Good morning!

1. "Here"
2. Notes on Independent and Dependent Events
3. Practice

Independent	Dependent
Self-sustaining	Not " "
Do something on your own	Rely on something
Does not affect anything else	Does affect it " "

Probability

Independent vs. Dependent events

Independent Events

- Two events A and B, are independent if the fact that A occurs does not affect the probability of B occurring.

- Examples- EX 1. Landing on heads from two different coins; EX 2. rolling a 4 on a die, then rolling a 3 on a second roll of the die.
- Probability of A and B occurring:

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

Experiment 1

- A coin is tossed and a 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

$P(H \text{ and } 3) = P(H) \cdot P(3)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$

$P(H \cap 3) = \frac{1}{12}$ or 0.08

Experiment 2

is this independent?

- A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and an eight?

$$P(J \cap 8) = P(J) \cdot P(8)$$

$$= \frac{4}{52} \cdot \frac{4}{52}$$

$$= \frac{16}{2704}$$

$$P(J \cap 8) = \frac{1}{169} \text{ or } 0.006$$

Experiment 3

16 total marbles

- A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and a yellow marble?

$$P(G \cap Y) = \frac{5}{16} \cdot \frac{6}{16}$$

$$= \frac{30}{256} = \frac{15}{128} \text{ or } .12$$

Experiment 4

P(like pizza) = 9/10

- A school survey found that 9 out of 10 students like pizza. If three students are chosen at random ~~with replacement~~, what is the probability that all three students like pizza?

$P(L \cap L \cap L) = \left(\frac{9}{10}\right)\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)$

$= \frac{729}{1000}$ or .729

Dependent Events

- Two events A and B, are dependent if the fact that A occurs affects the probability of B occurring.
- Examples- Picking a blue marble and then picking another blue marble if I don't replace the first one.
- Probability of A and B occurring:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Dependent

Experiment 1

16 total marbles

- A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. A second marble is chosen without replacing the first one. What is the probability of choosing a green and a yellow marble?

$P(G \text{ and } Y) = P(G) \cdot P(Y|G)$

$\frac{5}{16} \cdot \frac{6}{15}$ ← Denominator changed!

$\frac{30}{240} = \frac{1}{8}$ or .125

Experiment 2

10 total

- An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, do not replace it, and then randomly select a second fish. What is the probability that both fish are male?

$P(M \text{ and } M) = \frac{6}{10} \cdot \frac{5}{9}$

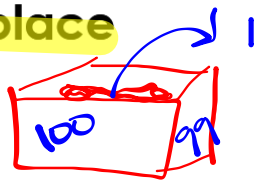
$= \frac{1}{3}$ or .33

Experiment 3

- A random sample of parts coming off a machine is done by an inspector. He found that 5 out of 100 parts are bad on average. If he were to do a new sample, what is the probability that he picks a bad part and then, picks another bad part if he doesn't replace the first?

$$P(B \cap B) = \frac{5}{100} \cdot \frac{4}{99}$$

$$= \frac{20}{9900} = \boxed{\frac{1}{495} \text{ or } .002}$$



Independent vs. Dependent

Determining if 2 events are independent

Conditional: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Independent Events

- Two events are independent if the following are true:

With replacement

$P(A \mid B) = P(A)$

$P(B \mid A) = P(B)$

$P(A \text{ AND } B) = P(A) \cdot P(B)$

$P(B) \cdot P(A|B)$

- To show 2 events are independent, you must prove one of the above conditions.

Experiment 1

- Let event G = taking a math class. Let event H = taking a science class. Then, G AND H = taking a math class and a science class.
- Suppose P(G) = 0.6, P(H) = 0.5, and P(G AND H) = 0.3.
- Are G and H independent?

$P(G \cap H) = P(G) \cdot P(H)$

$.3 = .6 \cdot .5$

$.3 = .3$ ✓ Independent!

Experiment 2

- In a particular college class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair.
- Let F be the event that the student is female. Let L be the event that the student has long hair.
- One student is picked randomly. Are the events of being female and having long hair independent?

