Good morning!

1. "Here"

2. Notes on Quadratic Word Problems

3. Test on Wednesday


Young Glory

Math Protocols for Word Problems

(and everything else)

1. Draw picture
2. Highlight information
3. Set up equation/formula
4. Solve for QUESTION

Converting Between Forms Practice

Part One: Convert from standard form to vertex form.

1) $y = x^2 - 8x + 15$

2) $y = x^2 - 4x$

3) $y = 2x^2 + 12x + 7$

4) $y = 2x^2 - 8x + 17$

Part Two: Convert from vertex form to standard form.

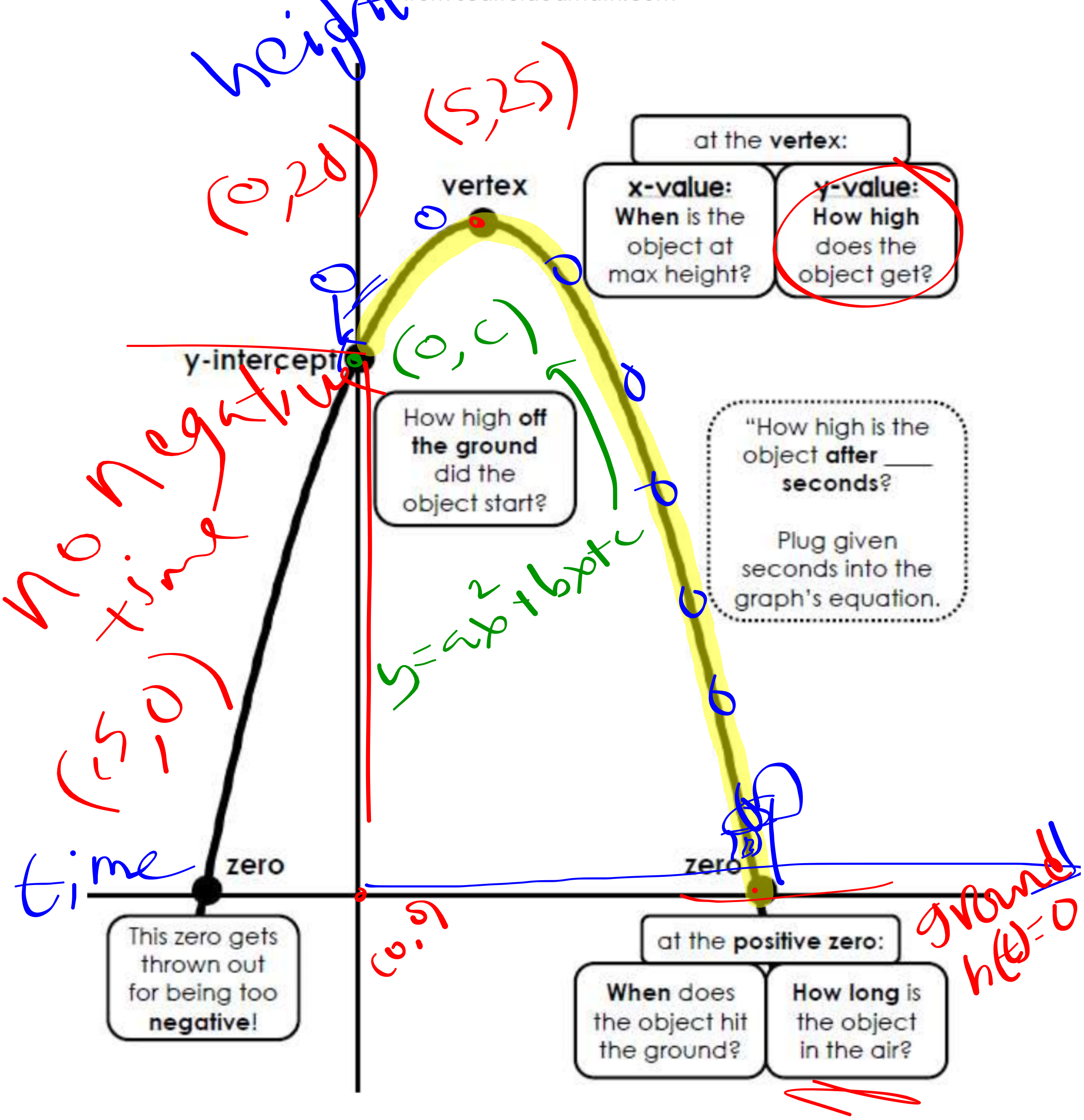
5) $y = (x + 4)^2 + 5$

6) $y = -(x + 3)^2 - 2$

7) $y = 2(x - 2)^2 - 3$

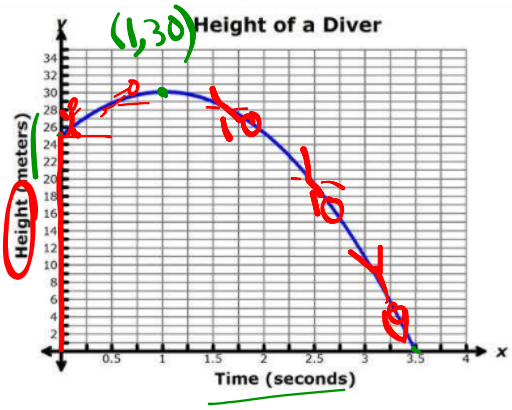
8) $y = \frac{1}{2}(x + 8)^2 + 6$

Quadratic Keywords
from scaffoldedmath.com



Applications of Quadratic Functions

