Good morning!

1. First and Last name as participant.
2. Type "here" for attendance.
3. Test is on Tuesday next week (after Labor Day).
4. Notes and practice for Congruent Triangles.

Corresponding Parts of Congruent Triangles are Congruent

A **Congruence Statement** tells us how the parts of one triangle match up with another triangle. The order of the letters is super important.

Example: Without even having a picture, if we have the statement $\triangle ABC \cong \triangle DEF$ then we know...

"same" congruent

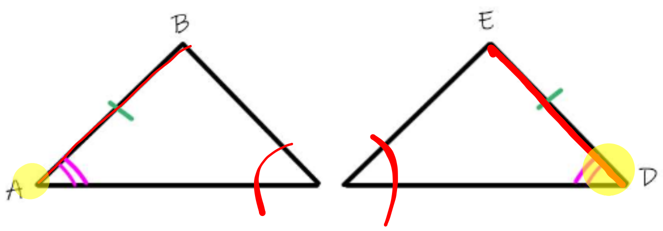
$\angle A \cong \angle D$ $\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$ $\overline{BC} \cong \overline{EF}$

$\angle C \cong \angle F$ $\overline{BC} \cong \overline{EF}$

Congruence Markings:

- In a diagram, when two angles are congruent, they will be marked with the same number of arches.
- When two side lengths are congruent, they will have the same number of tick marks.



- In the diagram to the right, the markings show that $\angle A \cong \angle D$, and $\overline{AB} \cong \overline{DE}$.

<u>You Try!</u> Based on the congruence statements or markings in the figure, determine the pairs of corresponding congruent angles, and corresponding congruent sides.

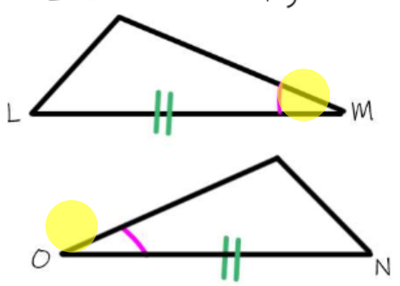
1. $\triangle CAT \cong \triangle DOG$

$\angle C \cong \angle D$ $\overline{CA} \cong \overline{DO}$

$\angle A \cong \angle O$ $\overline{AT} \cong \overline{OG}$

$\angle T \cong \angle G$ $\overline{CT} \cong \overline{DG}$

2. Based on the figure:



$\angle M \cong \angle O$

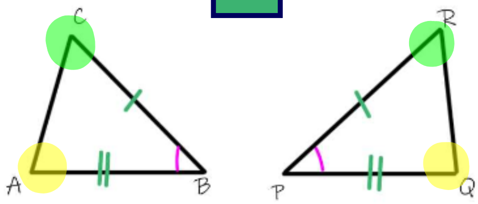
$\overline{ON} \cong \overline{ML}$

You Try!

3. $\triangle IJK \cong \triangle LMN$

$\angle I \cong \angle L$	$\overline{LM} \cong \overline{IJ}$
$\angle J \cong \angle M$	$\overline{MN} \cong \overline{JK}$
$\angle K \cong \angle N$	$\overline{LN} \cong \overline{IK}$

2. Based on the figure:



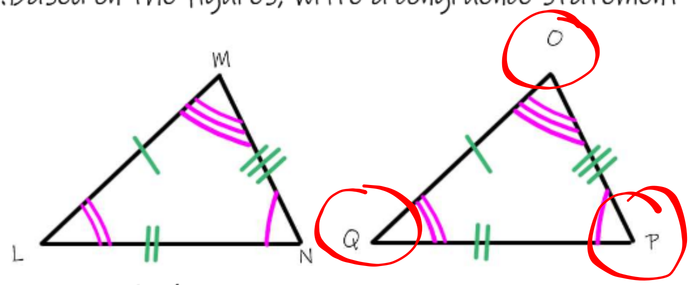
$\angle B \cong \angle P$

$\overline{AB} \cong \overline{QP}$

4. If $\triangle XYZ \cong \triangle TAC$ ...

$\angle X \cong \angle T$	$\overline{TA} \cong \overline{XY}$
$\angle Y \cong \angle A$	$\overline{AC} \cong \overline{YZ}$
$\angle Z \cong \angle C$	$\overline{TC} \cong \overline{XZ}$

