

Midpoint Formula

$$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

Ellipse

$$c^2 = a^2 - b^2$$

Hyperbola

$$c^2 = a^2 + b^2$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

①

$$16x^2 + 9y^2 - 32x + 72y + 16 = 0$$

$-16 \quad -16$

$$16x^2 - 32x + 9y^2 + 72y = -16$$

$$16(x^2 - 2x + \boxed{1}) + 9(y^2 + 8y + \boxed{16}) = -16 + 16\boxed{1} + 9\boxed{16}$$

$-16 + 16 + 144$

$$\begin{array}{l} 16(x-1)^2 + 9(y+4)^2 = 144 \\ \hline 144 \div 16 \qquad \qquad 144 \div 9 \qquad \qquad \qquad 144 \\ \hline \end{array}$$

$$\frac{(x-1)^2}{9} + \frac{(y+4)^2}{16} = 1$$

$$b=3$$

$$a=4$$

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 9$$

$$c = 7 \quad c = \sqrt{7}$$

center $(1, -4)$
 vertices $(1, -4 \pm 4)$
 co-v $(1 \pm 3, -4)$
 foci $(1, -4 \pm \sqrt{7})$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \text{Standard form}$$

② ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

vertices $(-3, 0)$, $(7, 0)$

foci: $(0, 0)$, $(4, 0)$

center $\left(\frac{-3+7}{2}, \frac{0+0}{2} \right)$

$$(2, 0)$$

$$\frac{(x-2)^2}{25} + \frac{y^2}{21} = 1$$

$$a=5$$

$$c=2$$

$$c^2 = a^2 - b^2$$

$$4 = 25 - b^2$$

$$b^2 = 25 - 4$$

$$b^2 = 21$$

③ $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ center
ver
foci
asym
+151 +151

$9x^2 - 18x - 16y^2 - 32y = 151$

$9(x^2 - 2x + \square) - 16(y^2 + 2y + \square) = 151 + 9\square - 16\square$

$\frac{9(x-1)^2}{144} - \frac{16(y+1)^2}{144} = \frac{144}{144}$

$\frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$ center $(1, -1)$

$a = 4$

$b = 3$

$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{16 + 9}$
 $= \sqrt{25}$

$c = 5$

vert $(1 \pm 4, -1)$

foci $(1 \pm 5, -1)$

asym $\pm \frac{b}{a} = \frac{3}{4}x$

④ vertices $(-10, 3)$, $(6, 3)$ major axis is horizontal
 foci $(-12, 3)$, $(8, 3)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\text{center} \left(\frac{-10+6}{2}, \frac{3+3}{2} \right)$$

$$(-2, 3)$$

$$a = 8, c = 10$$

$$\frac{(x+2)^2}{64} - \frac{(y-3)^2}{36} = 1$$

$$c^2 = a^2 + b^2$$

$$100 = 64 + b^2$$

$$36 = b^2$$

⑤ center $(0,0)$
major axis = 120 = $2a$
focus $(1,0)$

$$c = 1$$
$$a = 60$$

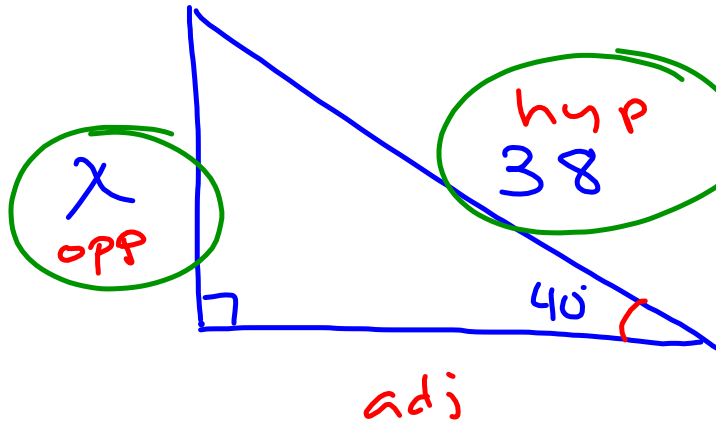
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{3600} - \frac{y^2}{3599} = 1$$

$$c^2 = a^2 - b^2$$
$$b^2 = a^2 - c^2$$

$$3600 - 1$$
$$b^2 = 3599$$

⑥

 $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$$38 \cdot \sin(40^\circ) = \frac{x}{38} \cdot 38$$

$$38 \sin(40^\circ) = x$$

$$x = 24.42$$

⑦ $0^\circ \leq \theta < 360^\circ$ Degree Mode

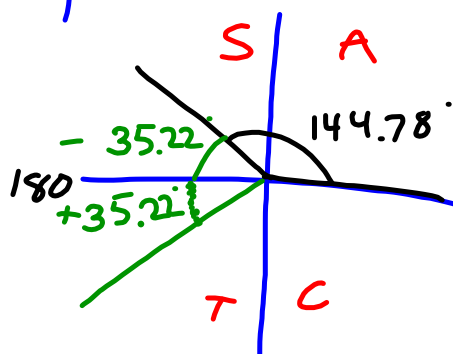
$$\sec \theta = -1.2241$$

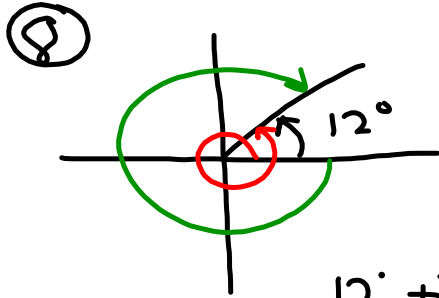
$$\underbrace{(\cos \theta)}_{\cos^{-1}} = \frac{1}{\underbrace{\sec \theta}_{\cos^{-1}}} = \left(\frac{1}{-1.2241} \right)$$

