Name:_________________________
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DISCOVERING THE PYTHAGOREAN IDENTITIES LEARNING TASK:

An **identity** is an equation that is valid for all values of the variable for which the expressions in the equation are defined.

You should already be familiar with some identities. For example, in Mathematics I, you learned that the equation $x^2 - y^2 = (x+y)(x-y)$ is valid for all values of x and y.

1. You will complete the table below by first randomly choosing values for the *x*'s and *y*'s, then evaluating the expressions $x^2 - y^2$ and $(x+y)(x-y)$. The first row is completed as an example.
   a. Since $x^2 - y^2 = (x+y)(x-y)$ is an identity, what should be true about the relationship between the numbers in the last two columns of each row?

   b. Complete the table below.

x	y	$x^2 - y^2$	$(x+y)(x-y)$

2. An identity is a specific type of equation. Many equations are not identities, however, because an equation is not necessarily true for all values of the involved variables. Of the eight equations that follow, only four are identities. Label the equations that are identities as such and provide a counterexample for the equations that are not identities.

   a. $(x-5)(x+5) = x^2 - 25$

   b. $(x+5)^2 = x^2 + 25$

   c. $\sqrt{x^2} = |x|$

   d. $\sqrt{x^4} = x^2$

   e. $\sqrt{x^6} = x^3$

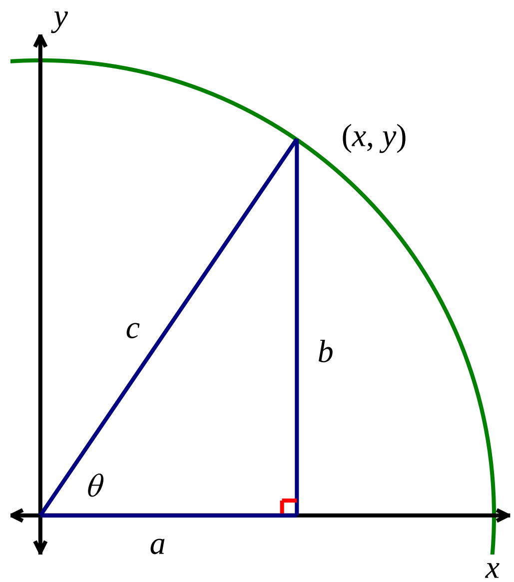
   f. $\sqrt{x^2 + y^2} = x + y$

   g. $(a+b)(a^2 - ab + b^2) = a^3 + b^3$

   h. $\dfrac{x+y}{x} = y$

3. In this unit you will investigate several trigonometric identities. This task looks at the **Pythagorean Identities**, which are three of the most commonly used trigonometric identities, so-named because they can be established directly from the Pythagorean Theorem.

In the figure below, the point (x, y) is a point on a circle with radius c. By working with some of the relationships that exist between the quantities in this figure, you will arrive at the first of the Pythagorean Identities



a. Use the Pythagorean Theorem to write an equation that relates a, b, and c.

b. What ratio is equal to $\cos\theta$?

c. What ratio is equal to $\sin\theta$?

d. Using substitution and simplification, combine the three equations from parts a-c into a single equation that is only in terms of θ. This equation is the first of the three Pythagorean identities. ***Hint**: Solve for a & b and substitute those values into the Pythagorean Theorem.*

4. Since the equation from 3d is an identity, it should be true no matter what θ is. Complete the table below, picking a value for θ that is in the appropriate quadrant. Use your calculator to round values to the nearest hundredth if the angle you choose is not found on the unit circle.

	θ	$\sin^2\theta$ *	$\cos^2\theta$	$\sin^2\theta+\cos^2\theta$
QI	$\frac{\pi}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1
QII				
QIII				
QIV				

* $\sin^2\theta=(\sin\theta)^2$, $\cos^2\theta=(\cos\theta)^2$, and so on. This is just a notational convention mathematicians use to avoid writing too many parentheses!

5. How can you use the data in problem 4 to verify that the identity is valid for the four values of θ that you chose?

6. The other two Pythagorean identities can be derived directly from the first. In order to make these simplifications, you will need to recall the definitions of the other four trigonometric functions:

$$\tan\theta=\frac{\sin\theta}{\cos\theta} \qquad \cot\theta=\frac{\cos\theta}{\sin\theta} \qquad \sec\theta=\frac{1}{\cos\theta} \qquad \csc\theta=\frac{1}{\sin\theta}$$

a. Divide both sides of the first Pythagorean identity by $\cos^2\theta$ and simplify. The result is the second Pythagorean identity.

b. Divide both sides of the first Pythagorean identity by $\sin^2\theta$ and simplify. The result is the third and final Pythagorean identity.

7. Since the equations from 6a and 6b are identities, they should be true no matter what θ is. Complete the table below, picking a value for θ that is in the appropriate quadrant. Use your calculator to round values to the nearest hundredth if the angle you choose is not found on the unit circle.

	θ	$1+\tan^2\theta$	$\sec^2\theta$	$1+\cot^2\theta$	$\csc^2\theta$
QI					
QII					
QIII					
QIV					

8. How can you use this data to verify that identities found in 6a and 6b are both valid for the four values of θ that you chose?

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DISCOVERING THE PYTHAGOREAN IDENTITIES LEARNING TASK – **ANSWER SHEET**

1.) a. ____

____

____

b.

x	y	$x^2 - y^2$	$(x+y)(x-y)$

2.) a. ____________________ e. ____________________

b. ____________________ f. ____________________

c. ____________________ g. ____________________

d. ____________________ h. ____________________

3.) a. _______________________

b. _________________________

c. _________________________

d. _________________________

4.)

	θ	$\sin^2\theta$ *	$\cos^2\theta$	$\sin^2\theta + \cos^2\theta$
QI	$\frac{\pi}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1
QII				
QIII				
QIV				

5.) ____________________

____

____

____

6.) a. ________________________

b. ___________________________

7.)

	θ	$1+\tan^2\theta$	$\sec^2\theta$	$1+\cot^2\theta$	$\csc^2\theta$
QI					
QII					
QIII					
QIV					

8.) ____

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