1) This graph represents the height of a diver above the water vs. the time after the diver jumps from a springboard. Answer the following questions based on the information.



a) How long did it take the diver to hit the water? seconds

3.5 seconds

b) How tall was the diving board? height

25 m

c) What was the maximum height reached by the diver?

30m

But what do we do if the graph isn't given to us? If we are not given a graph, we will be given the equation that represents the scenario.

You will need to determine whether the problem is asking you to find the vertex, the x-intercept(s), or the y-intercept.

- ▶ Vertex: maximum, minimum, highest, lowest

  Vertex form: $y = a(x-h)^2 + k \rightarrow$ vertex at (h, k)

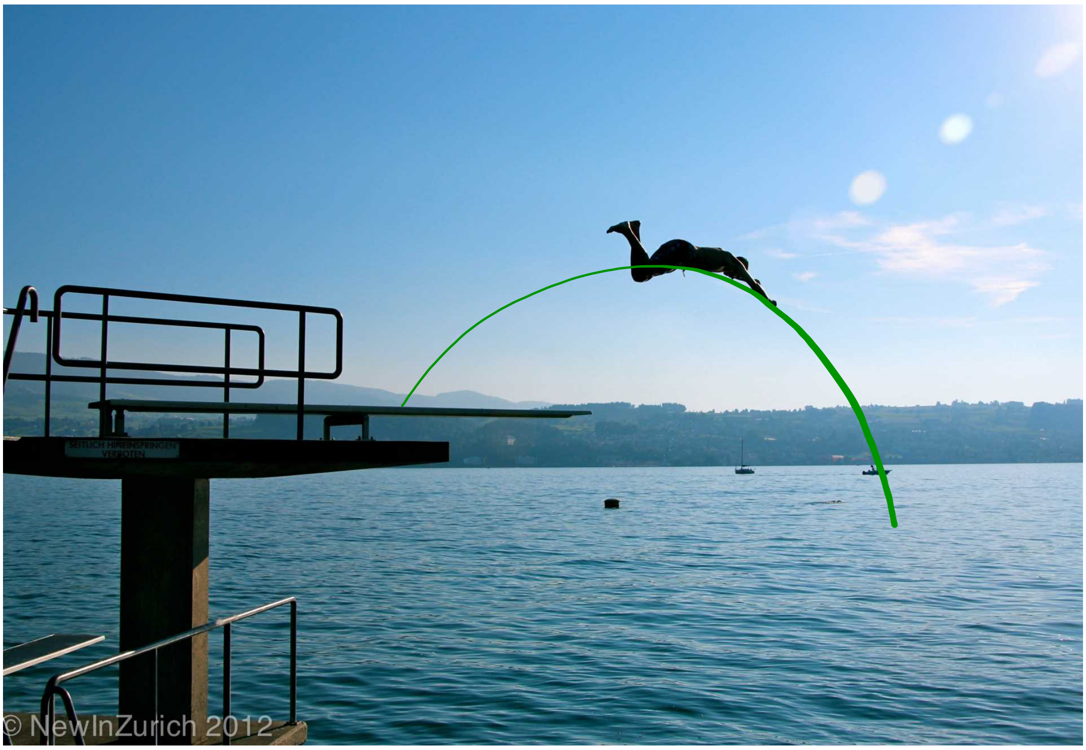
  Standard form: $y = ax^2 + bx + c \rightarrow$ x-value of vertex found using $x = \frac{-b}{2a}$ and then plug that in to find y-value