$$\theta = 144.78^\circ$$

and

$$215.22^\circ$$





$$12^\circ + 360^\circ = \underline{372^\circ}$$

$$12^\circ - 360^\circ = \underline{-348^\circ}$$

⑨

 $\csc 5.23$

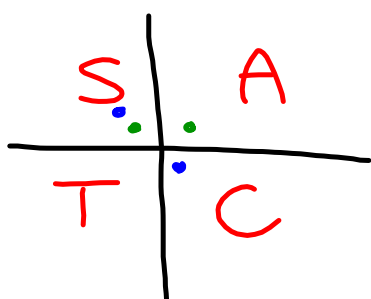
Radian Mode

$$= \frac{1}{\sin(5.23)} = -1.1507$$

⑩

$$\tan \theta < 0 : \text{II, IV}$$

$$\sin \theta > 0 : \text{I, II}$$

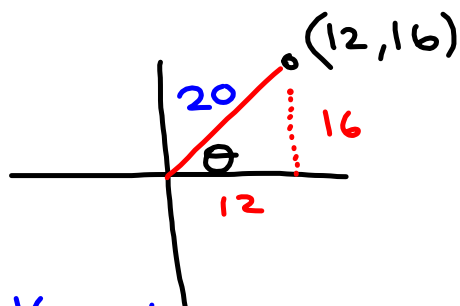


II

$$\textcircled{=} \quad \frac{5\pi}{7} \left(\frac{180}{\pi} \right) = \textcircled{\frac{900\pi}{7}}$$

$$\textcircled{12} \quad 480^\circ \left(\frac{\pi}{180} \right) = \textcircled{\frac{8\pi}{3}}$$

⑬



$$12^2 + 16^2 = c^2$$

$$144 + 256 = c^2$$

$$\sqrt{400} = \sqrt{c^2}$$

$$20 = c$$

$$\sin \theta = \frac{16}{20} = \frac{4}{5}$$

$$\csc \theta = \frac{5}{4}$$

$$\cos \theta = \frac{12}{20} = \frac{3}{5}$$

$$\sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{16}{12} = \frac{4}{3}$$

$$\cot \theta = \frac{3}{4}$$

⑭

$$\sin \theta = \frac{12 \cdot \sqrt{61}}{2\sqrt{61} \cdot \sqrt{61}} = \frac{6\sqrt{61}}{61}$$

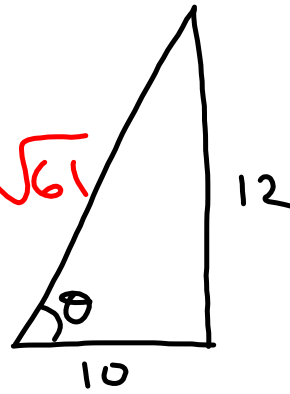
$$\csc \theta = \frac{\sqrt{61}}{6}$$

$$\cos \theta = \frac{10}{2\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{5\sqrt{61}}{61}$$

$$\sec \theta = \frac{\sqrt{61}}{5}$$

$$\tan \theta = \frac{12}{10} = \frac{6}{5}$$

$$\cot \theta = \frac{5}{6}$$



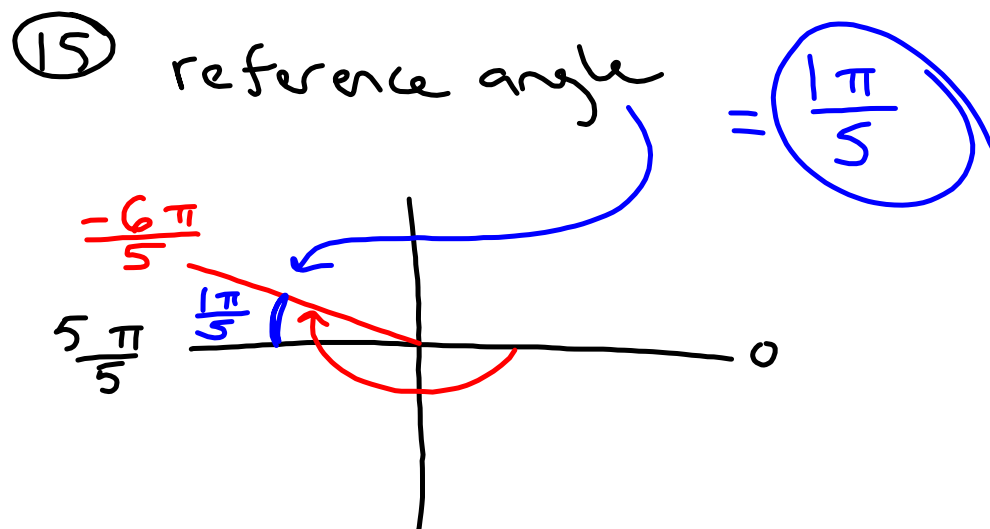
$$10^2 + 12^2 = c^2$$

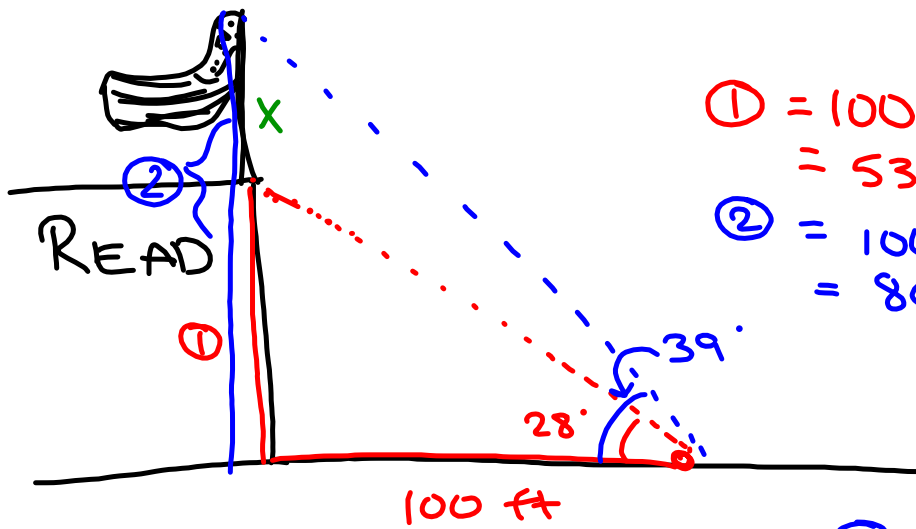
$$100 + 144 = c^2$$

$$\sqrt{244} = \sqrt{c^2}$$

$$61 \cdot 4 = c^2$$

$$c = 2\sqrt{61}$$





$$\textcircled{1} = 100 \tan(28^\circ)$$
$$= 53.17$$

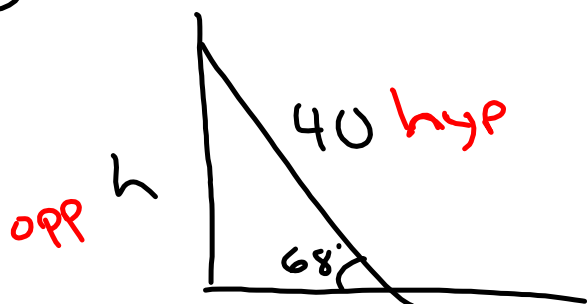
$$\textcircled{2} = 100 \tan(39^\circ)$$
$$= 80.98$$

