Warm-Up

February 2, 2017

Solve using the inverse.

$$\begin{bmatrix} 3 & 0 \\ 6 & -4 \end{bmatrix} X = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & -5 \end{bmatrix}$$

① Find Inverse
- det
$-\frac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

② Multiply

$3(-4) - 0(6)$
-12

$$-\frac{1}{12}\begin{bmatrix} -4 & 0 \\ -6 & 3 \end{bmatrix} X = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & -5 \end{bmatrix}$$

$2\times 2 \quad \cdot \quad 2\times 3$

$$\begin{bmatrix} -4(-1)+0(3) & -4(0)+0(4) & -4(2)+0(-5) \\ -6(-1)+3(3) & -6(0)+3(4) & -6(2)+3(-5) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & -8 \\ 15 & 12 & -27 \end{bmatrix}$$

$$X = -\frac{1}{12}\begin{bmatrix} 4 & 0 & -8 \\ 15 & 12 & -27 \end{bmatrix}$$

Parabolas and Circles: General → Standard

Step 1: Move constants and whatever letter isn't squared to one side.

Step 2: Factor out coefficient of squared term. Add yo' box.

Step 3: Half and square middle term.

Step 4: Factor out coefficient of unsquared term.

Step 5: Divide both sides by coefficient of squared binomial.

$$x^2 + y^2 - 4x + 8y - 5 = 0$$

$$+5 \quad +5$$

$$x^2 - 4x + y^2 + 8y = 5$$

$$1(x^2 - 4x + \boxed{4}) + 1(y^2 + 8y + \boxed{16}) = 5 + \boxed{4} + \boxed{16}$$

$$1(x-2)^2 + 1(y+4)^2 = 25$$

$$(x-2)^2 + (y+4)^2 = 25$$

$$4x^2+4y^2+36y+5=0$$

$$-5 \quad -5$$

$$4x^2+4y^2+36y=-5$$

$$4(x^2+0x+\boxed{0})+4(y^2+9y+\boxed{\tfrac{81}{4}})=-5+4\boxed{0}+4\boxed{\tfrac{81}{4}}$$

$$4(x+0)^2+4(y+\tfrac{9}{2})^2=-5+81$$

$$\frac{4x^2+4(y+\frac{9}{2})^2}{4}=\frac{76}{4}$$

$$x^2+(y+\tfrac{9}{2})^2=19$$

$$-x^2+6x+y-4=0$$

$$-y+4 \quad +4-y$$

$$-x^2+6x=-y+4$$

$$-1(x^2-6x+\boxed{9})=-y+4-\boxed{9}$$

$$-1(x-3)^2=-y-5$$

$$\frac{-1(x-3)^2}{-1}=\frac{-1(y+5)}{-1}$$

$$(x-3)^2=y+5$$

$$x^2+y^2+26x+8y+160=0$$
$$-160 \quad -160$$

$$x^2+26x+y^2+8y=-160$$

$$1(x^2+26x+\boxed{169})+1(y^2+8y+\boxed{16})$$

$$1\left(x^2+\frac{26}{2}\right)^2+1\left(y^2+\frac{8}{2}\right)^2=-160+169+16$$

$$+1\left(y^2+\frac{8}{2}\right)^2=25$$

$$\text{v.}\quad .)^2+1(y^2+4)^2=25$$

$$x^2 + y^2 - 4x + 30y + 213 = 0$$
$$-213 \quad -213$$

$$\left(x^2 - \frac{4}{2}x + \boxed{4}\right) + \left(y^2 + \frac{30}{2}y + \boxed{225}\right) = -213 + \boxed{4} + \boxed{225}$$