- ▶ X-Intercept: ending, landing, ground level, sea level

  Solve by: factoring, compl. the square, taking square roots, quadratic formula

- ▶ Y-Intercept: starting value (time is 0 seconds)

  Plug in 0 for x in the given equation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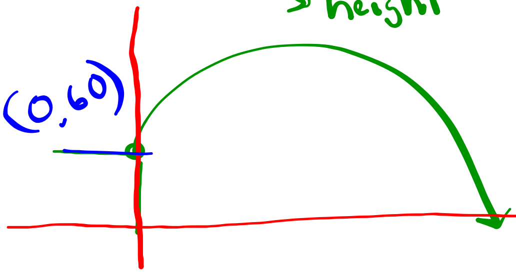
2) An object is launched from a platform. Its height (in meters), x seconds after the launch, is modeled by: $h(x) = -5x^2 + 20x + 60$. What is the height of the object at the time of launch?

time

height

$h(x) = -5x^2 + 20x + 60$

y-intercept



60 m at time of launch.

3) The height, h, in feet of an object above the ground is given by $\cancel{x}$ $= -16t^2 + 64t + 190$, $t \geq 0$, where t is the time in seconds.

$h(t) = -16t^2 + 64t + 190$

$0 = -16t^2 + 64t + 190$

$-190 \qquad -190$

$-16(t^2 - 4t + \boxed{4}) = -190 \boxed{-64}$

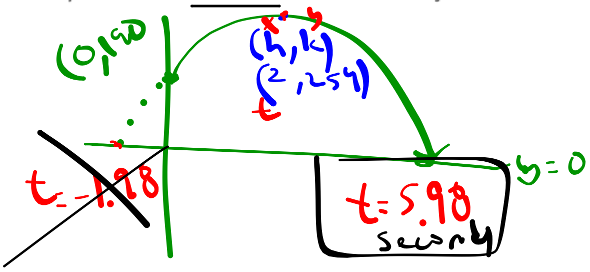
$\frac{-16(t-2)^2}{-16} = \frac{-254}{-16}$

$\sqrt{(t-2)^2} = \sqrt{15.875}$

$t - 2 = \pm 3.98$

$+2 \quad +2$

a) Find the time it takes the object to strike the ground.



b) Find the maximum height of the object.

vertex

$h = \frac{-b}{2a} = \frac{-(64)}{2(-16)} = \frac{-64}{-32} = 2$

$k = -16(2)^2 + 64(2) + 190$

$-16(4) + 128 + 190$

$-64 + 128 + 190$

k = 254 feet max height

Any object that is thrown or launched into the air, such as a baseball, basketball, or soccer ball, is a *projectile*. The general function that approximates the height h in feet of a projectile on Earth after t seconds is given.

