

ELLIPSE

Warm-Up

February 10, 2017

What conic section is this (circle, ellipse, parabola, or hyperbola?)

$$\textcircled{1}x^2 + 6x + \textcircled{4}y^2 - 8y = 6$$

$$1(x^2 + 6x + \boxed{9}) + 4(y^2 - 2y + \boxed{1}) = 6$$

$$\frac{1(x+3)^2}{19} + \frac{4(y-1)^2}{19} = \frac{19}{19}$$

$$\frac{(x+3)^2}{19} + \frac{(y-1)^2}{\frac{19}{4}} = 1$$

$$\left(\frac{1}{4/19}\right) \cdot \frac{4}{19} = \frac{4}{19}$$

$$\frac{4}{100} = \frac{1}{\frac{100}{4}} = \frac{1}{25}$$

$$\underbrace{4}_{A}x^2 + 5y + \underbrace{4}_{C}y^2 - 3x + 8 = 0$$

A=C
Circle

$$\underbrace{5}_{C}y^2 - 8y + \underbrace{4}_{A}x^2 - 3 = 0$$

Both pos, but different
ellipse

$$\underbrace{3}_{A}x^2 + 4y - 5x + 8 = 0$$

Don't see 1 squared

Parabola

$$\underbrace{3}_{C}y^2 + 4y - 5x + 8 = 0$$

no A
Parabola

$$\textcircled{5}x^2 - 2x + \textcircled{5}y^2 - 7y - 3 = 0$$

circle

$$\textcircled{4}x^2 + \textcircled{1}y^2 - 3x = 0$$

ellipse

$$\textcircled{7}x^2 + 7y + 7x + 7 = 0$$

parabola

Cle' yo' dess.

Formula sheet okay.

$$\textcircled{1}x^2 + 6x - \textcircled{4}y^2 - 8y = 6$$

$A=1$ $C=-4$

hyperbola

A, b, x, y

Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$c^2 = a^2 - b^2$$

Major axis (a): Whatever is greater

Hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

$$c^2 = a^2 + b^2$$

Major axis (a): Whatever is first

$$\textcircled{1} \frac{(x-0)^2}{81} - \frac{(y-0)^2}{4} = 1$$

Horizontal

Center $(0, 0)$

$$a = \sqrt{81} = 9$$

$$b = \sqrt{4} = 2$$

$$c^2 = a^2 + b^2$$

$$\sqrt{c^2} = \sqrt{81 + 4}$$

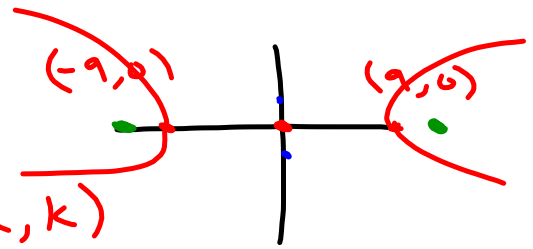
$$c = \sqrt{85}$$

$$v (h \pm a, k)$$

$$v (0 \pm 9, 0)$$

$$c (0 \pm \sqrt{85}, 0)$$

$$\text{"co-v"} (0, 0 \pm 2)$$



$$1. \frac{x^2}{81} - \frac{y^2}{4} = 1$$

opens: horizontal

center: (0,0)

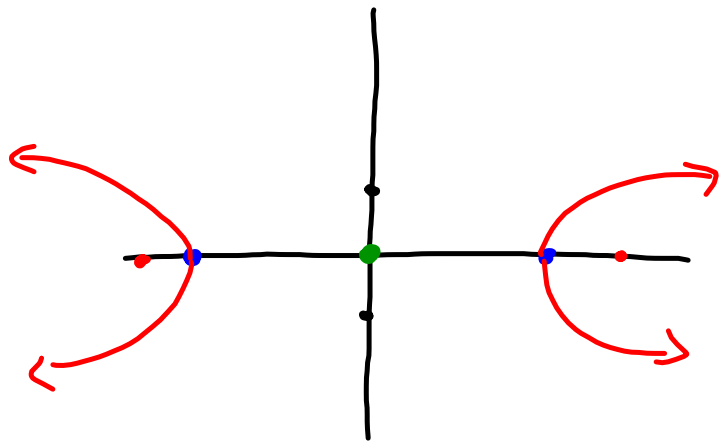
a = 9 b = 2

vertices: (0 ± 9, 0)

co-vertices: (0, 0 ± 2)

c = √85

foci: (0 ± √85, 0)