$$(x^2 - 4) + (y^2 + 30y) = 16$$

$$\boxed{(x-2)^2 + (y+15)^2 = 16}$$

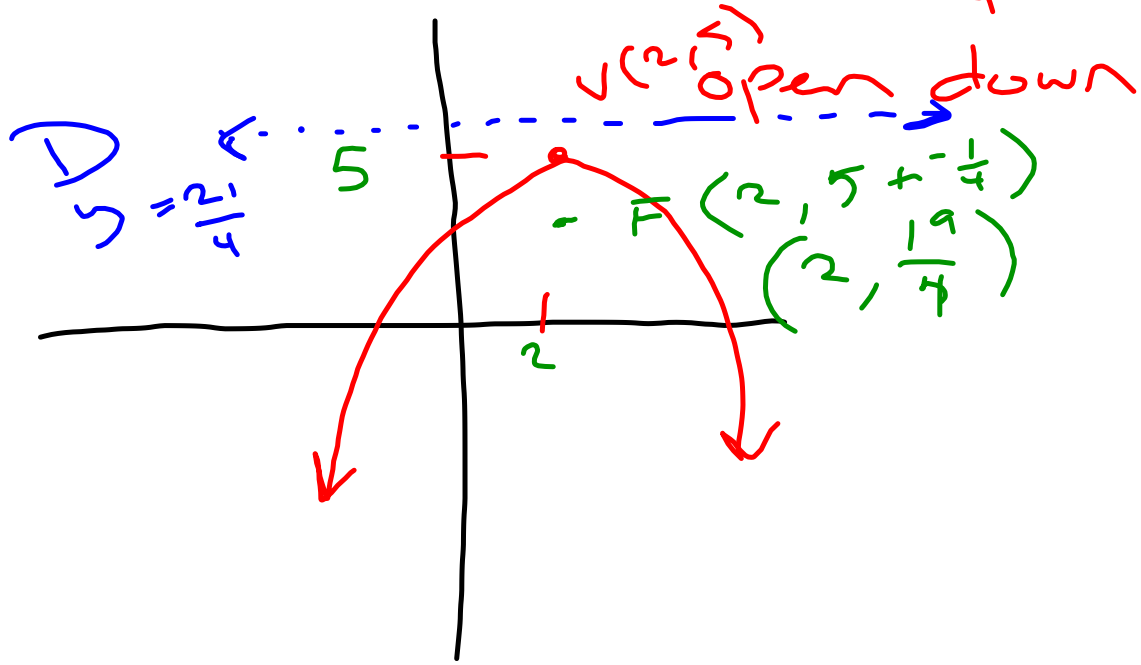
V (2, 5.4) $\frac{20}{4}$ (h, k)