$$x = \textcircled{2} - \textcircled{1}$$

$$x = 80.98 - 53.17$$

$$x = 27.81 \text{ feet}$$

⑪



$$40 \cdot \sin(68) = \frac{h}{40} \cdot 40$$

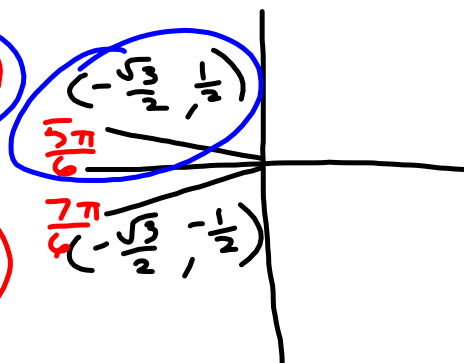
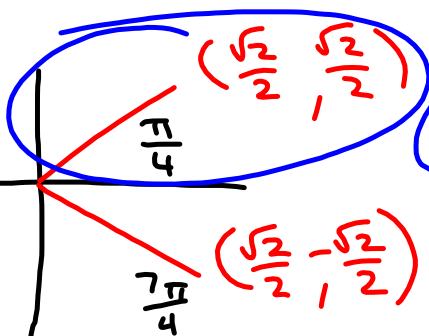
$$h = 37.09 \text{ feet}$$

18

a) $\arccos\left(\frac{\sqrt{2}}{2}\right)$

b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

range of
arccos
is $[0, \pi]$



①

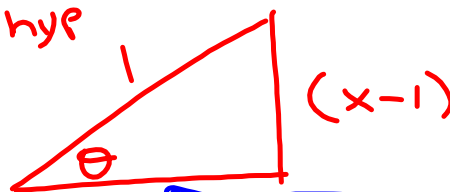
$$\sec \left[\arcsin \left(\frac{x-1}{1} \right) \right]$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$= \frac{1}{\sqrt{-x^2+2x}}$$

$$= \frac{\sqrt{-x^2+2x}}{-x^2+2x}$$

$$= \frac{-\sqrt{-x^2+2x}}{x^2-2x}$$



$$a^2 + b^2 = c^2$$

$$(x-1)^2 + b^2 = 1^2$$

$$x^2 - 2x + 1 + b^2 = 1$$

$$b^2 = -x^2 + 2x$$

$$b = \sqrt{-x^2 + 2x}$$

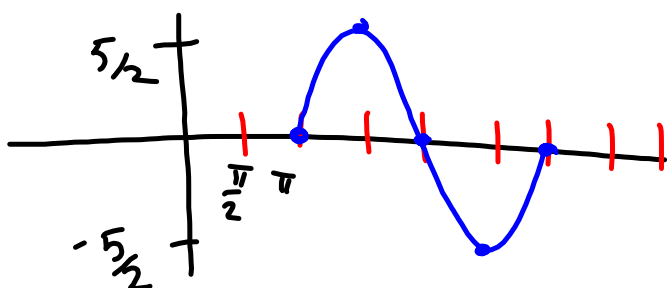
$$\textcircled{20} \quad y = \frac{5}{2} \sin(x - 180^\circ) = \frac{5}{2} \sin(x - \pi)$$

$$\text{amplitude} = \frac{5}{2}$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

$$\text{HS} \rightarrow \frac{c}{b} = \frac{\pi}{1}$$

π to right



$$\textcircled{21} \quad g(x) = \frac{1}{4} \tan(x - 90^\circ) = \frac{1}{4} \tan\left(x - \frac{\pi}{2}\right)$$

$$\text{amp} = \frac{1}{4}$$

$$\text{Per} = \frac{\pi}{b} = \frac{\pi}{1} = \pi$$

$$\textcircled{22} \quad y = \sin x$$

$$y = \frac{1}{2} \sin(\pi x) - 3$$

- ① vertical shrink by $\frac{1}{2}$.
- ② horizontal shrink by $\frac{1}{\pi}$.
- ③ vertical shift down 3.

$$\textcircled{23} \quad y = \tan x$$
$$y = \tan\left(x + \frac{\pi}{4}\right)$$

① horizontal shift left $\frac{\pi}{4}$.

$$(24) \quad y = -\tan\left(x - \frac{\pi}{3}\right)$$

Vertical asymptote

$$\tan: \quad x = \frac{\pi}{3} + n\pi$$

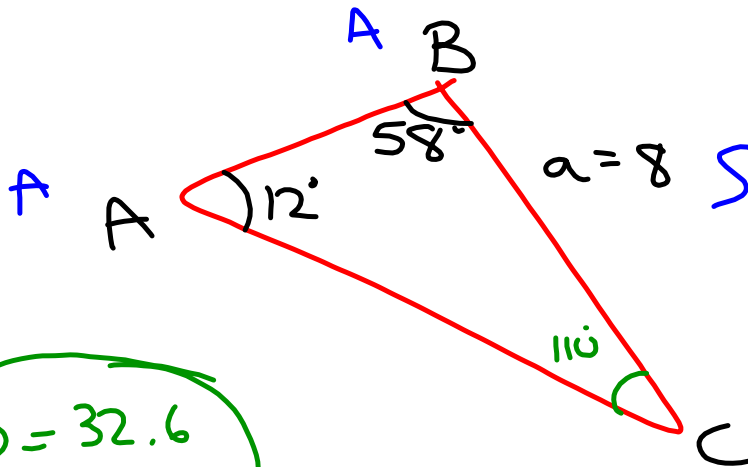
$$x = (\text{first asy}) + n(\text{period})$$

$$\begin{array}{r} \downarrow \\ x - \frac{\pi}{3} = 0 \\ + \frac{\pi}{3} \quad + \frac{\pi}{3} \\ \hline x = \frac{\pi}{3} \end{array}$$

$$\frac{\pi}{b} = \frac{\pi}{1} = \pi$$

What are six trig functions of
triangle with terminal side through $(-1, -3)$?