5. Based on the figures, write a congruence statement

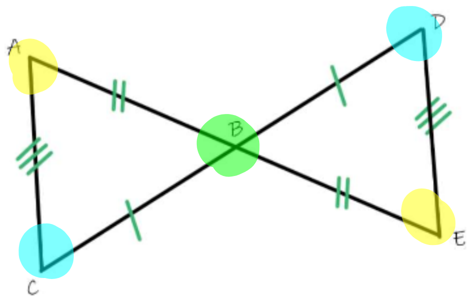


$\triangle LMN \cong QOP$

2 3 1   2 3 1

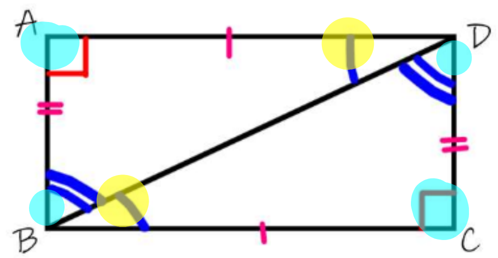
6. Based on the figures, write a congruence statement.

a.



$\triangle ABC \cong EBD$

b.



$\triangle ABD \cong CDB$

Proving Triangles Congruent

So far, we have answered questions about triangles that we have been told are congruent. **But what if we are not told whether or not they are the same?** There are a few ways that we can show that the triangles MUST be the same. These ways are called **theorems** or **postulates**. If we have enough information to show that one of the theorems or postulates is represented, **we can PROVE that two triangles are congruent.**

Congruence Postulate #1

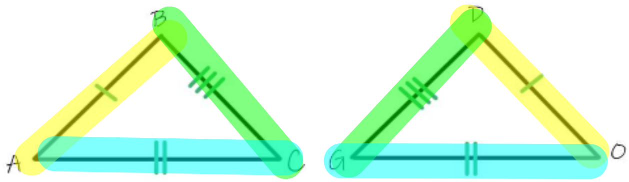
Side Side Side (SSS): If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

Example 1:

a. From the diagram we see:

$\overline{AB} \cong \overline{OD}$ and

$\overline{BC} \cong \overline{DG}$ and $\overline{CA} \cong \overline{OG}$



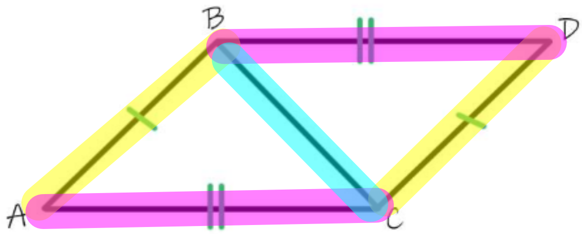
Therefore... $\triangle ABC \cong \triangle ODG$ by SSS

b. Let's take a look:

From the diagram, we know that

$\overline{AB} \cong \overline{DC}$ and $\overline{AC} \cong \overline{DB}$

But that's only 2 sides, and we need three.



What do you notice about the third side of each triangle? The two triangles are sharing the third side.

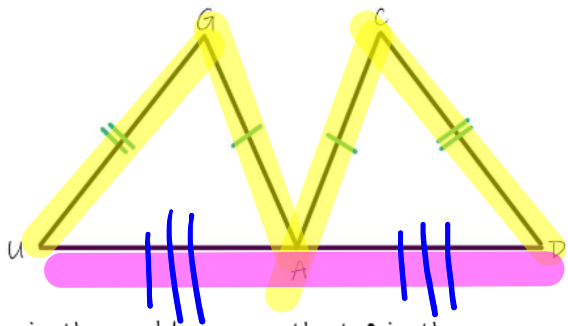
Reflex Prop. of ≅

Anytime we add something to a diagram, we **must** have a property or justification!