$$h(t) = -16t^2 + v_0 t + h_0$$

OR $(-4.9\ m/s^2)$

Constant due to Earth's gravity in ft/s^2

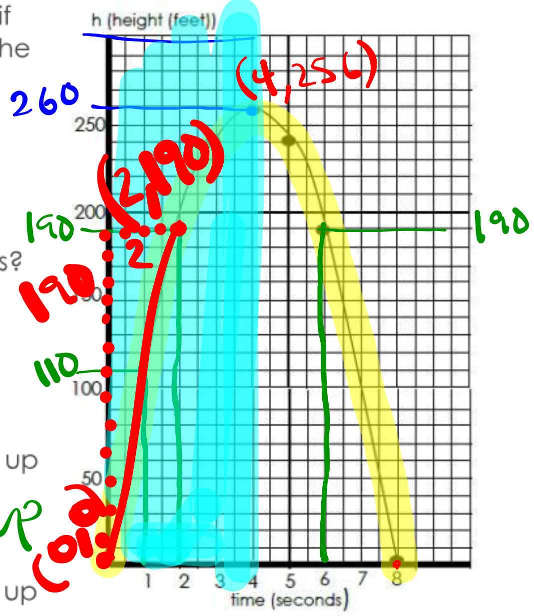
Initial vertical velocity in ft/s (at $t = 0$)

Initial height in ft (at $t = 0$)

Note that this model has limitations because it does not account for air resistance, wind, and other real-world factors.

Applications of Quadratic Functions Practice

1) Using the graph at the right, it shows the height, h, if feet of a small rocket t seconds after it is launched. The path of the rocket is given by the equation $h(t) = -16t^2 + 128t$. + height



a) How long is the rocket in the air?

8 sec

b) What is the greatest height that the rocket reaches?

260 ft

c) About how high is the rocket after 1 second?

110 ft

d) After 2 seconds, how high is the rocket? Is it going up or going down?

190 ft going up

e) After 6 seconds, how high is the rocket? Is it going up or going down?

190ft going down

f) What is the average speed between $t = 0$ seconds and $t = 2$ seconds.

$ARОC = \frac{\Delta y}{\Delta x} = \frac{190\text{ ft}}{2\text{ s}} = \boxed{95 \text{ ft/sec}}$

g) Using the equation, find the exact height of the rocket at 6.5 seconds.

$h(6.5) = -16(6.5)^2 + 128(6.5)$

$-676 + 832$

$\boxed{h(6.5) = 156 \text{ ft}}$

h) What is the domain of the graph?

x interval [0, 8]

i) What is the range of the graph?

y interval [0, 260]

j) Identify the vertex.

Point (4,256)

k) Identify the axis of symmetry.

$x = 4$

Quadratic Formula Word Problems

1) Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach his maximum height?

$x = \frac{-b}{2a} = \frac{-(16)}{2(-16)} = \frac{16}{32} = \frac{1}{2}$ second

or .5

b. What was the highest point that Jason reached?

$y = -16(.5)^2 + 16(.5) + 480$

$y = 484$ ft

c. Jason hit the water after how many seconds?

$0 = -16t^2 + 16t + 480$

$a = -16 \quad b = 16 \quad c = 480$

$x = \frac{-(16) \pm \sqrt{(16)^2 - 4(-16)(480)}}{2(-16)}$

$+x = -5$

$-x = 6$

$\boxed{t = 6 \text{ seconds}}$

2) If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height h after t seconds is given by the equation $h(t) = -16t^2 + 218t$ (if air resistance is neglected).

+0

a. How long will it take for the rocket to return to the ground?

$0 = -16t^2 + 218t$

factor GCF

$0 = 2t(-8t + 109)$

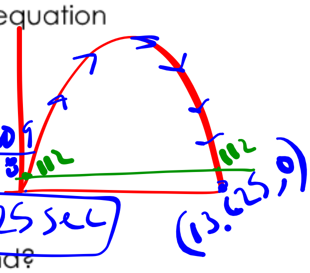
$\frac{2t}{2} = \frac{0}{2} \quad t = 0$ sec

$-8t + 109 = 0$

$-109 \quad -109$

$\frac{-8t}{-8} = \frac{-109}{-8}$

$\boxed{t = 13.625 \text{ sec}}$



b. After how many seconds will the rocket be 112 feet above the ground?