25) $A = 12^\circ$ $B = 58^\circ$ $a = 8$



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$b = \frac{8 \cdot \sin(58)}{\sin(12)} = 32.6$$

$$c = \frac{8 \cdot \sin(110)}{\sin(12)} = 36.2$$

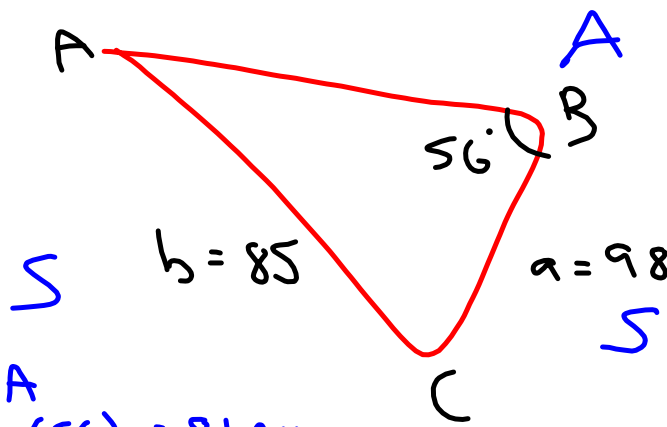
$$b = 32.6$$

$$c = 36.2$$

26

$B = 56^\circ$ $a = 98$ $b = 85$
"a" "b"

Find C.

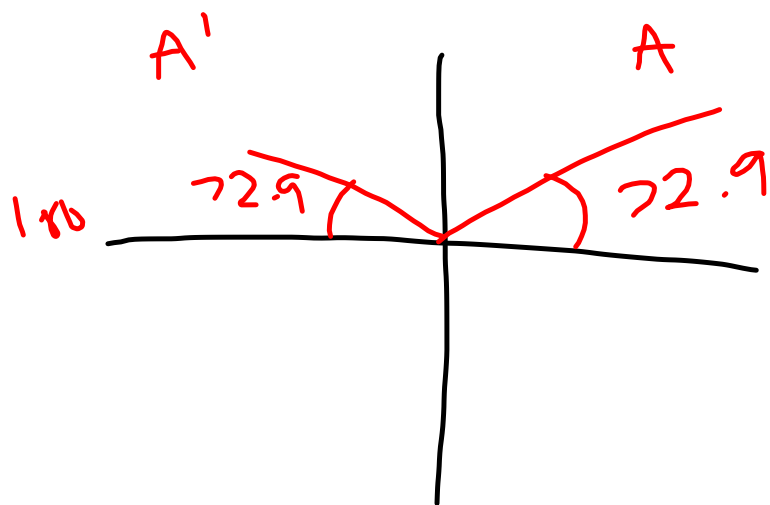


$C = 51.1^\circ$
 $C' = 16.9^\circ$

$b = 85$ $a = 98$

$h = b \sin A$
 $= 98 \sin(56) = 81.24$

$81.24 < 85 < 98$
 $h < "a" < "b"$
 2 solutions



$$\frac{\sin A}{98} = \frac{\sin 56}{85} = \frac{\sin C}{c}$$

$$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{98 \cdot \sin(56)}{85}\right) = 72.90^\circ = A$$

$$\Delta - B - A = C$$

$$C = 180 - 56 - 72.9 = 51.1^\circ$$

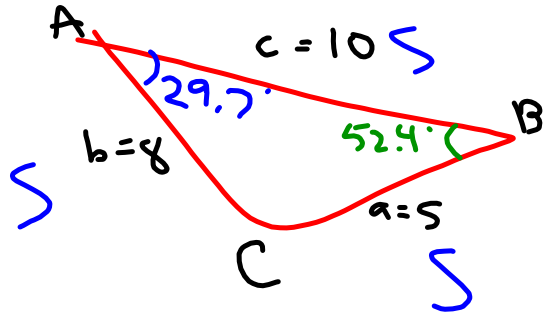
$$A' = 180 - A = 180 - 72.9$$

$$\Delta - B - A' = C'$$

$$C' = 180 - 56 - 107.1 = 16.9^\circ$$

(27) $a = 5, b = 8, c = 10$

Solve the triangle.



Law of Cosines

$$A = \cos^{-1} \left(\frac{(8^2 + 10^2 - 5^2)}{(2(8)(10))} \right)$$

$A = 29.69^\circ$

$$B = \cos^{-1} \left(\frac{(5^2 + 10^2 - 8^2)}{(2(5)(10))} \right)$$

$B = 52.41^\circ$

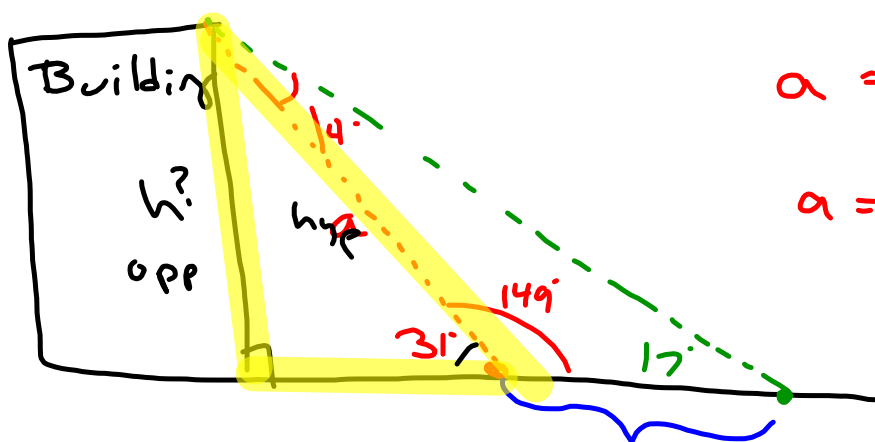
$$\begin{array}{r} 180 \\ - 52.41 \\ - 29.69 \\ \hline 97.9^\circ = C \end{array}$$

②⑧ $A=71^\circ$ $b=10$ $c=19$, Find area.

