

Good morning!

1. "Here"
2. Review discriminant
3. Develop the Quadratic Formula
4. Practice p. 22 of packet
5. Homework is on DeltaMath:)

BRACE YOURSELF

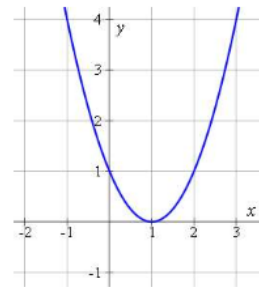
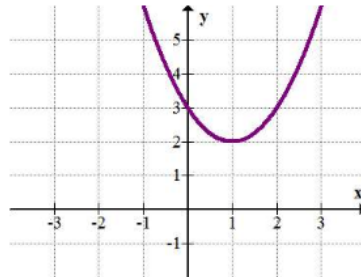
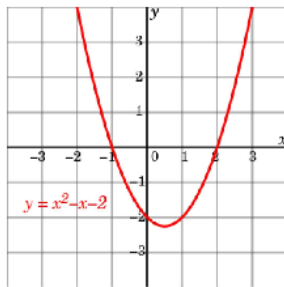
**THE QUADRATIC FORMULA IS
UPON US**



Quadratic Formula and the Discriminant

Remember, solutions to quadratic functions are also known as **zeroes, roots, and x-intercepts**.

How many solutions does each graph below have? (think about the sentence above)



The Discriminant

The discriminant is part of the quadratic formula. When you simplify the discriminant, it becomes a number that will tell you the number of solutions a quadratic function has.

Before finding the discriminant, you must make sure your equation is equal to zero

discriminant: $b^2 - 4ac$
 If the discriminant is negative, there is/are no real solution

If the discriminant is zero, there is/are one solution

If the discriminant is positive, there is/are two solutions

For each equation below, determine the number of solutions.

1) $x^2 + 6x + 4 = 0$

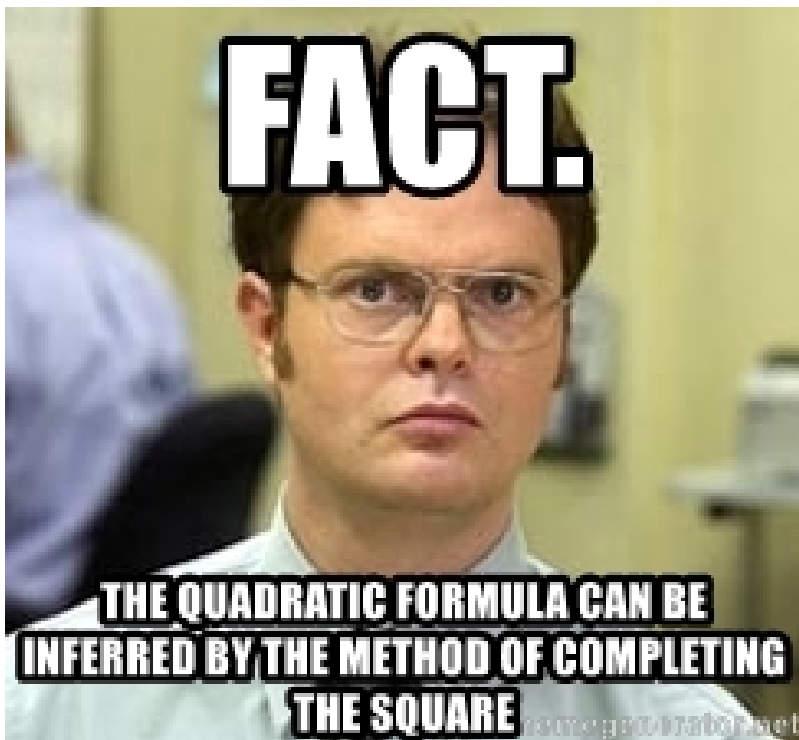
2) $-3x^2 + 17x - 2 = 3$

3) $3x + 7 = -5x^2 - 4$

4) $x^2 - 5x - 34 = 0$ $b^2 - 4ac$ 2 sol's
 $a = 1$
 $b = -5$
 $c = -34$
 $(-5)^2 - 4(1)(-34)$
 $25 + 136$
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5) $2x^2 - 3x + 2 = 0$