$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 81 + 4 \\ c^2 &= 85 \end{aligned} \quad c = \sqrt{85}$$

$$1. \frac{x^2}{81} - \frac{y^2}{4} = 1$$

nothing in paren 0
nothing in denom 1

center: $(0, 0)$

vertices: $(0 \pm 9, 0)$

foci: $(0 \pm \sqrt{85}, 0)$

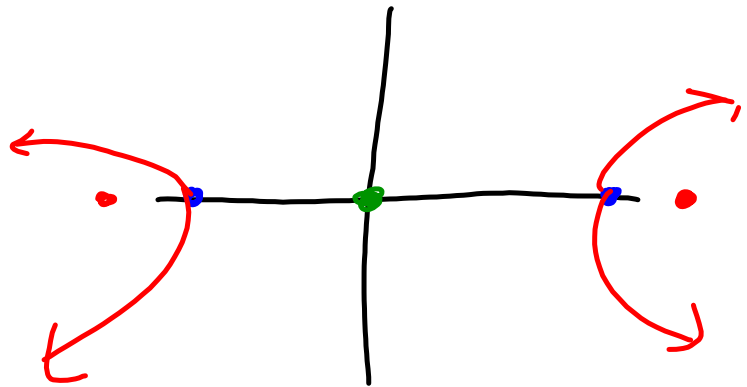
opens: horizontal

$$a^2 = 81 \quad b^2 = 4$$

$$a = 9 \quad b = 2$$

$$c^2 = a^2 + b^2 \\ = 81 + 4$$

$$c^2 = 85 \\ c = \sqrt{85}$$



Equation

Characteristics

Graph

$$\textcircled{1} \quad \frac{x^2}{81} - \frac{y^2}{4} = 1$$

hyperbola
a is always first

center (0, 0)

$$\sqrt{a^2} = \sqrt{81}$$

$$a = 9$$

horizontal

$$\begin{matrix} \sqrt{(h \pm a, k)} \\ (0 \pm 9, 0) \end{matrix}$$

$$(9, 0) \quad (-9, 0)$$

$$\frac{(x-0)^2}{81} - \frac{(y-0)^2}{4} = 1$$

$$\sqrt{b^2} = \sqrt{4}$$

$$b = 2$$

$$a^2 + b^2 = c^2$$

$$81 + 4$$

$$\sqrt{85} = \sqrt{c^2}$$

$$c = \sqrt{85}$$

$$f (h \pm c, k)$$

$$(0 \pm \sqrt{85}, 0)$$

⑭ $v(-3,0)$ $(3,0)$ x changed
 "co-v" $(0,-5)$ $(0,5)$ horizontal

Center $(0,0)$
 h,k

$$\frac{(x-h)^2}{a^2 9} - \frac{(y-k)^2}{b^2 25} = 1$$

$$a=3$$

$$a^2=9$$

$$b=5$$

$$b^2=25$$

$$\begin{pmatrix} -3, 0 \\ x_1 \end{pmatrix}$$

$$\begin{pmatrix} 3, 0 \\ x_2 \end{pmatrix}$$

$$\left(\frac{x_1 + x_2}{2} \right)$$

$$\left(\frac{y_1 + y_2}{2} \right)$$

$$-3 + 3 = 0$$

$$0 + 0$$

$$\frac{0}{2}$$

$$\frac{0}{2}$$

$$\begin{pmatrix} 0, 0 \end{pmatrix}$$

$$\begin{array}{l} 15 \quad v \quad (0, -2) \quad (0, 2) \\ \quad \quad c \quad o \quad v \quad (-4, 0) \quad (4, 0) \end{array}$$

$$c \quad (0, 0)$$

$$\textcircled{1} \quad \frac{x^2}{81} - \frac{y^2}{4} = 1$$

↑
hyperbola

x is first \rightarrow horizontal

center $(0, 0)$
h, k

$$\sqrt{a^2} = \sqrt{81}$$
$$a = 9$$

$$v(h \pm a, k)$$
$$v(0 \pm 9, 0)$$

↙ ↘

$$(9, 0) \quad (-9, 0)$$

$$\frac{(x-0)^2}{81} - \frac{(y-0)^2}{4} = 1$$

$$c^2 = a^2 + b^2$$

81 + 4

$$\sqrt{c^2} = \sqrt{85}$$

$$c = \sqrt{85}$$

$$f(h \pm c, k)$$

$$(0 \pm \sqrt{85}, 0)$$

$$(\sqrt{85}, 0)$$

$$(-\sqrt{85}, 0)$$

$$\textcircled{2} \quad \frac{x^2}{121} - \frac{y^2}{81} = 1$$

horizontal $c(0,0)$

$$\sqrt{a^2} = \sqrt{121}$$

$$a = 11 \quad v(h \pm a, k)$$
$$(0 \pm 11, 0)$$

$$\swarrow \quad \searrow$$
$$(11, 0) \quad (-11, 0)$$

$$c^2 = a^2 + b^2$$
$$121 + 81$$

$$\sqrt{c^2} = \sqrt{202}$$

$$c = \sqrt{202}$$

$$f(h \pm c, k)$$
$$0 \pm \sqrt{202}, 0$$

$$(\sqrt{202}, 0) \quad (-\sqrt{202}, 0)$$

⑭ $\sqrt{x_1, y_1}$ $\sqrt{x_2, y_2}$ x changes
 horizontal
 $\sqrt{(-3, 0)}$ $\sqrt{(3, 0)}$
 "co-vertices" $(0, -5)$ $(0, 5)$