$$\begin{aligned} \text{Area} &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} (10)(19) \sin(71) \\ &= 89.82 \text{ cm}^2 \end{aligned}$$

$$\textcircled{29} \quad A \cdot E = 17^\circ$$

$$A \cdot E = 31^\circ$$



$$\sin(31) = \frac{\text{height}}{60.43} \quad 50\text{m}$$

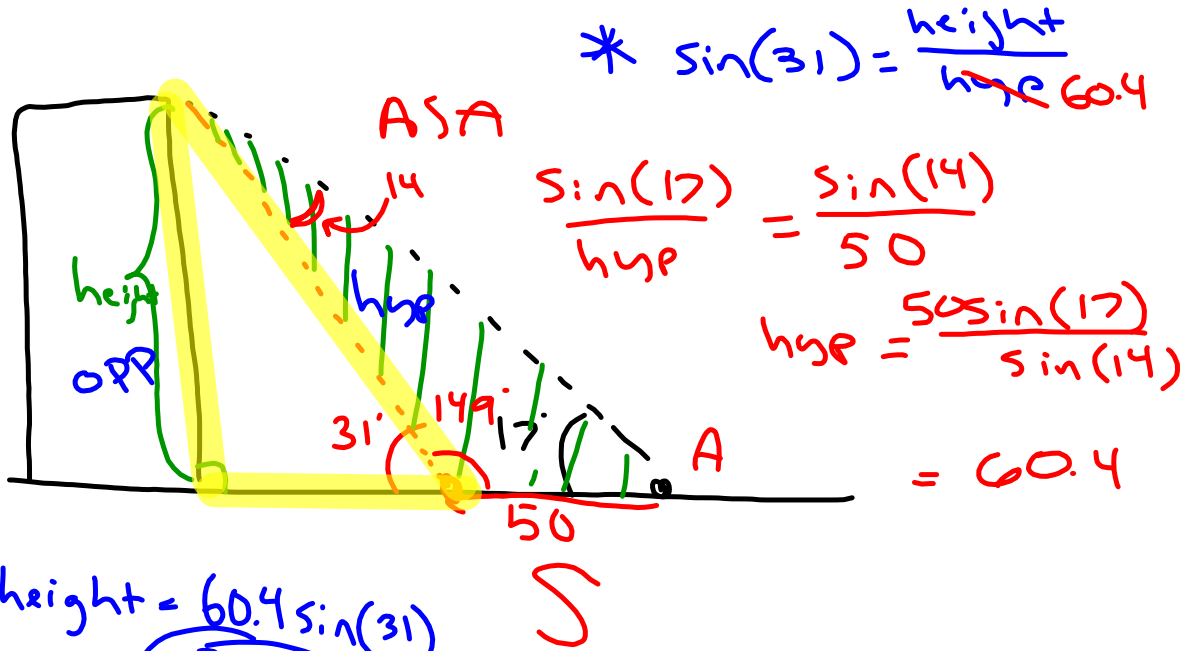
$$60.43 \sin(31) = \textcircled{31.12 \text{ m}}$$

Law of Sines
ASA =

$$\frac{\sin(14)}{50} = \frac{\sin(17)}{a}$$

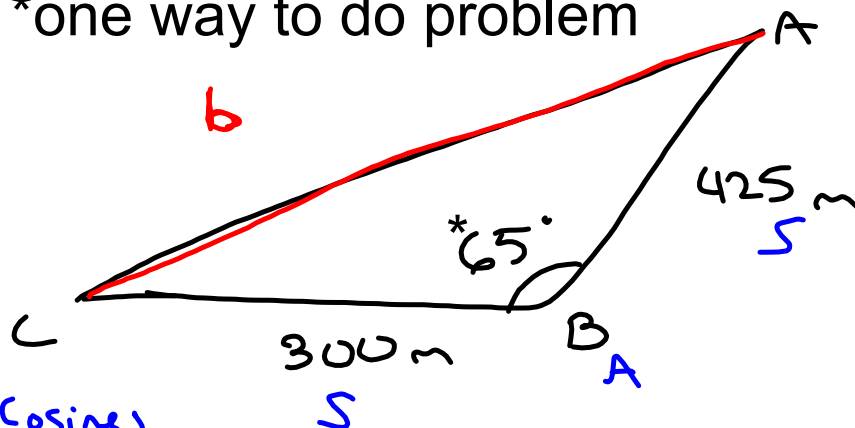
$$a = \frac{50 \sin(17)}{\sin(14)}$$

$$a = \underline{\underline{60.43 \text{ m}}}$$



* $\text{height} = 60.4 \sin(31)$
 $= 31.1 \text{ m}$

30 *one way to do problem



Law of Cosines

$$b = \sqrt{300^2 + 425^2 - 2(300)(425)\cos(65)}$$

$$= 403.5 \text{ m}$$

$$\textcircled{31} \quad \sin X = \frac{2\sqrt{2}}{3} \begin{array}{l} \text{opp} \\ \text{hyp} \end{array}$$

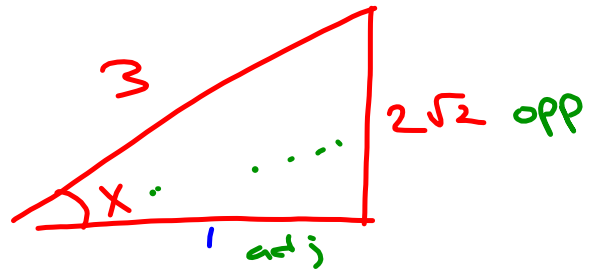
$$\cos X = \frac{1}{3}$$

$$\sec X = 3$$

$$\csc X = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan X = \frac{2\sqrt{2}}{1}$$

$$\cot X = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$



$$a^2 + b^2 = c^2$$

$$(2\sqrt{2})^2 + b^2 = 3^2$$

$$(4 \cdot 2) + b^2 = 9$$

$$8 + b^2 = 9$$

$$-8 \quad -8$$

$$\hline \sqrt{b^2} = \sqrt{1}$$

$$b = 1$$

Skip 32

$$\textcircled{33} \frac{\cos x}{\cos x} \frac{1+\sin x}{1+\sin x} + \frac{1+\sin x}{\cos x} \frac{1+\sin x}{1+\sin x}$$

common den.

$$\frac{(\cos^2 x + \sin^2 x) + 2\sin x + 1}{\cos x(1+\sin x)}$$

$$\begin{array}{|c|c|} \hline 1 & 1+\sin x \\ \hline +\sin x & \hline \hline \end{array}$$

$$\frac{1+1+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2(1+\sin x)}{\cos x(1+\sin x)}$$

$$2 \left(\frac{1}{\cos x} \right) = 2 \sec x$$

34

$$\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$$

$$\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \cdot \sin^2 x}{1 \cdot \sin^2 x}$$

$$\frac{\cos^2 x - \cos^2 \sin^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x \sin^2 x (1 - \sin^2 x)}{\sin^2 x}$$

$$\frac{\cos^2 x \sin^2 x (\cos^2 x)}{\sin^2 x}$$

$$\cot^2 x \cos^2 x = \cot^2 x \cos^2 x$$

Rewrite
Add fract.