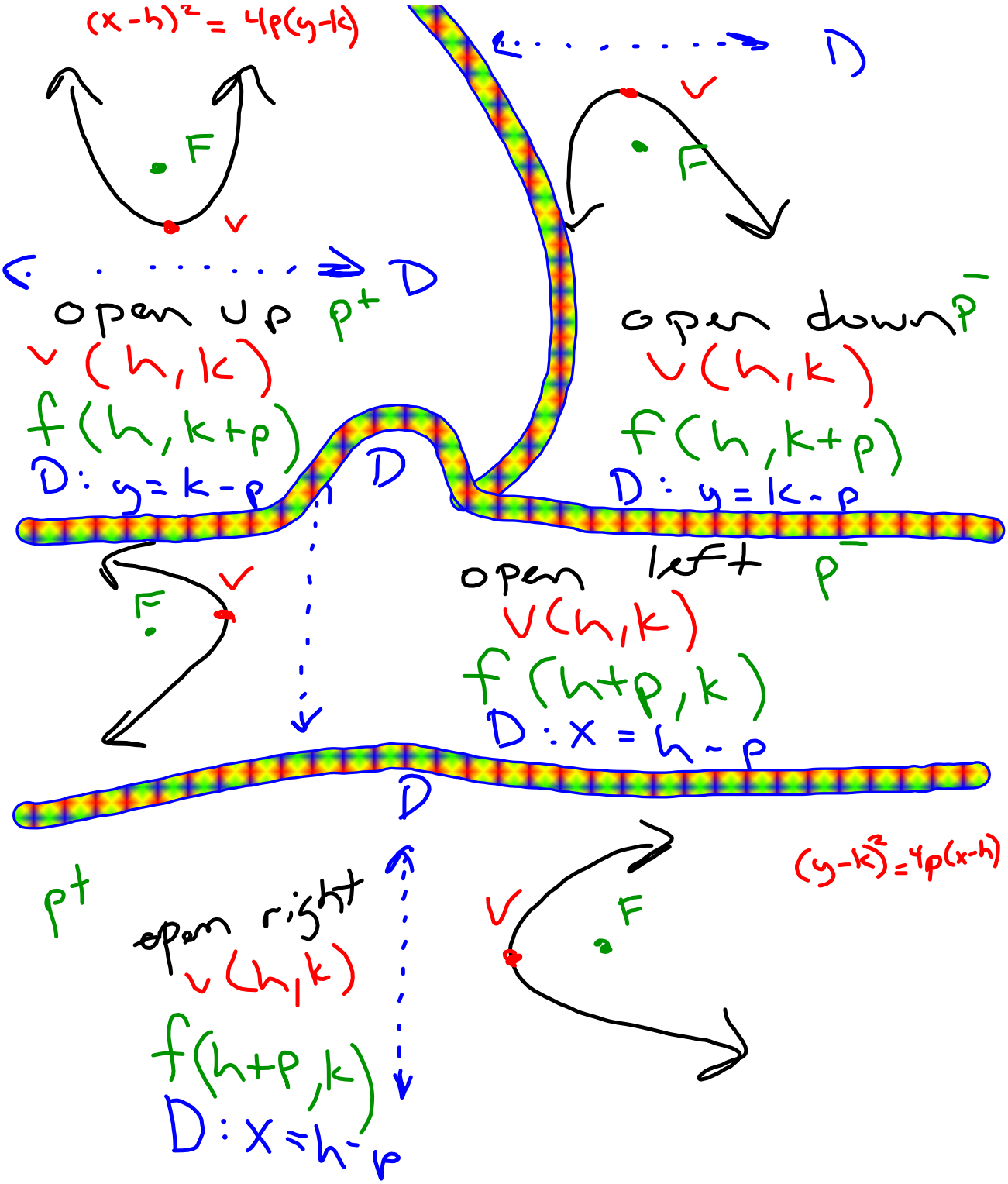
f

D

$\frac{4p}{4} = \frac{-1}{4}$

$p = -\frac{1}{4}$




$(x-h)^2 = 4p(y-k)$
F
V
D
open up p^+
V (h, k)
f (h, k+p)
D: y = k - p
F
V
D
open down p^-
V (h, k)
f (h, k+p)
D: y = k - p
D
F
V
open left p^-
V (h, k)
f (h+p, k)
D: x = h - p
D
p^+
open right
V (h, k)
f (h+p, k)
D: x = h - p
V
F
$(y-k)^2 = 4p(x-h)$

$Ax^2 + Cy^2 + Dx + Ey + F = 0$

$A = C$ if circle

$A \neq 0$ No circle

$4x^2 - 16x + 4y - 4 = 0$ constant on the right

$+4 \quad +4$

$4x^2 - 16x + 4y = 4$

$-4y \quad -4y$

$4x^2 - 16x = -4y + 4$

$4(x^2 - 4x + \boxed{4}) = -4y + 4 + 4\boxed{4}$

$4(x-2)^2 = -4y + 4 + 16$

$4(x-2)^2 = -4y + 20$

$\frac{4(x-2)^2}{4} = \frac{-4(y-5)}{4}$

$(x-2)^2 = -1(y-5)$

$Ax^2 + Cy^2 + Dx + Ey + F = 0$

$A = C$, → circle

$A \neq 0$

Not circle!!!

Identify the vertex, axis of symmetry, and direction of opening of each.

1) $y = 2(x+10)^2 + 1$

V

F

D

2) $y = -\frac{1}{3}(x-7)^2 + 1$

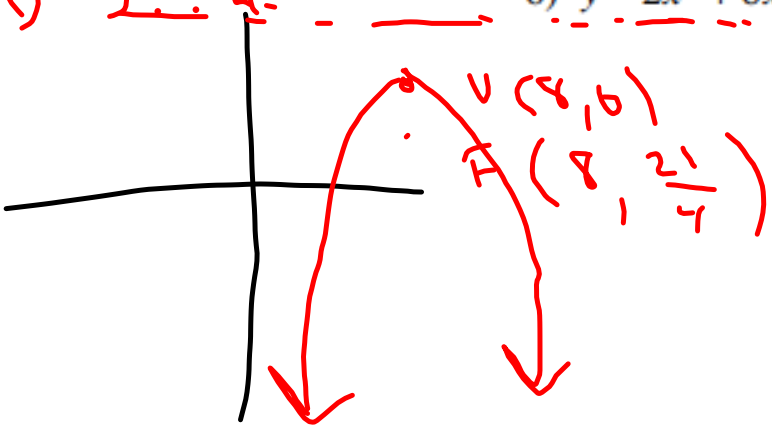
3) $y = -\frac{1}{3}x^2 + \frac{16}{3}x - \frac{46}{3}$