$112 = -16t^2 + 218t$

$-112 \qquad -112$

$0 = -16t^2 + 218t - 112$

$x = \frac{-(218) \pm \sqrt{(218)^2 - 4(-16)(-112)}}{2(-16)}$

$+x = 0.53$ sec

$-x = 13.1$ sec

c. How long will it take the rocket to hit its maximum height?

$-16t^2 + 218t$

a b

$h = \frac{-b}{2a} = \frac{-218}{2(-16)} = \frac{218}{32} = 6.8$ sec

d. What is the maximum height?

$h(6.8) = -16(6.8)^2 + 218(6.8)$

$\boxed{h(6.8) = 742.6 \text{ ft}}$

3) A rocket is launched from atop a 101 – foot cliff with an initial velocity of 116 ft/s. It's height is represented by the quadratic equation $y = -16x^2 + 116x + 101$, where x represents the amount of time since launch in seconds and y represents the height of the rocket in feet.

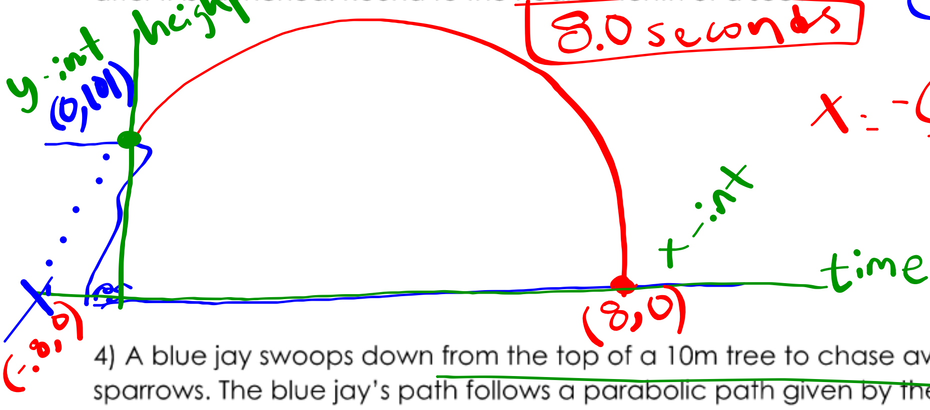
Use the quadratic formula to find out how long the rocket will take to hit the ground after it is launched. Round to the nearest tenth of a second.

$\boxed{8.0 \text{ seconds}}$

$y = -16x^2 + 116x + 101$

$a = -16 \quad b = 116 \quad c = 101$

$$x = \frac{-(116) \pm \sqrt{(116)^2 - 4(-16)(101)}}{2(-16)}$$



4) A blue jay swoops down from the top of a 10m tree to chase away some house sparrows. The blue jay's path follows a parabolic path given by the function $h(t) = 2t^2 - 8t + 10$ where t is time in seconds and $h(t)$ is height in meters.

a) What is the blue jay's lowest height? When did the blue jay reach the lowest height?

k $\quad a = 2 \quad b = -8 \quad c = 10 \quad$ h

$h = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2 \text{ sec}$

$k = 2(2)^2 - 8(2) + 10$

$8 - 16 + 10$

$-8 + 10 = \boxed{2 \text{ m}}$

b) What is the blue jay's starting height?

10 meters

c) Does the blue jay ever touch the ground? If so, at what time?

Nah, bruh.

Math Protocols for Word Problems

(and everything else)

1. Draw picture
2. Highlight information
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4. Solve for QUESTION

Good morning!

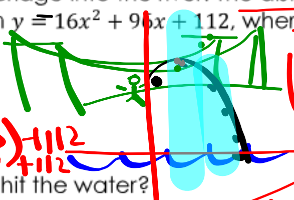
1. "Here"
2. Continue to practice Word problems
3. Test tomorrow:)

Quadratic Word Problems

Ex 1. Mr. Case tosses a coin off the City of Gotham Bridge into the river. The distance, in feet, the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$, where x represents time in seconds.

distance above

$a = -16 \quad b = 96 \quad c = 112$



$\boxed{\text{max height} = 256\text{ ft}}$

time

a) What is the greatest height of the coin?

max

$h = \frac{-b}{2a} = \frac{-(96)}{2(-16)} = 3 \qquad k = -16(3)^2 + 96(3) + 112$

$-144 + 288 + 112$

b) How much time will it take for the coin to hit the water?

