

DETERMINANTS

NOTE: DETERMINANTS ARE ONLY DONE ON SQUARE MATRICES

Ex 1. 2x2 MATRIX

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

$ad - bc$

$$\begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix}$$

$2 - (-12) = 14$

|| mean the determinant

Ex 2. 3x3 MATRIX

$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & 0 & 4 \\ 4 & 2 & 2 \end{vmatrix}$$

$$\underline{0} + \underline{48} + \underline{4} - \underline{0} - \underline{8} - \underline{12}$$

downs ups

$= 32$

1. We rewrite the first two columns.
2. we multiply the 3 down diagonals (+)
3. we multiply the 3 up diagonals (-)
4. add (subtract) the 6 products.

Assignment

Evaluate each determinant.

1) $\begin{vmatrix} 1 & 0 \\ 2 & -4 \end{vmatrix}$

2) $\begin{vmatrix} 4 & -5 \\ -5 & 0 \end{vmatrix}$

3) $\begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}$

4) $\begin{vmatrix} -4 & 3 \\ 3 & 0 \end{vmatrix}$

5) $\begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix}$

6) $\begin{vmatrix} 0 & 5 \\ 3 & -4 \end{vmatrix}$

7) $\begin{vmatrix} -5 & 2 \\ -2 & -5 \end{vmatrix}$

8) $\begin{vmatrix} -3 & -2 \\ 1 & 1 \end{vmatrix}$

9) $\begin{vmatrix} -3 & -3 \\ -5 & -4 \end{vmatrix}$

10) $\begin{vmatrix} -2 & 3 \\ 5 & -4 \end{vmatrix}$

11) $\begin{vmatrix} 2 & 5 \\ -1 & -3 \end{vmatrix}$

12) $\begin{vmatrix} 3 & 0 \\ -3 & -2 \end{vmatrix}$

13) $\begin{vmatrix} -1 & 4 \\ -2 & 2 \end{vmatrix}$

14) $\begin{vmatrix} 5 & -3 \\ -1 & 5 \end{vmatrix}$

15) $\begin{vmatrix} 1 & 4 \\ 5 & 3 \end{vmatrix}$

16) $\begin{vmatrix} 5 & 3 & -4 \\ -2 & -2 & -3 \\ 4 & 5 & 1 \end{vmatrix}$

17) $\begin{vmatrix} -2 & -1 & -5 \\ -5 & 5 & -4 \\ 5 & -4 & -2 \end{vmatrix}$

18) $\begin{vmatrix} -5 & -2 & 5 \\ -1 & -3 & -3 \\ 5 & -1 & -2 \end{vmatrix}$

19) $\begin{vmatrix} 3 & -5 & 4 \\ 3 & 0 & -3 \\ -5 & 0 & -4 \end{vmatrix}$

20) $\begin{vmatrix} 2 & 0 & -5 \\ 3 & 2 & -4 \\ 4 & -5 & 0 \end{vmatrix}$

$\begin{matrix} -2 & -1 \\ -5 & 5 \\ 5 & -4 \end{matrix}$
 $\begin{matrix} -2 & -1 \\ -5 & 5 \\ 5 & -4 \end{matrix}$

 $\underline{20} + \dots$

Matrix Inverses

2x2 matrices

Ex. 1 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$

Remember
-1 means
Inverse...

$$\frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$|A| = \det(A)$$

Ex. 2 $\begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}^{-1}$

$$\begin{aligned} \det(A) &= \\ &= (4)(3) - (2)(-1) \\ &= 12 - (-2) \\ &= 14 \end{aligned}$$

$$\frac{1}{14} \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$$

Ex. 3 $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$

$$\begin{aligned} \det(A) &= \\ &= (2)(4) - (3)(3) \\ &= 8 - 9 \\ &= -1 \end{aligned}$$

$$\frac{1}{-1} \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

Assignment

Find the inverse of each matrix.

1) $\begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}$

2) $\begin{bmatrix} 7 & 6 \\ 7 & 6 \end{bmatrix}$

3) $\begin{bmatrix} -2 & -5 \\ 4 & 12 \end{bmatrix}$

4) $\begin{bmatrix} -6 & 5 \\ 6 & -4 \end{bmatrix}$

5) $\begin{bmatrix} 1 & 0 \\ -2 & -9 \end{bmatrix}$

6) $\begin{bmatrix} -3 & -1 \\ 0 & -2 \end{bmatrix}$

7) $\begin{bmatrix} 4 & -1 \\ -9 & 3 \end{bmatrix}$

8) $\begin{bmatrix} 1 & 12 \\ -1 & -6 \end{bmatrix}$

9) $\begin{bmatrix} -2 & 10 \\ -1 & 3 \end{bmatrix}$

10) $\begin{bmatrix} -2 & 5 \\ -4 & 9 \end{bmatrix}$

11) $\begin{bmatrix} -1 & 0 \\ 7 & 8 \end{bmatrix}$

12) $\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix}$

13) $\begin{bmatrix} 2 & 10 \\ -2 & -9 \end{bmatrix}$

14) $\begin{bmatrix} -1 & -6 \\ -1 & -2 \end{bmatrix}$

15) $\begin{bmatrix} 5 & -10 \\ 1 & -2 \end{bmatrix}$

16) $\begin{bmatrix} 7 & 7 \\ 2 & 2 \end{bmatrix}$

17) $\begin{bmatrix} 4 & -3 \\ 7 & -4 \end{bmatrix}$

18) $\begin{bmatrix} 5 & -5 \\ 7 & -6 \end{bmatrix}$

19) $\begin{bmatrix} 1 & 1 \\ 3 & 5 \end{bmatrix}$

20) $\begin{bmatrix} 8 & -1 \\ 1 & 0 \end{bmatrix}$