When two triangles are sharing a side length we can use the **Reflexive** property to show that it is congruent to itself! Therefore: $\overline{BC} \cong \overline{CB}$

Now we can prove: $\triangle ABC \cong \triangle DCB$ by the SSS postulate.

c. Another property we'll see with side lengths...



Given: A is the midpoint of $\overline{DU}$, can we prove that $\triangle UGA \cong \triangle DCA$?

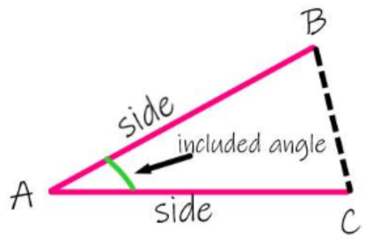
So... we know that: $\overline{UG} \cong \overline{DC}$ and $\overline{GA} \cong \overline{CA}$.
That's only two sides, so we are going to need another side length. The **given** information in the problem says that **A** is the **midpoint** of $\overline{DU}$.

Define Midpoint: Divides a segment into two equal pieces. (The halfway point)

Based on this definition, know we know $\overline{UA} \cong \overline{DA}$.

Therefore $\triangle UGA \cong \triangle DCA$ by SSS.

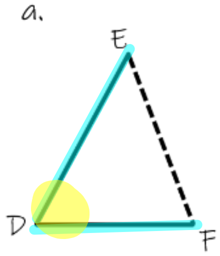
Our next postulate with involve using some angles, so we need to understand some vocabulary first. When two sides of a triangle meet, they form an angle. The angle where two sides meet is called their "**included**" angle.



- In the diagram to the right, angle A is the included angle of sides AB and AC.

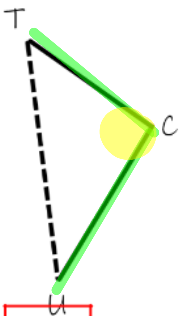
You try: Identify the included angle of the solid sides of each triangle.

2. a.



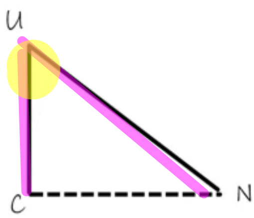
$\angle D$

b.



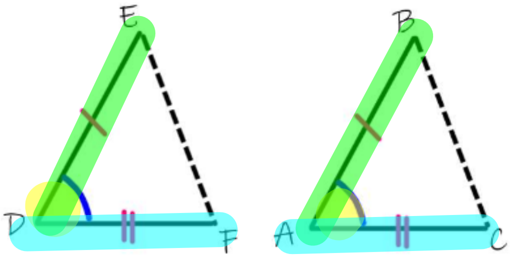
$\angle C$

c.



$\angle U$

Let's Take a closer look:



- From the diagram we can see that $\overline{DE} \cong \overline{AB}$ and $\overline{DF} \cong \overline{AC}$
- We can also see that the **Included angle** between the sides is marked as congruent.
- Anytime you have **two fixed distances** (your solid sides) bound by the **same angle** (the included angle) the distance it takes to connect those endpoints (E to F) or (B to C) will always be the same!

This means anytime you see two **congruent sides** with **congruent included angles** you know that the two triangles MUST be congruent.

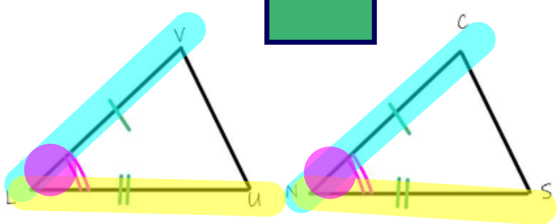
When we prove triangles congruent this way, we are using our second postulate:

Congruence Postulate #2

<u>Side Angle Side (SAS):</u> If two sides and the included angle of one triangle are congruent two sides and the include angle of another triangle, the two triangles will be congruent.