GCF
Pyth. Id.
Rewrite
 $\cos^2 x + \sin^2 x = 1$
 $-\sin^2 x \rightarrow$

$$\cos^2 x = 1 - \sin^2 x$$

$$\textcircled{35} \quad [0, 2\pi)$$

$$3 + \tan^2 x - 1 = 0$$

+1 +1

$$\frac{3 + \tan^2 x}{3} = \frac{1}{3}$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan^{-1}(\tan x) = \pm \frac{1}{\sqrt{3}} \quad \frac{5}{x}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

36

b. $2\sin x - 1 = 0$

$$\frac{\quad + 1 \quad + 1}{\quad}$$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

(37)

$$2 \cos^2 x - \cos x = 1$$

$$\overset{a}{2} \cos^2 x - \overset{b}{\cos x} - \overset{c}{1} = 0$$

$$\left(\cos x - \frac{1}{2} \right) \left(\cos x + \frac{1}{2} \right) = 0$$

$$\cos x - \frac{1}{2} = 0$$

$$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$x = 0$$

$$\cos x + \frac{1}{2} = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\begin{array}{cc} a \cdot c & \\ -2 & +1 \\ -2 & -1 \\ & b \end{array}$$

③⑧

$$\begin{aligned} & \sin(105^\circ) \\ \sin(45+60) &= \sin(45)\cos(60) + \cos(45)\sin(60) \\ & \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ & \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ & \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

39) $\tan u = \frac{3}{4}$, $0 < u < \frac{\pi}{2}$

$\cos(u-v)$

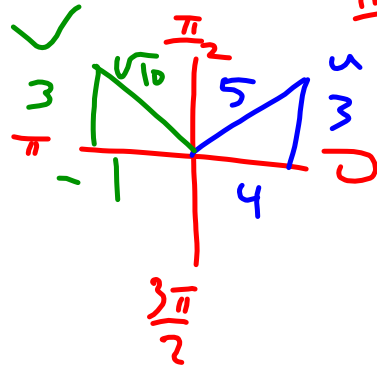
$= \cos u \cos v + \sin u \sin v$
 $\left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{10}}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{\sqrt{10}}\right)$
 $\frac{-4}{5\sqrt{10}} + \frac{9}{5\sqrt{10}}$

$= \frac{-4 + 9}{5\sqrt{10}}$

$= \frac{5}{5\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\csc v = \frac{\sqrt{10}}{3}$