Center $(0, 0)$

$$\frac{(x - \cancel{h})^2}{\cancel{a^2} 9} - \frac{(y - \cancel{k})^2}{\cancel{b^2} 25} = 1$$

$$a = 3$$

$$a^2 = 9$$

$$b = 5$$

$$b^2 = 25$$

Mittelpunkt

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\frac{-3 + 3}{2}$$

$$0 + 0$$

$$\frac{0}{2}$$

$$\frac{0}{2}$$

$$(0, 0)$$

$$\textcircled{16} \quad \begin{array}{l} v \quad (0, -4) \quad (0, 4) \\ f \quad (0, -5) \quad (0, 5) \end{array} \left. \begin{array}{l} \text{y change} \\ \text{vertical} \end{array} \right\}$$

center

$$(0 \quad 0)$$

$$\frac{(y - \overset{0}{\cancel{k}})^2}{\cancel{a^2} 16} - \frac{(x - \overset{0}{\cancel{h}})^2}{\cancel{b^2} 9} = 1$$

$$a = 4$$

$$a^2 = 16$$

$$\begin{aligned}c^2 &= a^2 + b^2 \\5^2 &= 4^2 + b^2 \\25 &= 16 + b^2 \\-16 & \quad -16 \\ \hline 9 &= b^2\end{aligned}$$

