

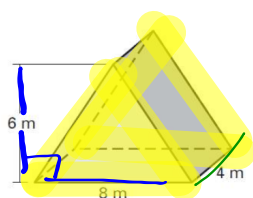
Good morning!

1. "Here" Unit 3b Test this THURSDAY!
2. Notes on Spheres and Cavalieri's Principle
3. Sphere Practice for homework - turn in to CTLS
4. Quiz opens today at 2:00 PM

Find the Volumes

Rectangular  
Prism

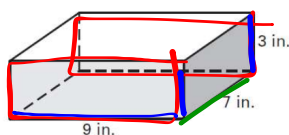
4.



$$V = Bh$$
$$\downarrow$$
$$\frac{1}{2}bh$$
$$\frac{1}{2}(8 \times 6) \cdot 4$$

$$V = 96 \text{ m}^3$$

5.

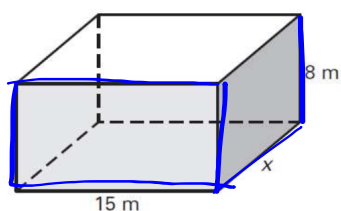


$$V = Bh$$
$$\downarrow$$
$$bh$$
$$9 \cdot 7 \cdot 3$$

$$V = 189 \text{ in}^3$$

### Working Backwards

$$V = 1440 \text{ m}^3$$



Rectangular  
Prism

$$V = Bh$$

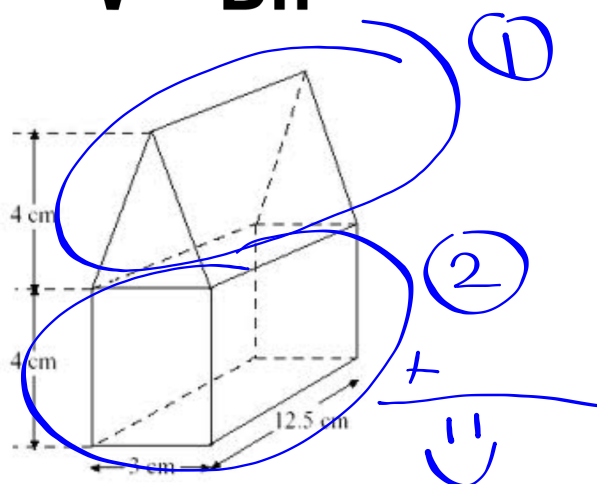
$$1440 = \underbrace{b \cdot h}_{15 \cdot 8} \cdot \underbrace{h}_x$$

$$x = 12 \text{ m}$$

## Composite Solids

EX: Find the volume.

$$V = Bh$$





# Volume of Pyramids

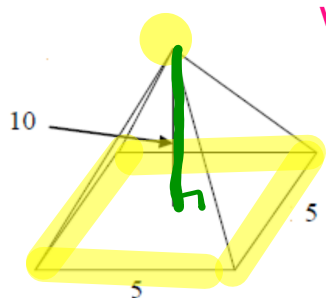
## Volume of Pyramids

$$V = \frac{1}{3}Bh$$

*B stands for the area of the base*

Find the volume and round to the nearest tenth.

Square  
Pyramid



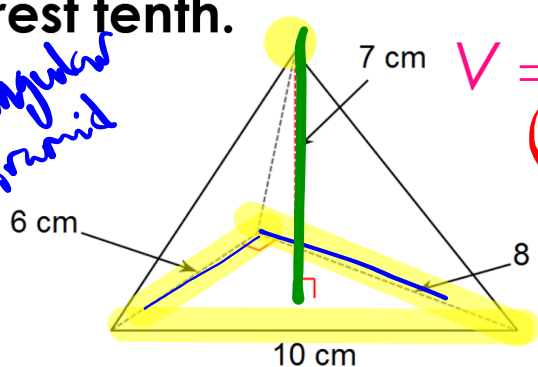
$$V = \frac{1}{3} Bh$$

Handwritten annotations: A green arrow points from the 'h' in the formula to the number 10. A blue bracket is drawn under the '1/3' and '5.5' (which is half of 11, though the diagram shows 10). The number 5.5 is written in blue.

$$V = \frac{250}{3} = 83.3 \text{ units}^3$$

Find the volume and round to the nearest tenth.