$\frac{\pi}{2} < v < \pi$



~~~~~ 5

$$\textcircled{48} \quad P(3,2) \quad Q(-2,-5)$$


$$\vec{PQ} = \left\langle \overset{Q-P}{(-2-3)}, (-5-2) \right\rangle$$

$$\langle -5, -7 \rangle$$

$$|\vec{PQ}| = \sqrt{(-5)^2 + (-7)^2}$$

$$= \sqrt{25 + 49}$$

$$= \sqrt{74}$$



$$\theta = \tan^{-1}\left(\frac{-7}{-5}\right) = 54.4^\circ$$

$$+ 180^\circ$$

$$\underline{\underline{234.4^\circ}}$$

(49)

$4 - 8i$

in polar form

$\langle 4, -8 \rangle$

~~$\frac{-8}{4}$~~

$r (\text{cis } \theta)$   
mag  $\rightarrow$   $r$   $\uparrow$  dir  $\theta$

$r = \sqrt{4^2 + (-8)^2}$

$= \sqrt{80} = 2\sqrt{20} = 4\sqrt{5}$

$\theta = \tan^{-1}\left(\frac{-8}{4}\right) = -63.4$   
 $+360$   
 $\hline 296.6^\circ$

$4\sqrt{5} (\cos(297) + i\sin(297))$

$4\sqrt{5} (\text{cis}(296.6))$

Skip 32 |

com den.

$$\textcircled{33} \frac{\cos x}{\cos x(1+\sin x)} + \frac{1+\sin x}{\cos x(1+\sin x)}$$

$$\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)} = \frac{2 + 2\sin x}{\cos(1+\sin x)}$$

$$\frac{2(1+\sin x)}{\cos x(1+\sin x)} = 2 \left( \frac{1}{\cos x} \right)$$

$$= 2 \sec x$$

34)  $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

$$\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x (\sin^2 x)}{1 (\sin^2 x)}$$

$$\frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x (\overset{\sin^2 x}{1 - \sin^2 x})}{\sin^2 x}$$

$$\frac{\cos^2 x (\cos^2 x + \sin^2 x - \sin^2 x)}{\sin^2 x}$$

$$\frac{(\cos^2 x)(\cos^2 x)}{\sin^2 x}$$

$$\cot^2 x \cos^2 x = \cot^2 x \cos^2 x$$

Rewrite  
common den.

GCF

Pyth. ID  
 $\cos^2 x + \sin^2 x = 1$

Rewrite

(35)

$$3 \tan^2 x - 1 = 0$$

$$3 \tan^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan^{-1}(\tan x) = \left( \pm \frac{1}{\sqrt{3}} \right) \times 15$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

36

$$b. \quad 2\sin x - 1 = 0$$

$$\hline$$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$



$$\textcircled{37} \quad 2\cos^2 x - \cos x = 1$$


---


$$2\cos^2 x - \cos x - 1 = 0$$

$$\left(\cos x - \frac{2}{2}\right)\left(\cos x + \frac{1}{2}\right) = 0$$

$$\cos x - 1 = 0$$


---


$$\cos x = 1$$

$$x = 0$$

$$\cos x + \frac{1}{2} = 0$$


---


$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

a.c

~~$$\begin{array}{r} -2 \\ -2 \end{array} \quad \begin{array}{r} 1 \\ -1 \\ b \end{array}$$~~

$$\begin{aligned} & \textcircled{38} \sin(105^\circ) \\ &= \sin(\underset{1}{60} + \underset{2}{45}) = \sin(60)\cos(45) + \cos(60)\sin(45) \\ & \quad \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ & \quad \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\ & \quad \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

39)  $\tan u = \frac{3}{4}$   $0 < u < \frac{\pi}{2}$   
 $\csc v = \frac{\sqrt{10}}{3}$   $\frac{\pi}{2} < v < \pi$

$\cos(u-v)$

$= \cos u \cos v + \sin u \sin v$

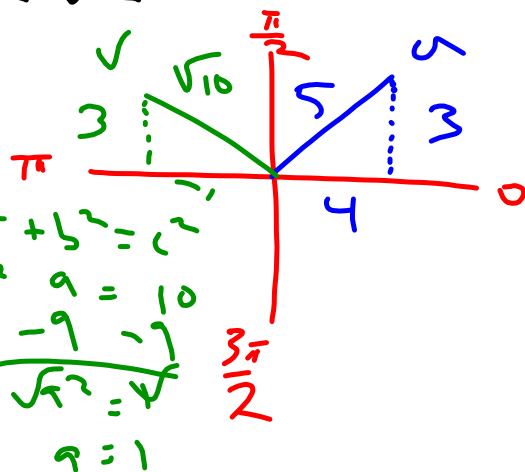
$\left(\frac{4}{5}\right)\left(\frac{-1}{\sqrt{10}}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{\sqrt{10}}\right)$

$\frac{-4}{5\sqrt{10}} + \frac{9}{5\sqrt{10}}$

$\frac{5}{5\sqrt{10}}$

$= \frac{1}{\sqrt{10}} \cdot \sqrt{10}$

$= \frac{\sqrt{10}}{10}$



$$\textcircled{40} \begin{bmatrix} 2 & 3 & 4 & 5 \\ -1 & 0 & 4 & -6 \end{bmatrix}$$

$$2 \times 4$$

④ Solve for  $y$ .

$$x + 2y + z = 5$$

$$2x - y - 3z = 5$$

$$-2x + 3y + z = -11$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ -11 \end{bmatrix}$$

$$y = \frac{D_y}{D} = \frac{-20}{20}$$

$$y = -1$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & -1 & -3 & 2 & -1 \\ -2 & 3 & 1 & -2 & 3 \end{vmatrix}$$

$$= -1 + 2 + 6 - 2 + 9 - 4$$

$$= 20$$

$$D_y = \begin{vmatrix} 1 & 5 & 1 & 1 & 5 \\ 2 & 5 & -3 & 2 & 5 \\ -2 & -11 & 1 & -2 & -11 \end{vmatrix}$$

$$= 5 + 30 - 22 + 10 - 33 - 10$$

$$= -20$$

$$\textcircled{47} \text{ Find area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 7 & 2 & 1 \\ 9 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2} ( \underline{2} + \underline{27} + \underline{35} - \underline{18} - \underline{5} - \underline{21} )$$

$$= \frac{1}{2} ( 20 )$$

$$\textcircled{= 10} \text{ AREA IS POSITIVE!}$$

50

$$5(\cos(38^\circ) + i \sin(38^\circ))$$

$r = 5$   
magnitude

$\theta = 38^\circ$

Direction

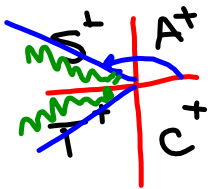
$$\langle 5 \cos(38), 5 \sin(38) \rangle$$

$$\langle 3.94, 3.07 \rangle$$

⑦ 2 values  $\theta$  ( $0^\circ \leq \theta < 360^\circ$ )

Degree Mode

$$\frac{1}{\sec \theta} = -1.2241$$



$$\cos \theta = \frac{1}{\sec \theta} = -\frac{1}{1.2241}$$