$$\textcircled{7} \quad \frac{x^2}{20} - \frac{(y+1)^2}{10} = 1$$

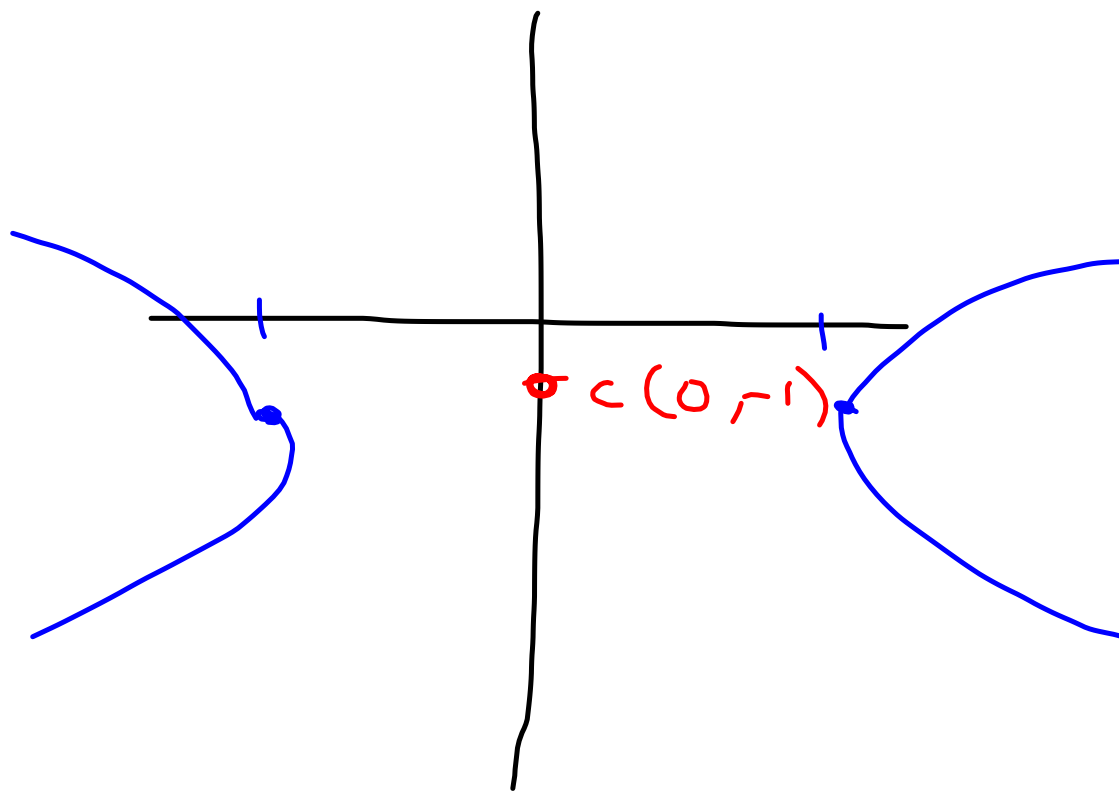
① center $(0, -1)$

② x first \Rightarrow horizontal

$$v(h \pm a, k)$$

$$(0 \pm \sqrt{20}, -1)$$

$$(\pm 4.5, -1)$$



$$\textcircled{1} \quad \frac{x^2}{81} - \frac{y^2}{4} = 1$$

hyperbola

x is first \rightarrow horizontal

center is $(0, 0)$

$$\sqrt{a^2} = \sqrt{81}$$

$$a = 9$$

$$v \left(\begin{array}{c} h \pm a, k \\ 0 \pm 9, 0 \end{array} \right)$$

$$\swarrow$$
$$(9, 0)$$

$$\searrow$$
$$(-9, 0)$$

$$\frac{(x-0)^2}{81} - \frac{(y-0)^2}{4} = 1$$

$$c^2 = a^2 + b^2$$

81 + 4

$$\sqrt{c^2} = \sqrt{85}$$
$$c = \sqrt{85}$$

$$f \left(h \pm c, k \right)$$

$0 \pm \sqrt{85}, 0$

↙

$$(\sqrt{85}, 0)$$

↘

$$(-\sqrt{85}, 0)$$

$\textcircled{14}$
 $\overset{x \text{ changes}}{\text{horizontal}}$
 $\overset{x_1, y_1}{V} (-3, 0)$
 $\overset{x_2, y_2}{(3, 0)}$
 $\overset{\text{co-}V}{(0, -5)}$
 $(0, 5)$

① center $(0, 0)$

② x changes on vertices
 \hookrightarrow horizontal

③

$$\frac{(x - \cancel{h})^2}{a^2 9} - \frac{(y - \cancel{k})^2}{b^2 25} = 1$$

$$a = 3$$

$$b = 5$$

$$a^2 = 9$$

$$b^2 = 25$$

Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

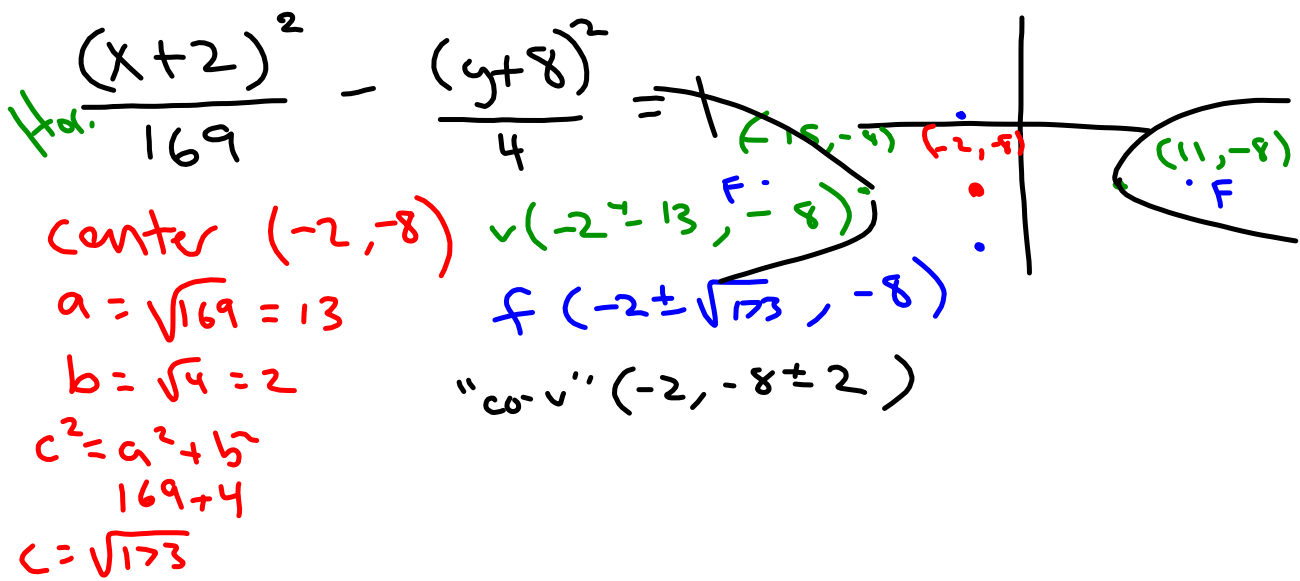
$$\frac{3+3}{2}$$

$$\frac{0+0}{2}$$

$$\frac{2}{2}$$

$$\frac{0}{2}$$

$$1, 0$$



⑭ a v $(-3, 0)$ $(3, 0)$ Hor.

b c o v $(0, -5)$ $(0, 5)$

center $(\frac{-5+3}{2}, \frac{0+0}{2})$

$(0, 0)$

$$\frac{(x-0)^2}{9} - \frac{(y-0)^2}{25} = 1$$

$a=3$

$b=5$