6) $9x^2 + 24x + 10 = -6$



$$ax^2 + bx + c = 0$$

Standard form

$$ax^2 + bx = -c$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + a\frac{b^2}{4a^2}$$

$$a\left(x + \frac{b}{2a}\right)^2 = -c + \frac{ab^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-c}{a} + \frac{b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\sqrt{\frac{b^2}{4a^2} - \frac{c \cdot 4a}{a \cdot 4a}}}{1}$$

$$x + \frac{b}{2a} = \frac{\sqrt{\frac{b^2 - 4ac}{4a^2}}}{1}$$

$$x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} \quad -\frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

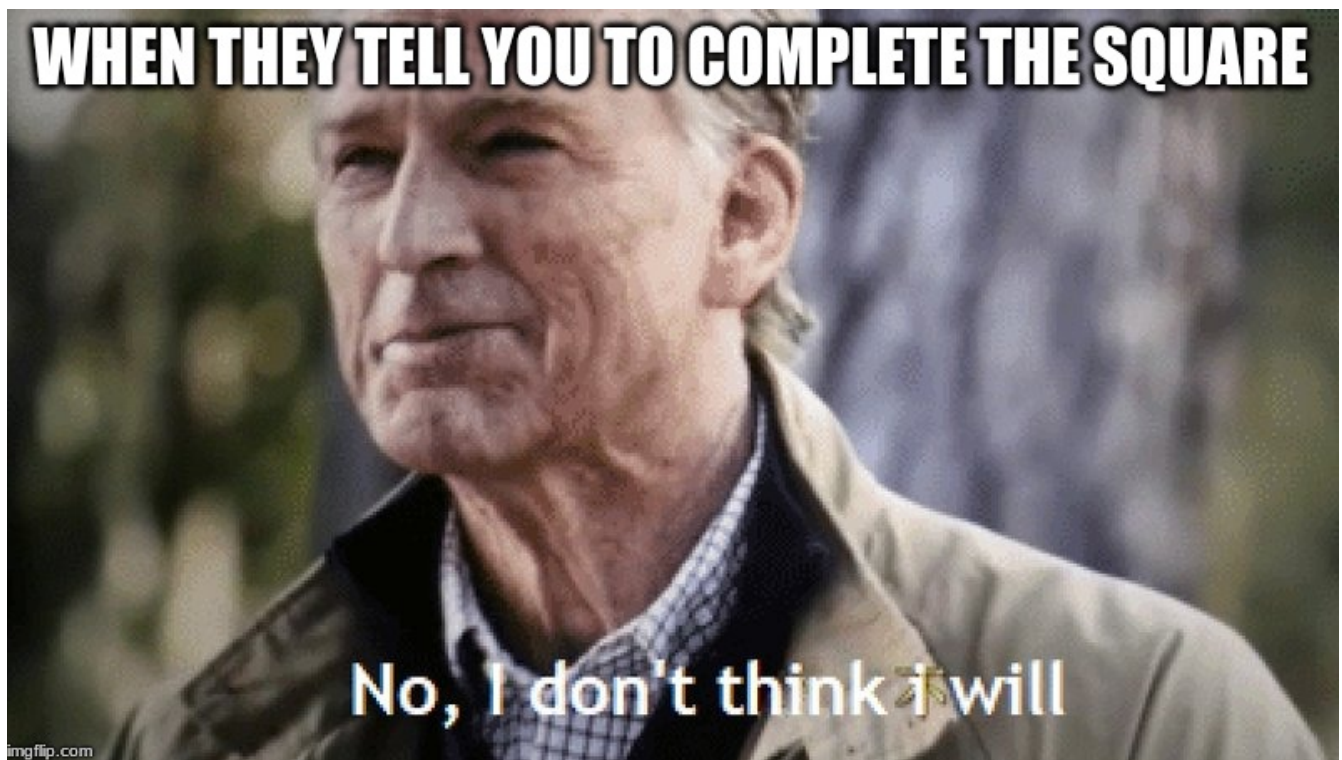
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula

$$\frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$$

$$\frac{20}{5} = 4$$

$$\frac{20}{2 \cdot 5} = \frac{20}{10} = 2$$



The Quadratic Formula

You can use the quadratic formula anytime that a quadratic equation is in general form. The quadratic formula is one method that will always work when solving quadratics.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $4x^2 - 13x + 3 = 0$

Steps	Example
1) Find a, b, and c **make sure equation is set equal to zero**	$a = 4 \quad b = -13 \quad c = 3$
2) Plug a, b, and c into the quadratic formula	$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(3)}}{2(4)}$
3) Simplify the discriminant and denominator	$\frac{13 \pm \sqrt{121}}{8}$ $= \frac{13 \pm 11}{8}$ <i>169 - 16(3) 169 - 48 121</i>
4) Separate into two equations and simplify	$x = \frac{13+11}{8} = \frac{24}{8} = 3$ $x = \frac{13-11}{8} = \frac{2}{8} = \frac{1}{4}$ <i>2 sol's</i>

Practice:

$a = 1 \quad b = 11 \quad c = 10$

1) $x^2 + 11x + 10 = 0$

Discriminant: 81 which means 2 solutions

Root(s): -1, -10

$$x = \frac{-(11) \pm \sqrt{(11)^2 - 4(1)(10)}}{2(1)}$$

$$\frac{-11 \pm \sqrt{81}}{2}$$

$$\frac{-11 \pm 9}{2}$$

$$x = \frac{-11+9}{2} = \frac{-2}{2} = -1$$

$$x = \frac{-11-9}{2} = \frac{-20}{2} = -10$$



$a=7$ $b=-8$ $c=1$ ✓

2) $7x^2 + 8x + 1 = 0$

Discriminant: $b^2 - 4ac = 64 - 28 = 36$

of solutions: 2 solutions

Root(s): $-\frac{1}{7}, -1$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(7)(1)}}{2(7)}$$

$$x = \frac{-8 \pm \sqrt{36}}{14} \quad x = \frac{-8 - 6}{14}$$

$$x = \frac{-8 + 6}{14} = \frac{-2}{14}$$

$$x = \frac{-2}{14} = \boxed{\frac{-1}{7}}$$

$a=9$ $b=6$ $c=1$

4) $9x^2 + 6x + 1 = 0$

Discriminant: $b^2 - 4ac = 0$

of solutions: 1 solution

Root(s): $-\frac{1}{3}$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(9)(1)}}{2(9)}$$

$$x = \frac{-6 \pm \sqrt{0}}{18}$$

$$x = \frac{-6}{18} = \boxed{\frac{-1}{3} = x}$$

$a=-3$ $b=2$ $c=8$

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3) $-3x^2 + 2x = -8$ $-3x^2 + 2x + 8 = 0$

Discriminant: $b^2 - 4ac = 4 - 96 = -92$ *2 solutions*

of solutions: 2 solutions

Root(s): $-\frac{4}{3}, 2$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(-3)(8)}}{2(-3)}$$