Triangular  
Pyramid



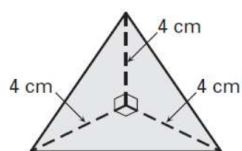
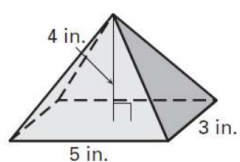
$$V = \frac{1}{3} Bh$$

$\left( \frac{1}{2}bh \right)$

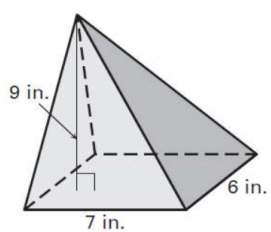
$$\frac{1}{3} \cdot \frac{1}{2} (6)(8)(7)$$

$$\frac{6 \cdot 8 \cdot 7}{3 \cdot 2} = 56 \text{ cm}^3$$

Find the Volumes



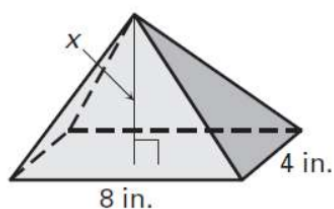
Find the Volume



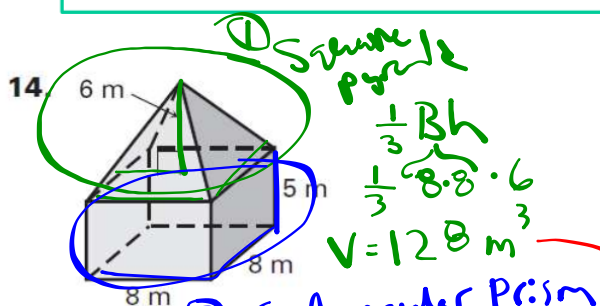
## Working Backwards

**Find the value of  $x$ .**

$$V = 64 \text{ in.}^3$$



# Composite Solids



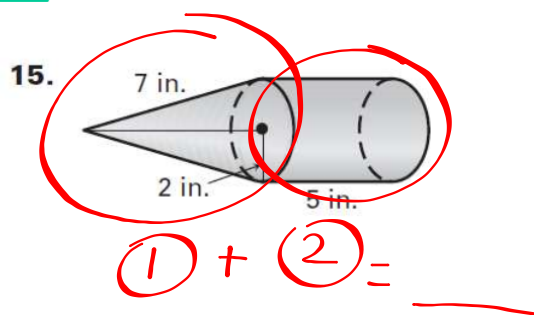
② rectangular Prism

$$V = Bh$$

$8 \cdot 8 \cdot 5$

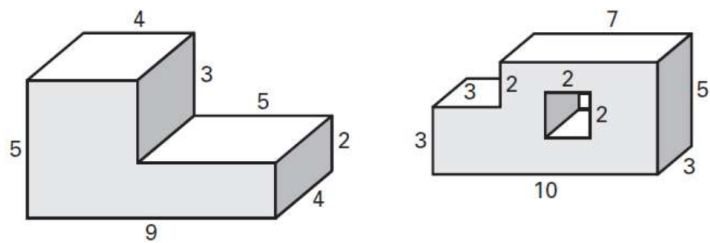
$$V = 320 m^3$$

$$\begin{array}{r} 128 \\ + 320 \\ \hline V = 448 m^3 \end{array}$$



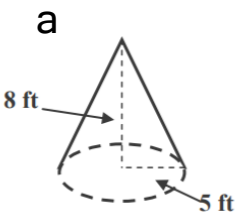


Composite Solids

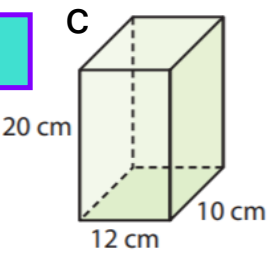




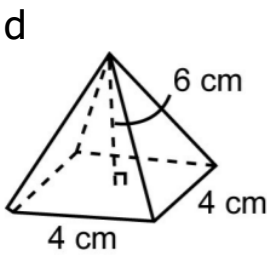
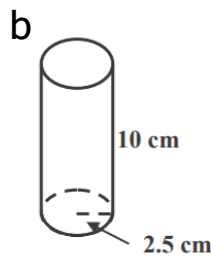
# Volume of Spheres Cavalieri's Principle



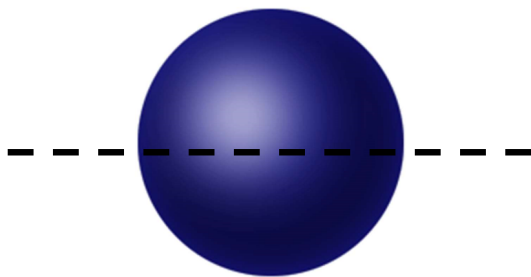
Warm UP



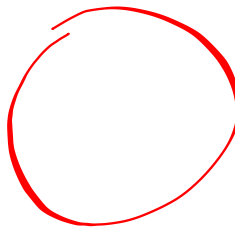
round all to the nearest  
tenth :)



If you cut a sphere right down the middle you would create two congruent halves called HEMISPHERES.