$P(L|F) = .75$

$P(F) = .6 \quad P(L) = .5 \quad P(F \cap L) = .45$

① $P(L|F) = P(L)$

$.75 = .5$ X

② $P(F \cap L) = P(F) \cdot P(L)$

$.45 = .6 \cdot .5$

$.45 = .30$ X

Dependent

Approach #2

- If they are independent, $P(L \mid F)$ should equal $P(L)$.
- $0.75 \neq 0.5$

Independent Probability Name:

Two events are said to be **Independent** if the occurrence of the first event does **NOT** affect the probability of the second event and events are independent if **P(A) • P(B) = P(A and B)**

INDEPENDENT PROBABILITY

1. Determine the following probabilities if each of the following are **independent**.

GIVEN:	P(A) = 0.8	P(B) = 0.25	P(C) = 0.6

a. P(A and C) = $P(A)\cdot P(C)$ $= .8 \cdot .6$ Decimal: .48

b. P(A and B and C) = $P(A)\cdot P(B)\cdot P(C)$ $.8 \cdot .6 \cdot .25$ Decimal: .12

c. P(Rolling a 4 on a standard die and B) = $P(4) \cdot P(B)$ $\frac{1}{6} \cdot .25$ Decimal: 0.04

d. Find P(D) if D is an independent event and P(C and D) = 0.10 $P(C \text{ and } D) = P(C)\cdot P(D)$ $.1 = .6 \cdot x$ $x = \frac{1}{6} = .17$ Decimal:

e. P(Rolling a 2 on a standard die and picking a card with a "7" on it from a standard deck of cards) = $\left(\frac{1}{6}\right)\left(\frac{4}{52}\right) = \frac{1}{78}$ or 0.01 Decimal:

f. If your chances of losing the shell game if you randomly pick is 2 in 3. What are the chances that you would lose 5 games in a row?

$P(L \cap L \cap L \cap L \cap L) = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) =$ Decimal: $\frac{32}{243} = .13$

g. If the Atlanta Hawks free throw percentage is 82%, what is the probability that a player for the Hawks will make 2 free shots in a row?

$P(M \cap M) = (.82)(.82) = .67$ Percentage: .67

h. The chance of rain on a random day in May in Gwinnett is about 30%. Using this empirical probability, what would you estimate the probability of having ~~NO rain~~ for an entire week (7 days)?

$(.7)(.7)(.7)(.7)(.7)(.7)(.7)$ Percentage: .08 > .70

2.

GIVEN:	P(M) = 0.8	P(N) = 0.25	P(R) = 0.6

a. If the probability of **P(M and N) = 0.2**, are M and N independent?

b. If the probability of **P(N and R) = 0.3**, are N and R independent?

DEPENDENT PROBABILITIES

3. Consider that 3 consecutive cards are drawn **without replacement** from a shuffled deck of cards

   A. What is the probability that the first two cards drawn are face cards?

   Decimal:

   B. What is the probability that the all three cards are hearts?

   Decimal:

   C. What is the probability that all three cards are a King?

   Decimal:

   D. What is the probability that all three cards are numbered?

   Decimal:

4. A bag contains 4 blue marbles, 4 red marbles, and 4 green marbles:

   A. What is the probability of drawing 2 green marbles **without replacement**?

   Decimal:

   B. What is the probability of drawing 3 marbles without replacement in a row of the same color **without replacement**?

   Decimal:

5. James has 3 dimes, 4 pennies, and 2 quarters in his pocket. If each coin is equally likely to be pulled out of his pocket in order **without replacement**, what is the probability that he will pull out the 2 quarters in a row first?

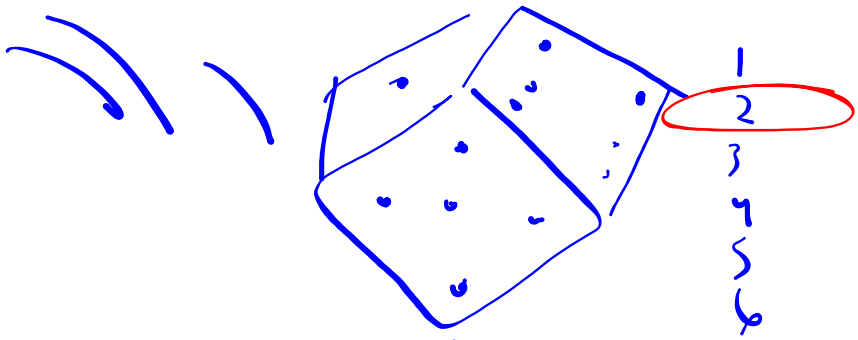
   Reduced Fraction:

6. In a cookie jar there are 10 chocolate chip cookies and 8 peanut butter cookies left. The cookies are randomly mixed together in the jar. What is the probability of pulling two of the same types of cookies out of the cookie jar in a row **without replacement**?

   DECIMAL:

7. In a classroom there are 7 male students and 11 female students that are taking a test. If each student is equally likely to turn in their test at any given time at the end of class, what is the probability that the first 3 students to turn in their test are female students?

   DECIMAL:



$P(4) = \frac{1}{6}$