$x = \frac{-(96) \pm \sqrt{(96)^2 - 4(-16)(112)}}{2(-16)}$ x-int

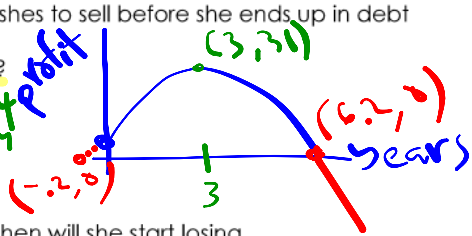
$\cancel{x = -1}$ $\boxed{x = 7}$ $\boxed{7 \text{ seconds}}$

c) What is the interval of decrease of the coin's path?

(3, 7)

$a = -3 \quad b = 18 \quad c = 4$

Ex 2. The profits of Mrs. Rainey's company can be represented by the equation $y = -3x^2 + 18x + 4$ where y is the amount of profit in hundreds of thousands of dollars and x is the number of years of operation. She realizes her company is on the downturn and wishes to sell before she ends up in debt



a) When will Rainey's business show the maximum profit?

$h = \frac{-b}{2a} = \frac{-(18)}{2(-3)} = \boxed{3 \text{ yrs}}$ $k = -3(3)^2 + 18(3) + 4$

$-27 + 54 + 4$

31

b) What is the maximum profit?

\$ 3,100,000.00

c) At what time will it be too late to sell her business? (When will she start losing money?) x-int

after 6.2 years

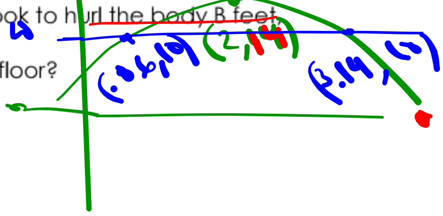
$x = \frac{-(18) \pm \sqrt{(18)^2 - 4(-3)(4)}}{2(-3)}$

d) What does the y-intercept represent in the profit model?

(0, 4) \$4 hundred thousand she started

Ex. 3 During the Middle Ages, victims of the bubonic plague were used for biological attacks, often by flinging fomites such as infected corpses and excrement over castle walls using catapults. Bodies would be tied along with ~~cannonballs and shot~~ towards the city area. If one of the bodies could be modeled by the function $B(t) = -3(t-2)^2 + 14$, where t is time it took to hurl the body B feet.

$h = 2 \quad k = 14$



a) How long would it take for the projectile to hit the castle floor?

4.1 seconds

$0 = -3(t-2)^2 + 14$

$-14 \qquad -14$

$\frac{-14}{-3} = \frac{-3(t-2)^2}{-3}$

$\sqrt{4.6} = \sqrt{(t-2)^2}$

$\pm 2.1 = t - 2$

$+2 \qquad +2$

$\boxed{t = 2 + 2.1 = 4.1}$

$t = 2 - 2.1 = -.1$

b) What was the max height of the body?

14 feet

c) How tall was the catapult before the body was launched?

y-int $\quad y = -3(0-2)^2 + 14$

$-3(4) + 14 = -12 + 14 = \boxed{2 \text{ feet}}$

d) How much time would it take for the body to have a height of 10 feet? Is that the only time at which it would be 10 feet off the ground?

$10 = -3(t-2)^2 + 14$

$-14 \qquad\qquad -14$

$\frac{-4}{-3} = \frac{-3(t-2)^2}{-3}$

$\sqrt{\frac{4}{3}} = \sqrt{(t-2)^2}$

$\pm 1.14 = t - 2$

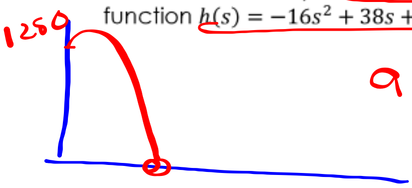
$+2 \qquad +2$

$t = 2 + 1.14 = 3.14$ sec

$t = 2 - 1.14 = 0.86$ sec

1. The Empire State Building is 1250 feet tall. If an object is thrown upward from the top of the building at an initial velocity of 38 feet per second, its height s seconds after it is thrown is given by the function $h(s) = -16s^2 + 38s + 1250$. How long will it take for the ball to hit the ground?