3. <u>a.</u> From the diagram we see: $\overline{LV} \cong \overline{NC}$ and $\overline{LU} \cong \overline{NS}$ and the included angles: $\angle L \cong \angle N$

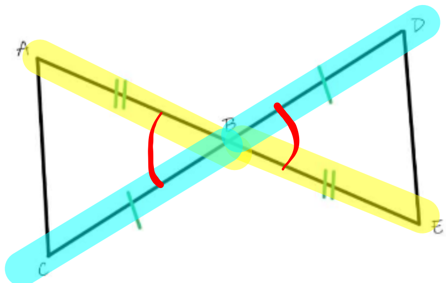
Therefore... $\triangle LVU \cong \triangle NCS$ by SAS



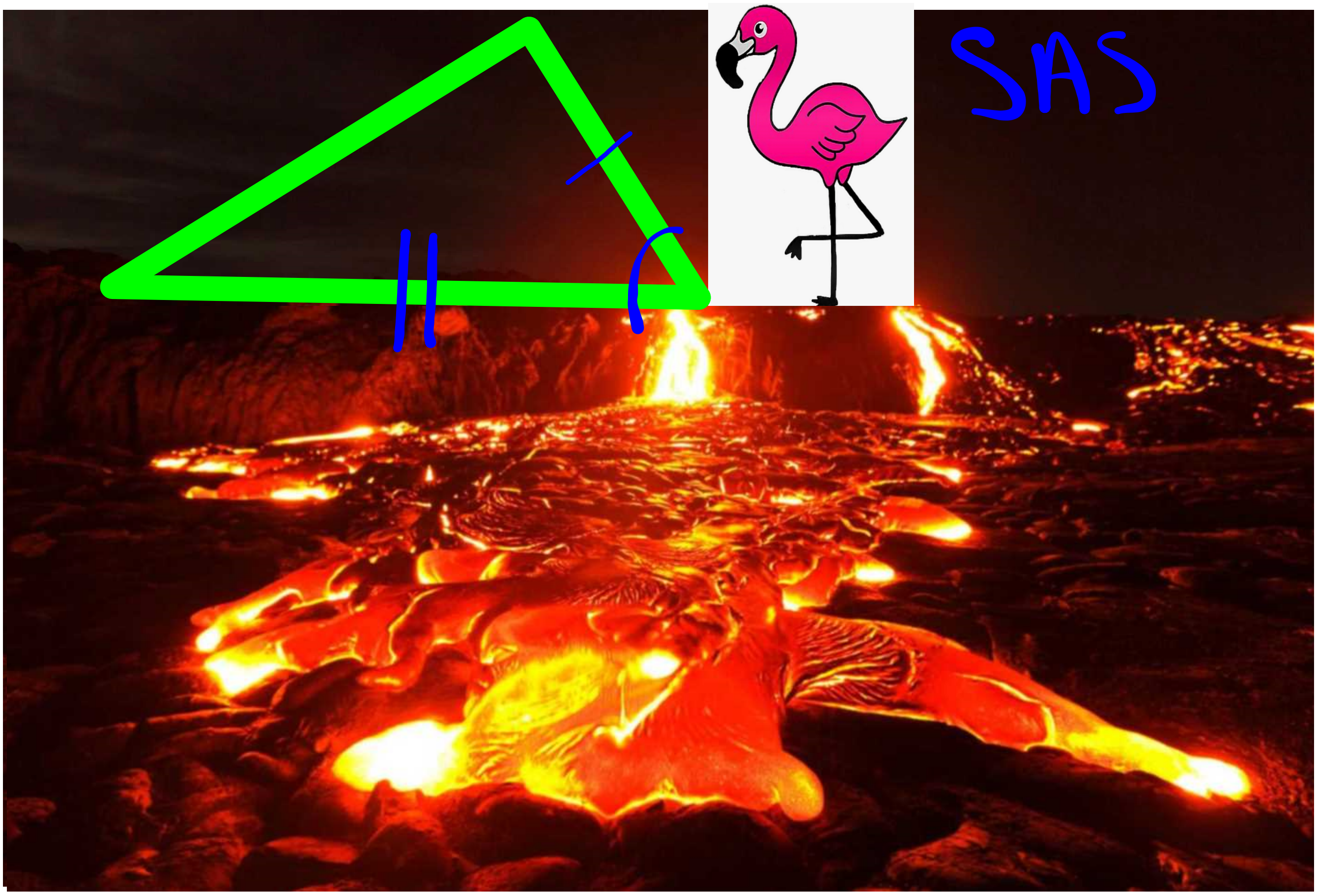
b. <u>Lets Take a Look:</u>

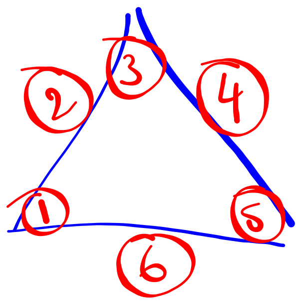
From the diagram, we know that

$\overline{AB} \cong \overline{EB}$ and $\overline{BC} \cong \overline{BD}$



But that's only 2 sides. We either need another side for SSS or the included angle for SAS. **Remember**, we MUST have a property or justification to add anything to our diagram.... Do you notice anything about the included angles that we know from previous lessons? They are Vertical Angles!

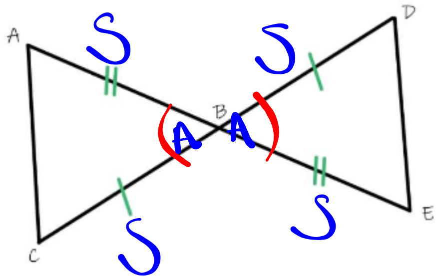

SAS


1
2
3
4
5
6

3b Continued:

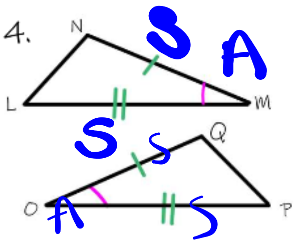
Any time you see vertical angles you can add a marking for them into your diagram!