4) $y = 2x^2 + 36x + 166$

5) $y = x^2 + 4x - 5$

$D = 2 \div \frac{27}{4}$

6) $y = 2x^2 + 8x + 16$



$\frac{6}{1} + \frac{3}{4} \div 6\frac{3}{4} = 6.75 \div$

$$(y+5)^2 = 3(x-4)$$

H

P +

3) $3y = 3\left(-\frac{1}{3}x^2 + \frac{16}{3}x - \frac{46}{3}\right)$

$3y = -x^2 + 16x - 46$

$+46 \qquad +46$

$-x^2 + 16x = 3y + 46$

$-1(x^2 - 16x + \boxed{64}) = 3y + 46 - \boxed{64}$

$-1(x-8)^2 = 3y - 18$

$\frac{-1}{-1}(x-8)^2 = \frac{3}{-1}(y-6)$

$(x-8)^2 = -3(y-6)$

$V(8,6)$

$F\left(8, \frac{21}{4}\right)$

$4p = \frac{-3}{4}$

$p = -\frac{3}{4}$

opens down

$D: y = k - p = 6 + \frac{3}{4}$

$y = 2\frac{3}{4}$

$k + p$

$6 + p$

$6 + -\frac{3}{4}$

$\frac{24}{4} - \frac{3}{4}$

$\frac{21}{4}$

6) $y = 2x^2 + 8x + 16$

$-16 \qquad -16$

$y - 16 = 2x^2 + 8x$

$y - 16 + 2[4] = 2(x^2 + 4x + [4])$

$y - 16 + 8 = 2(x+2)^2$

$\frac{2(x+2)^2}{2} = \frac{1}{2}(y-8)$

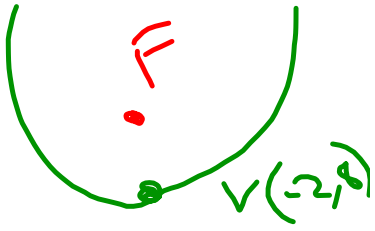
$(x+2)^2 = \frac{1}{2}(y-8)$

$V(-2, 8)$

$4p = \frac{1}{2} \cdot \frac{1}{4}$

$p = \frac{1}{8}$

open up.