$$\cos \theta = \left( \frac{1}{-1.2241} \right)$$

$$\theta = 35.22 + 180$$

$$= 215.22^\circ$$

$$\theta = \frac{-140}{-35.22} = 144.78^\circ$$



$$\textcircled{37} \quad \frac{2 \cos^2 x - \cos x = 1}{-1 \quad -1}$$

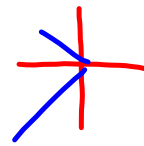
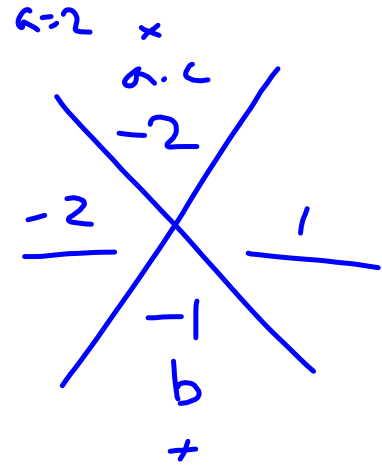
$$2 \cos^2 x - \cos x - 1 = 0$$

$$\left( \cos x - \frac{2}{2} \right) \left( \cos x + \frac{1}{2} \right)$$

$$(\cos x - 1) \left( \cos x + \frac{1}{2} \right) = 0$$

$$\begin{array}{l} \cos x - 1 = 0 \\ +1 \quad +1 \\ \hline \cos x = 1 \\ \cos^{-1}(\cos x) \\ x = 0 \end{array}$$

$$\begin{array}{l} \cos x + \frac{1}{2} = 0 \\ -\frac{1}{2} \quad + \\ \hline \cos x = -\frac{1}{2} \\ \cos^{-1}(\cos x) \\ x = \frac{2\pi}{3}, \frac{4\pi}{3} \end{array}$$



41) Solve for y.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -3 \\ -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ -11 \end{bmatrix}$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 2 & -1 & -3 & 2 & -1 \\ -2 & 3 & 1 & -2 & 3 \end{vmatrix} \begin{matrix} + \\ - \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix}$$

$$-1 + 12 + 6 - 2 - 9 - 4$$

$$-1 + 12 + 6 - 2 + 9 - 4 = 20$$

$$D_y = \begin{vmatrix} 1 & 5 & 1 & 1 & 5 \\ 2 & 5 & -3 & 2 & 5 \\ -2 & -1 & 1 & -2 & -1 \end{vmatrix} \begin{matrix} + \\ - \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix} \begin{matrix} + \\ - \\ + \end{matrix} \begin{matrix} - \\ + \\ - \end{matrix}$$

$$+5 + 30 - 22 - 10 - 33 - 10$$

$$5 + 30 - 22 + 10 - 33 - 10 = -20$$

$$y = \frac{D_y}{D} = \frac{-20}{20} = -1$$

$$y = -1$$

(46)

$$\begin{vmatrix} (5+2x) & -2 \\ (7-x) & 3 \end{vmatrix} = 5$$

$$3(5+2x) - (7-x)(-2) = 5$$

$$15 + 6x - (-14 + 2x) = 5$$

$$\underline{15} + \underline{6x} + \underline{14} - \underline{2x} = 5$$

$$\begin{array}{r} 29 + 4x = 5 \\ -29 \quad -29 \\ \hline \end{array}$$

$$\frac{4x}{4} = \frac{-24}{4}$$

$$x = -6$$

③④ verify.

$$\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$$

35

$$\frac{3 \tan^2 x - 1 = 0}{\substack{+1 \quad +1}}$$

$$\frac{3 \tan^2 x = 1}{3} = \frac{1}{3}$$

$$\tan^2 x = \frac{1}{3}$$

$$\sqrt{(\tan x)^2} = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$$

$$\cancel{\tan}(\tan x) = \left( \pm \frac{\sqrt{3}}{3} \right) \times \cancel{1}$$

$$\cancel{\tan}^{-1}$$

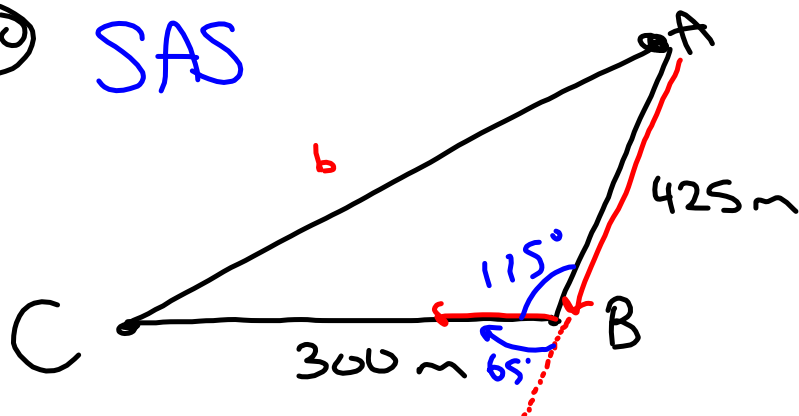
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{7\pi}{6}$$

$$\frac{\cancel{\frac{\pi}{6}}}{\cancel{2}} \cdot \frac{\cancel{\frac{\pi}{6}}}{\cancel{2}} = \frac{\pi}{3}$$

$$\frac{\cancel{\frac{5\pi}{6}}}{\cancel{2}} \cdot \frac{\cancel{\frac{5\pi}{6}}}{\cancel{2}} = \frac{5\pi}{3}$$

30

SAS



$$b = \sqrt{a^2 + c^2 - 2ac \cdot \cos B}$$

$$= \sqrt{(300^2 + 425^2 - 2 \cdot 300 \cdot 425 \cdot \cos(115^\circ))}$$

$$= 615.14 \text{ m}$$

$$\textcircled{31} \quad \csc\left(\frac{\pi}{2} - x\right) = 3$$

$$\sin x = \frac{2\sqrt{2}}{3} \quad \begin{array}{l} \text{opp} \\ \text{hyp} \end{array}$$

$$\begin{aligned} \csc(x) &= \frac{3 \cdot \sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} & 2\sqrt{2} &= \text{opp} \\ &= \frac{3\sqrt{2}}{4} \end{aligned}$$

$$\cos(x) = \frac{1}{3}$$

$$\sec(x) = \frac{3}{1}$$

$$\tan(x) = \frac{2\sqrt{2}}{1}$$

$$\cot(x) = \frac{1 \cdot \sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{4}$$



$$1 \quad \text{adj} = 1$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + (2\sqrt{2})^2 &= 3^2 \end{aligned}$$

$$a^2 + 4 \cdot 2$$

$$\begin{aligned} a^2 + 8 &= 9 \\ -8 & \quad -8 \end{aligned}$$

---


$$\sqrt{a^2} = \sqrt{1}$$

$$a = 1$$

28

$$A = 71^\circ \quad b = 10 \text{ m} \quad c = 19 \text{ m}$$

Area

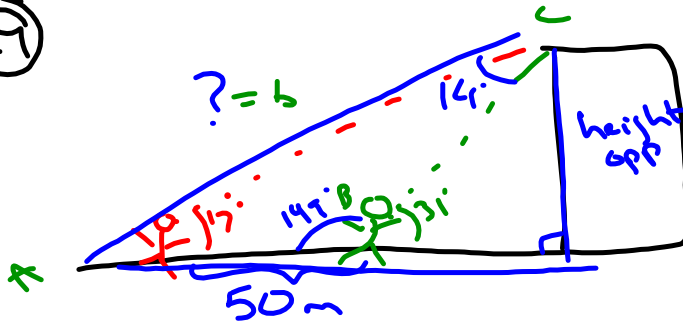
$$\frac{1}{2} a c \sin B = \frac{1}{2} b c \sin A = \frac{1}{2} a b \sin C$$

$$\frac{1}{2} (10)(19) \sin(71^\circ)$$

$$= 89.82 \text{ m}^2$$



29



ASA  $\rightarrow$  LoS

$$\frac{\sin 17^\circ}{a} = \frac{\sin 49^\circ}{b} = \frac{\sin 14^\circ}{50}$$

$$\frac{b \sin(17)}{\sin(17)} = \frac{50 \sin(49)}{\sin(17)}$$

$$b = 106.44 \text{ m}$$

$$\sin(17) = \frac{\text{opp}}{106.44}$$

$$106.44 \sin(17) = \text{opp}$$

$$31.12 \text{ m} = \text{opp}$$

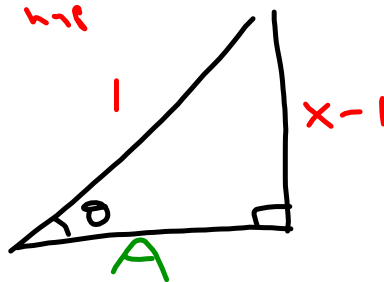
The building is 31.12 m tall.  
or 90.25 ft

19

$$\sec \left[ \arcsin \left( \frac{x-1}{1} \right) \right]$$

$$\sec \theta = \frac{H}{A}$$

$$\sec \theta = \frac{1}{x(2-x)}$$



$$a^2 + b^2 = c^2$$

$$a^2 + (x-1)^2 = 1^2$$

$$a^2 + x^2 - 2x + 1 = 1$$

$$a^2 = 2x - x^2$$