Now we know that ∠ABC ≅ ∠EBD. Since these are the included angles, we can now say that △ABC ≅ △EBD by the SAS Postulate.



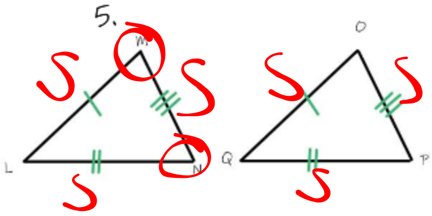
You Try: Decide which congruence postulate can be used for each pair of triangles below. If they are congruent, write a congruence statement. If neither postulate can be used, put an "X" in each blank.

4.



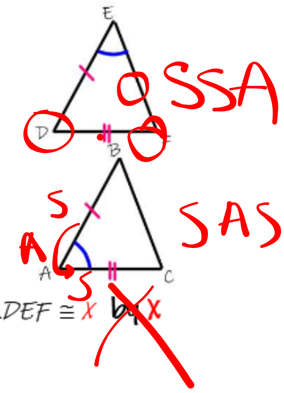
△LMN ≅ △POQ by SAS

5.



△LMN ≅ △QOP by SSS

6.



△DEF ≅ X by X

Challenge Problem! Putting it all together 😊

7. Given: B is the **Midpoint** of $\overline{CD}$.

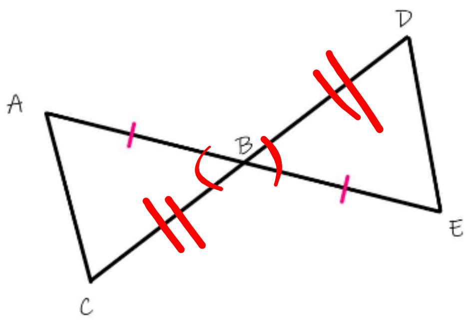
From the diagram we know.... $\overline{AB} \cong \overline{EB}$

- What can we mark because of the midpoint?

  $\overline{CB} \cong \overline{DB}$

- We can mark ∠ABC ≅ ∠EBD because they are **Vertical Angles**

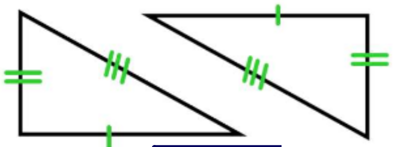
Therefore: △ABC ≅ △EBD by SAS.