You can think of the globe. The equator cuts the earth into the northern and southern hemisphere.



2-D  
Look at the cross section formed  
when you cut a sphere in half.

What shape is it?

**A circle!!! This is called the  
GREAT CIRCLE of the sphere.**

**The only thing  
flat-earthers have to fear**



**is sphere itself.**

## Formula for Volume of a Sphere

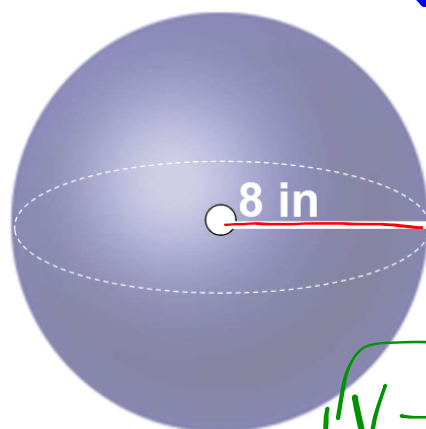
$$V = \frac{4}{3} \pi r^3$$

cubic units  
ft<sup>3</sup>, m<sup>3</sup>, in<sup>3</sup>, ...



Find the Exact Volume:

$$V = \frac{4}{3} \pi r^3$$



$r = 8$

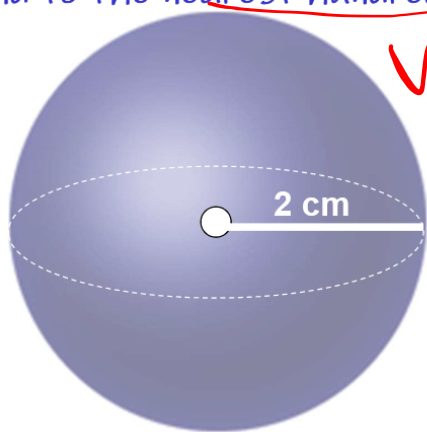
$$\frac{4\pi}{3} (8)^3$$

$$8 \cdot 8 \cdot 8 \cdot 4 \div 3 =$$

$$V = \frac{2048}{3} \pi \text{ in}^3$$

Find the volume

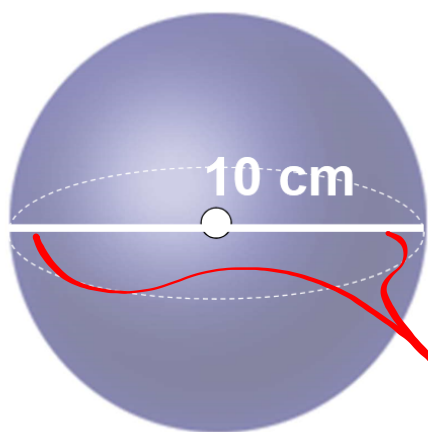
(round to the nearest hundredth):



$$V = \frac{4}{3} \pi (2)^3$$

$$V = 33.51 \text{ cm}^3$$

Find the Exact Volume  
Beware the diameter!

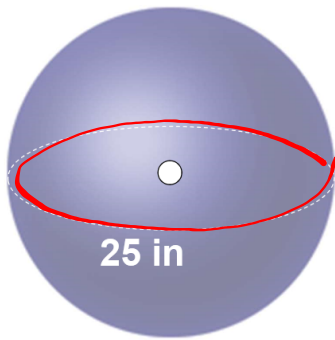


$$D = 10$$
$$r = 5$$

$$V = \frac{4}{3} \pi (5)^3$$

$$V = \frac{500}{3} \pi \text{ cm}^3$$

The circumference of a great circle of a sphere is 25 inches. Find the volume of the sphere. (Round to the nearest tenths.)



$$C = 2\pi r$$

$$\frac{25}{2\pi} = \frac{2\pi r}{2\pi} \quad \textcircled{1}$$

$$25 \div 2 \div \pi = r$$

$$4.0 = r$$

$$\textcircled{2} V = \frac{4}{3}\pi r^3$$

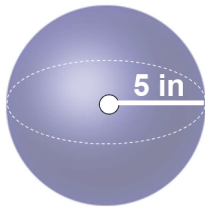
$$= \frac{4}{3}\pi (4)^3$$

$$V = 268.1 \text{ in}^3$$

### Volume of a Sphere

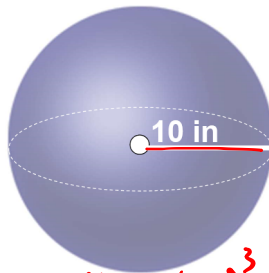
A spherical balloon has an initial radius of 5 in. When more air is added, the radius becomes 10 in. Explain how the volume changes as the radius changes.