$$x = \frac{-2 \pm \sqrt{100}}{-6}$$

$$x = \frac{-2 + 10}{-6} \quad x = \frac{-2 - 10}{-6}$$

$$x = \frac{8}{-6} = \frac{-4}{3} \quad x = \frac{-12}{-6} = 2$$

$$\boxed{x = -\frac{4}{3}}$$

$$\boxed{x = 2}$$

5) $9x^2 + 14x + 3 = 0$ $a=9$ $b=14$ $c=3$

Discriminant: $b^2 - 4ac = 196 - 108 = 88$

of solutions: 2 solutions

Root(s): $\frac{-7 \pm \sqrt{22}}{9}$

$$x = \frac{-(14) \pm \sqrt{(14)^2 - 4(9)(3)}}{2(9)}$$

$$x = \frac{-14 \pm \sqrt{88}}{18}$$

$$x = \frac{-14 \pm 2\sqrt{22}}{18}$$

$$x = \frac{-7 \pm \sqrt{22}}{9} = \boxed{\frac{-7 + \sqrt{22}}{9}} \text{ or } \frac{-7}{9} + \frac{\sqrt{22}}{9}$$

$$x = \frac{-7 - \sqrt{22}}{9} = \boxed{\frac{-7 - \sqrt{22}}{9}} \text{ or } \frac{-7}{9} - \frac{\sqrt{22}}{9}$$

The Work of Al-Khwarizmi

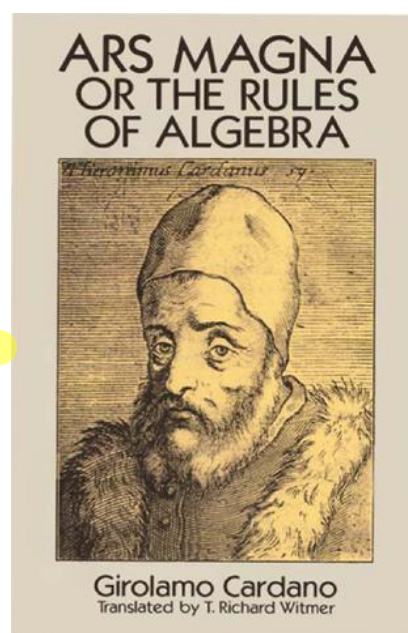
The work of the Babylonians was lost for many years. In 825 CE, about 2,500 years after the Babylonian tablets were created, a general method that is similar to today's Quadratic Formula was authored by the Arab mathematician Muhammad bin Musa al-Khwarizmi in a book titled *Hisab al-jabr w'al-muqabala*. Al-Khwarizmi's techniques were more general than those of the Babylonians. He gave a method to solve any equation of the form $ax^2 + bx = c$, where a , b , and c are positive numbers. His book was very influential. The word "al-jabr" in the title of his book led to our modern word "algebra." Our word "algorithm" comes from al-Khwarizmi's name.



Muhammad bin Musa
al-Khwarizmi

Neither the Babylonians nor al-Khwarizmi worked with an equation of the form $ax^2 + bx + c = 0$, because they considered only positive numbers, and if a , b , and c are positive, this equation has no positive solutions.

In 1545, a Renaissance scientist, Girolamo Cardano, blended al-Khwarizmi's solution with geometry to solve quadratic equations. He allowed negative solutions and even square roots of negative numbers that gave rise to complex numbers, a topic you will study in Advanced Algebra. In 1637, René Descartes published *La Géométrie* that contained the Quadratic Formula in the form we use today.



- ◆ 1500BC Egyptians made a table.
- ◆ 580 BC Pythagoras hates irrational numbers.
- ◆ 400 BC Babylonians solved quadratic equations.
- ◆ 300 BC Euclid developed a geometrical approach and proved that irrational numbers exist.
- ◆ 598-665AD Brahmagupta took the Babylonian method that allowed the use of negative numbers.
- ◆ 800AD Al-Khwarizmi removed the negative and wrote a book *Hisab al-jabr w-al-musqalah* (Science of the Reunion and the Opposition)
- ◆ 1145AD Abraham bar Hiyya Ha-Nasi (Savasaorda) wrote the book *Liber embadorum* – contained the complete solution to the quadratic equation.
- ◆ 1637AD Rene Descartes published *La Geometrie* containing the quadratic formula we know today.