x-int



$a = -16 \quad b = 38 \quad c = 1250$

$x = \frac{-(38) \pm \sqrt{(38)^2 - 4(-16)(1250)}}{2(-16)}$

$\boxed{x = 10.1 \text{ seconds}}$

$\cancel{x = -7.7}$

2. Ms. Walsh throws a basketball into the air, releasing it 5 feet above the ground with an initial velocity of 15 ft/sec. She rebounds the ball with her other hand when the ball returns to 5 feet above the ground (since she cannot jump). If the equation $h = -16t^2 + 15t$ gives the path of the ball from hand to hand, find how long the ball is in the air.

3. Mr. Sims is catapulting oranges into his neighbor's yard. The average height, in feet, of the fruit can be modeled by the function $h(t) = 40t - 16t^2$.

a) If the neighbor's kid is flying a drone at an altitude of 20 feet halfway between Mr. Sims and the growing pile of citrus, will it start raining orange juice?

b) What is the average height of the orange after 1 second?

d) After how many seconds will it return to the ground?

4. A ball is thrown upward from the ground. Its height (h, in feet) is given by the function $h(t) = -16t^2 + 64t + 3$, where t is the length of time (in seconds) that the ball has been in the air. What is the maximum height that the ball reaches?

5. The height, h(t), in feet, of an object shot from a cannon with initial velocity of 20 feet per second can be modeled by the equation $h = -16t^2 + 20t + 6$, hwer4er= t is the time, in seconds, after the cannon is fired. What is the maximum altitude that the object reaches?

6. Physicists tell us that altitude h in feet of a projectile t seconds after firing is $h = -16t^2 + v_0t + h_0$, where v_0 is the initial velocity in feet per second and h_0 is the altitude in feet from which it is fired. If a rocket is launched from a hilltop 2400 feet above the desert with an initial upward velocity of 400 feet per second, then when will it land on the desert?

7. The engine torque y (in foot-pounds) of one model of car is given by $y = -3.75x^2 + 23.2s + 38.8$ where x is the speed of the engine (in thousands of revolution per minute).

a) Find the engine speed that maximizes the torque.

b) What is the maximum torque?

8. In baseball, the flight of a pop-up may be described as $d = -16t^2 + 80t + 3.5$ where d gives the ball's height above the ground in feet as a function of time t. How long does the catcher have to get into position under the ball after the ball is hit?

9. When a gray kangaroo jumps, its path through the air can be modeled by $y = -0.0267x^2 + 0.8x$ where x is the kangaroo's horizontal distance traveled (in feet) and y is its corresponding height (in feet).

a) How high can a gray kangaroo jump?

d) How far can it jump?

10. Ms. Walsh threw a basketball from the basketball court toward the hoop. The quadratic function that models the height, in feet, of the ball after t seconds is $h(t) = -16t^2 + 12t + 6$. If the hoop is 8 feet high, how long is the ball in the air before the ball goes through the hoop?

11. Mr. Sims launches a kickball into the air. The distance, in feet, that the kickball is over the field is modeled by the equation $s(t) = -16t^2 + 70t$, where t is the horizontal distance in feet and s(t) is the vertical distance in feet.

a) How far does the ball travel before it reaches its maximum height?

b) What is the maximum height of the kickball?

c) How far did Mr. Sims kick the ball?

12. Jessie and Jayla simultaneously toss tennis balls from the window of an apartment building at a height of 150 feet into a bucket at a height of 15 feet, 3 inches on the roof of the building next door. The quadratic functions that model the height, in feet, of the balls after t seconds are
$h(t) = -16t^2 - 5t + 150$ for Jessie and $h(t) = -16t^2 + 5t + 150$ for Jayla.
Which ball lands in the bucket sooner? Examine the two equations to explain the difference in the times