$$V = \frac{4}{3} \pi r^3$$



$$V = \frac{4}{3} \pi (5)^3$$

$$V = 523.6 \text{ in}^3$$



$$V = \frac{4}{3} \pi (10)^3$$

$$V = 4188.8 \text{ in}^3$$

Doubled radius

$$5(2) = 10$$

$$25\pi(4) = 100\pi$$

$$523.6(8) = 4188.8$$

$$\frac{4188.8}{523.6} = 8$$

Perimeter  
sides

$$k=2$$

$$A:B$$

$$\text{Area } A^2:B^2 \quad k^2=2^2=4$$

$$\text{Volume } A^3:B^3 \quad k^3=2^3=8$$

radius      ① → ②  
14 → 56

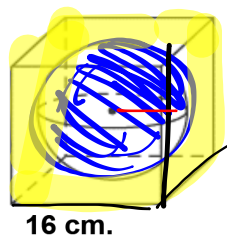
Volumes  
 $14^3 \rightarrow 56^3$

### Volume of a Sphere

A sphere has an initial volume of  $400 \text{ cm}^3$ . The sphere is made bigger by making the radius 4 times as big. What is the new volume of the sphere?

### Volume of a Sphere

A sphere is inscribed in a cube-shaped box as pictured below. To the nearest centimeter, what is the volume of the empty space in the box?



Beware the diameter

① Cube

$$V = 16 \cdot 16 \cdot 16$$

$$V = 4096$$

② Sphere

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi (8)^3 = 2144.7$$

$$r = \frac{16}{2} = 8$$

① - ②

$$4096 - 2144.7$$

$$= 1951.3 \text{ cm}^3$$

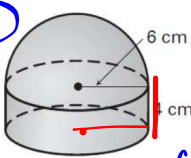
10th place



# Composites With HEMISPHERES

Beware the  
hemisphere

①

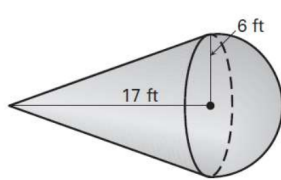


6 cm  
4 cm

②

$V = Bh$   
 $\pi r^2 h$   
 $\pi (6)^2 (4)$   
 $V = 144\pi$

$V = \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right)$   
 $V = \frac{2}{3} \pi r^3$   
 $V = \frac{2}{3} \pi (6)^3$   
 $V = 144\pi$



6 ft  
17 ft

$V = 144\pi$

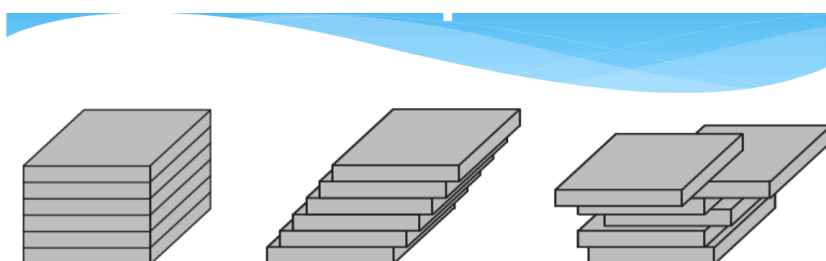
$V = 144\pi + 144\pi = 288\pi$

## Cavalieri's Principle

The volumes of two objects of the same height are equal if the areas of their corresponding cross sections are equal

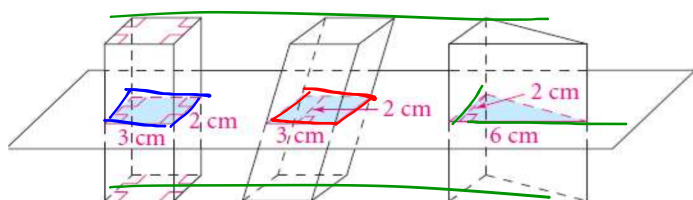


"The reviews are in. Math doesn't look any better in 3-D."



