form of the equation.
Write the equation of an hyperbola using given information.

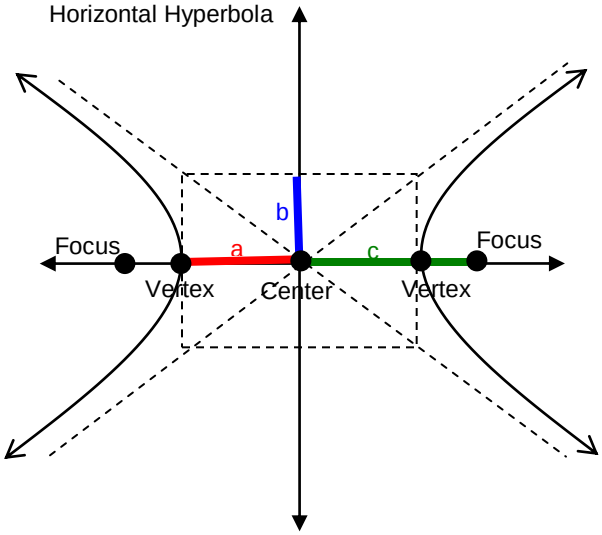
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$ *Opens horizontally (x is positive).*

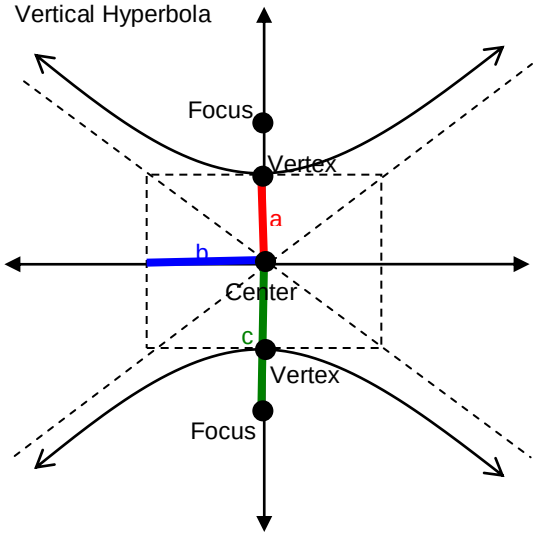
$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$ *Opens vertically (y is positive).*

Parts of an hyperbola:
center (h, k)

a – distance from the center to vertex

c – distance from the center to focus.
The foci lie on the transverse axis.
The foci lie inside the opening of the hyperbola.





Graphing Hyperbolas:

1. Find and plot the center (h, k). *Remember that the x-coordinate of the center is first.*
2. Sketch the rectangle and asymptotes.
   **The square root of the x-denominator tells you how far to move left and right from the center to the sides of the rectangle. The square root of the y-denominator tells you how far to move up and down from the center to the sides of the rectangle. The asymptotes pass through the diagonals of the rectangle. The hyperbola will approach the asymptotes.
3. Determine if the hyperbola is horizontal or vertical and sketch the graph.
4. Label the vertices and foci.
   How to plot FOCI:
   1) Find c, solve $a^2 + b^2 = c^2$
   2) Count from center c spaces each direction inside the opening of the hyperbola.

Examples:

1. $\frac{x^2}{9}-\frac{y^2}{4}=1$

2. $\frac{(y-2)^2}{4}-\frac{(x+1)^2}{5}=1$

Writing the standard form equation of a hyperbola

Examples:
Notice that the constant term in the standard form equation of a hyperbola is ONE. If an equation is already in the form $x^2 - y^2$ or $(x-h)^2 - (y-k)^2$, then you only need to divide by the constant and simplify the fractions to change the equation to standard form.

1. $9x^2 - 16y^2 = 144$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

**Simplify the fractions. How many times will 9 divide into 144? How many times will a6 divide into 144?*

2. $3(x+2)^2 - 4(y-1)^2 = 192$

$$\frac{3(x+2)^2}{192} - \frac{4(y-1)^2}{192} = \frac{192}{192}$$

$$\frac{(x+2)^2}{64} - \frac{(y-1)^2}{48} = 1$$

If the equation is in general form such as example #3, then you must complete the square to write the equation in standard form.

3. $4x^2 - 9y^2 - 16x + 18y - 65 = 0$

$$(4x^2 - 16x) + (-9y^2 + 18y) = 65$$

$$4(x^2 - 4x + \_\_\_) - 9(y^2 - 2y + \_\_\_) = 65$$

$$4(x^2 - 4x + 4) - 9(y^2 - 2y + 1) = 65 + 4(4) + -9(1)$$

$$4(x-2)^2 - 9(y-1)^2 = 72$$

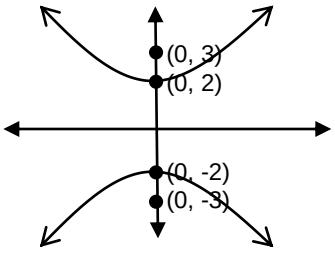
$$\frac{(x-2)^2}{18} - \frac{(y-1)^2}{8} = 1$$

1. Group x's and y's. Move the constant to the right.
2. Factor out the coefficient of x^2 and y^2.
3. Complete the square. **Remember that you must also add the same value on the right. You must multiply the value by the number in front of the parentheses. Don't forget the sign.*
4. Factor and simplify.
5. Divide by the constant and simplify the fractions. Write the positive term first.

Writing the equation of a hyperbola

1. Sketch the given information in needed.
2. Find the center(h, k), a and b. Use $a^2 + b^2 = c^2$ if needed.
3. Substitute into the standard form equations.

Example) Write the equation of a hyperbola with foci(0, -3) & (0, 3) and vertices (0, -2) & (0, 2).



The center is midway between the vertices – (0,0)
Since the hyperbola is vertical, use $y^2 - x^2$.

Plug-in the center: $\frac{y^2}{\_} - \frac{x^2}{\_} = 1$

The vertical distance from vertex to center is 2.
Square this value and write it under y: $\frac{y^2}{4} - \frac{x^2}{\_} = 1$

Use $a^2 + b^2 = c^2$ to find b^2.
$2^2 + b^2 = 3^2$, $b^2 = 5$
Substitute 5 into the equation.

$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$