(3) $3 \cdot y = \left(-\frac{1}{3}x^2 + \frac{16}{3}x - \frac{46}{3}\right)^3$

$3y = -x^2 + 16x - 46$

$+46 \qquad +46$

$-x^2 + 16x = 3y + 46$

$-1(x^2 - 16x + \boxed{64}) = 3y + 46 - \boxed{64}$

$-1(x-8)^2 = 3y - 18$

$-1(x-8)^2 = 3(y-6)$

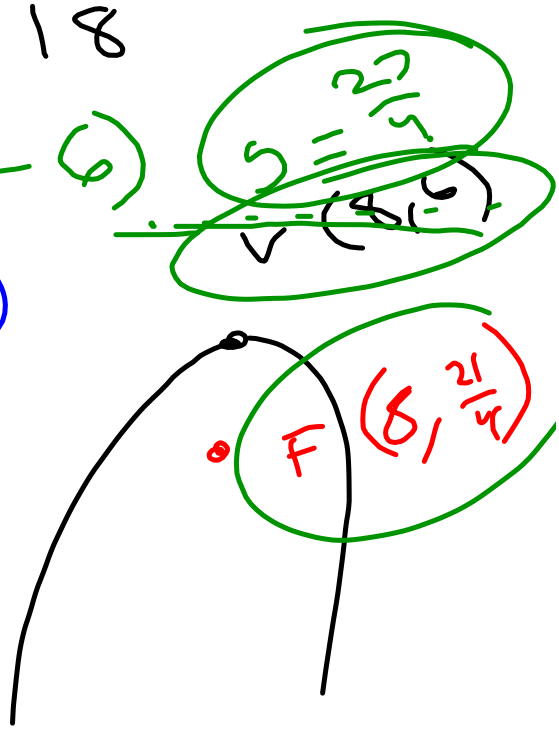
$(x-8)^2 = -3(y-6)$

$V(8,6)$

$4p = -3$

$p = -\frac{3}{4}$

open down



(4) $y = 2x^2 + 36x + 166$

$-166 \qquad -166$

$2x^2 + 36x = y - 166$

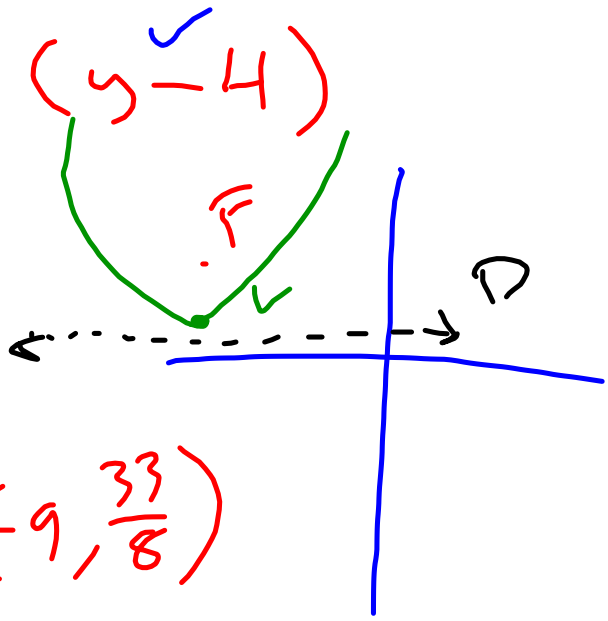
$2(x^2 + 18x + \boxed{81}) = y - 166 + 2\boxed{81}$

162

$\frac{2}{2}(x+9)^2 = \frac{1}{2}(y-4)$

$(x+9)^2 = \frac{1}{2}(y-4)$

$V(-9, 4)$



$4p = \frac{1}{2} \cdot \frac{1}{4}$

$\frac{4p}{4} = \frac{1}{8}$

$p = \frac{1}{8}$

open up

$f(-9, \frac{33}{8})$

$D: y = \frac{31}{8}$

① $y = 2(x+10)^2 + 1$

$-1 \qquad -1$

$\frac{1}{2}(y-1) = \frac{2(x+10)^2}{2}$

$(x+10)^2 = \frac{1}{2}(y-1)$

V

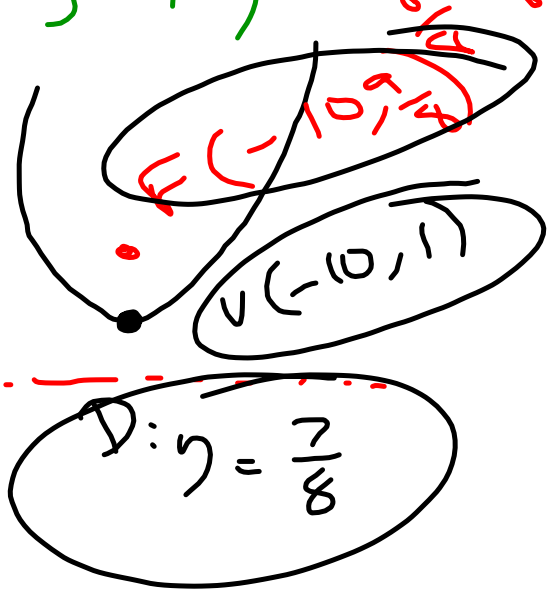
V(-10, 1)

Find p.

$4p = \frac{1}{2} \cdot \frac{1}{4}$

$p = \frac{1}{8}$

open up



$x^2 - 6x + 9 = 8y + 16$

$-8y - 16 \quad -8y - 16$

$x^2 - 6x - 8y - 7 = 0$

$+7 \quad +7$

$x^2 - 6x - 8y = 7$

$+8y \quad +8y$

$x^2 - 6x = 8y + 7$

$1(x^2 - 6x + \boxed{9}) = 8y + 7 + 1\boxed{9}$

$1(x-3)^2 = 8y + 16$

Factor out 8 on y

$(x-3)^2 = 8(y+2)$ $\quad v(h,k)$

$(x-h)^2 = 4p(y-k)$

$4x^2 - 2x + 4y - 4 = 0$ constants to 'da right

$+4 \quad +4$

$4x^2 - 2x + 4y = 4$ move $4y$ to 'da right

$-4y \quad -4y$

$4x^2 - 2x = -4y + 4$ Factor out x^2

$4\left(x^2 - \frac{1}{2}x + \boxed{\frac{1}{16}}\right) = -4y + 4 + 4\boxed{\frac{1}{16}}$