**These pieces maintain their SAME volume regardless of how they are moved.**

## Cavalieri's Principle Examples



$$A_{\square} = 3 \cdot 2 \\ = 6 \text{ cm}^2$$

$$A_{\square} = 3 \cdot 2 \\ = 6 \text{ cm}^2$$

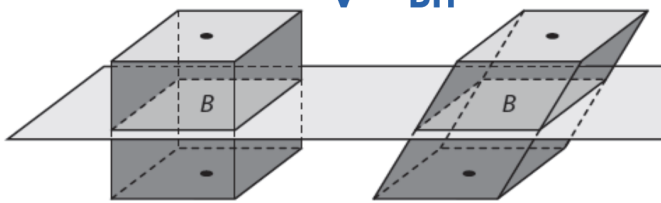
$$A_{\triangle} = \frac{1}{2}bh \\ = \frac{1}{2}2 \cdot 6 \\ = 6 \text{ cm}^2$$

SAME  
Volume

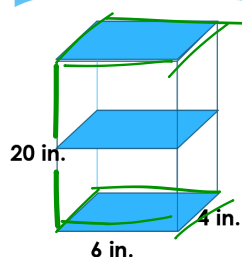
## Cavalieri's Principle Examples

The same volume formula applies whether it's a **right** prism or an **oblique** prism.

$$V = Bh$$



Example 1: Which solid has more volume? Use Cavalieri's Principle.

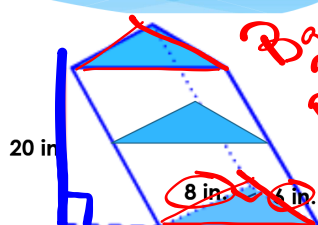


$$V = Bh$$

$$6 \cdot 4 \cdot 20$$

$$24 \cdot 20$$

$$480 \text{ in}^3$$



$$h = 20 \text{ in}$$

Base Triangular  
Base Prism

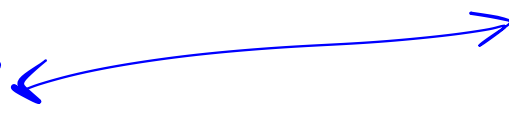
$$V = Bh$$

$$\frac{1}{2}bh$$

$$\frac{1}{2}(8)(6) \cdot 20$$

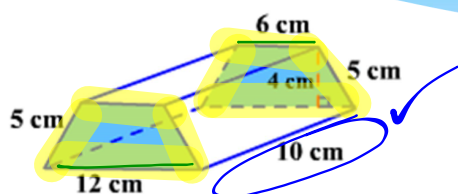
$$(24) \cdot 20$$

$$480 \text{ in}^3$$



Example 2: Which solid has more volume? Use Cavalieri's Principle.

$$A_{\triangle} = \frac{1}{2}(b_1 + b_2) \cdot h$$

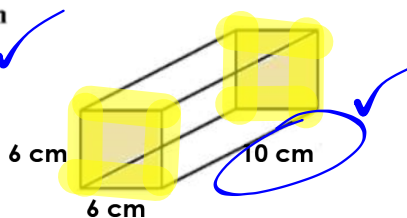


$$V = B \cdot h$$

$$\frac{1}{2}(6+12) \cdot 4 \cdot 10$$

$$36 \cdot 10$$

$$V = 360 \text{ cm}^3$$



$$V = B \cdot h$$

$$6 \cdot 6 \cdot 10$$

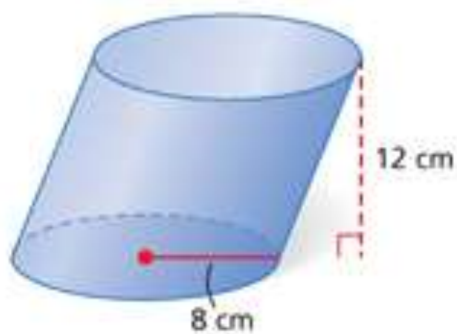
$$36 \cdot 10$$

$$360$$



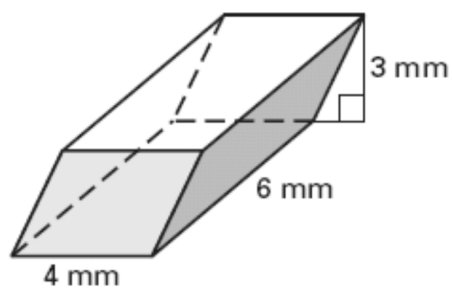
3. Find the volume to the nearest tenth.

$$V = Bh$$



4. Find the volume to the nearest tenth.

$$V = Bh$$



- ① Sphere Practice
- ② Volume Quiz @ 2:00 PM