$4\left(x - \frac{1}{4}\right)^2 = -4y + \frac{4}{4} + \frac{1}{4}$ $\cdot 4$

$4\left(x - \frac{1}{4}\right)^2 = -4y + \frac{17}{4}$

$1x^2 + 1y^2 - 5y + 2x = 3$

A C E D -F

A=C

circle

$Ax^2 + Cy^2 + Dx + Ey + F = C$

$4x^2 + 4y + x = 3$

A U E D

$4y^2 + 4y + x = 3$

C

$4x^2 - 2x + 4y - 4 = 0$

$+4 \quad +4$

$4x^2 - 2x + 4y = 4$

$-4y \quad -4y$

$4x^2 - 2x = -4y + 4$

① constant and y to one side.

② Group the x's, make sure coef. on $x^2 = 1$

$4\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) = -4y + 4 + 4\left(\frac{1}{16}\right)$

halfit square it.

$4\left(x - \frac{1}{4}\right)^2 = -4y + 4 + \frac{1}{4}$

$\frac{4(x-\frac{1}{4})^2}{4} = \frac{-4y + \frac{17}{4}}{4}$ $\frac{1}{4}$

$\left(x - \frac{1}{4}\right)^2 = -y + \frac{17}{16}$

$\left(x - \frac{1}{4}\right)^2 = -1\left(y - \frac{17}{16}\right)$

$$2x^2+4x-2y-1=0$$

$$+1 \quad +1$$

$$2x^2+4x-2y=1$$

$$+2y \quad +2y$$

$$2x^2+4x=2y+1$$

$$2(x^2+2x+\boxed{1})=2y+1+2\boxed{1}$$

$$2(x+1)^2=2y+3$$

$$\frac{2(x+1)^2}{2}=\frac{2\left(y+\frac{3}{2}\right)}{2}$$

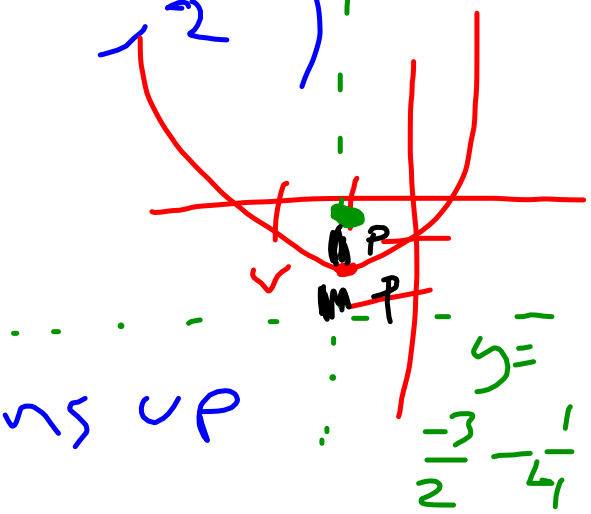
$$(x+1)^2=1\left(y+\frac{3}{2}\right)$$

$\updownarrow \quad (x-h)^2 = 4p(y-k)$

vertex $\left(-1, -\frac{3}{2}\right)$

$\frac{1}{4} = \frac{4p}{4}$

$p = \frac{1}{4}$ opens up



focus $\left(-1, \frac{-3}{2} + \frac{1}{4}\right)$

directrix

$$4x^2 - 2x + 6y - 4 = 0$$

$-6y \quad +4 \qquad -6y \quad +4$

$$4x^2 - 2x = -6y + 4$$

$$4\left(x^2 - \frac{1}{2}x + \boxed{\frac{1}{16}}\right) = -6y + 4 + 4\boxed{\frac{1}{16}}$$

$$4\left(x - \frac{1}{4}\right)^2 = -6y + 4 + \frac{1}{4}$$

$$\frac{4\left(x - \frac{1}{4}\right)^2}{4} = \frac{-6y + \frac{17}{4}}{4}$$

$$\left(x - \frac{1}{4}\right)^2 = -\frac{6y}{4} + \frac{17 \cdot 4}{16 \cdot 6}$$

$$\left(x - \frac{1}{4}\right)^2 = -\frac{6}{4}\left(y - \frac{68}{96}\right)$$