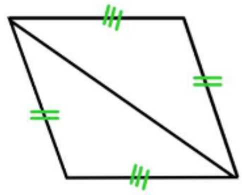
Triangle Congruence Practice - SSS and SAS

1. Which of the following examples does **NOT** show SSS congruence?

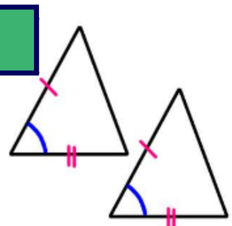
a.



b.

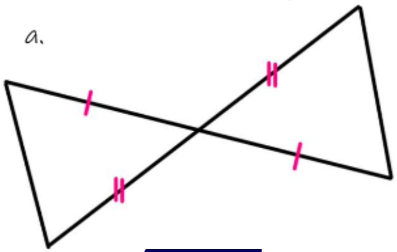


c.

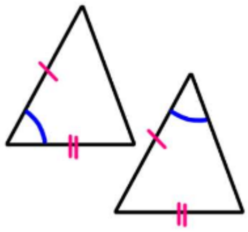


2. Which of the following does **NOT** show SAS congruence?

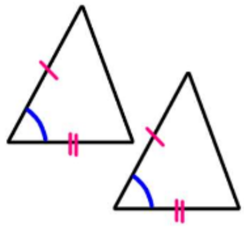
a.



b.

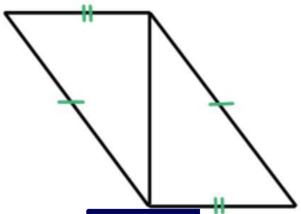


c.

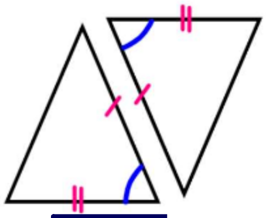


3. Determine if you can use SSS or SAS to prove the pairs of triangles below congruent. If it does not fit one of those postulates, write "neither."

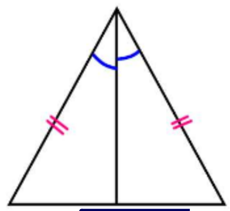
a.



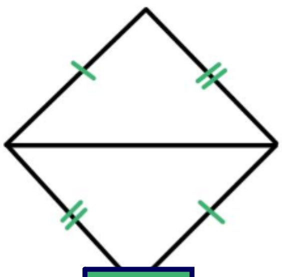
b.



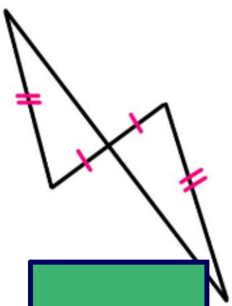
c.



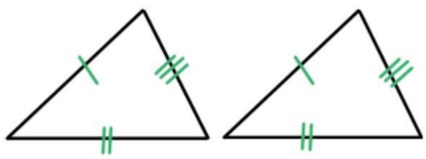
d.



e.

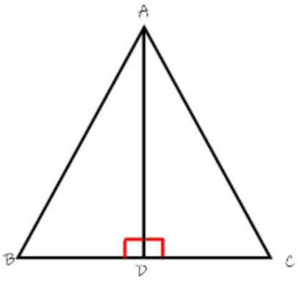


f.



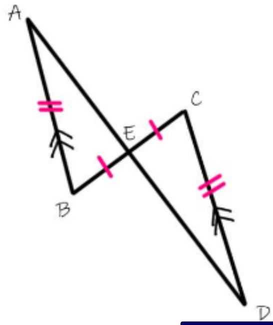
4. Given the information, determine which postulate you can use to prove the triangles congruent.

a. Given: D is the midpoint of BC.



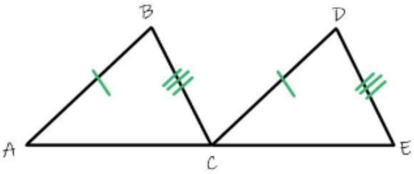
$\triangle BAD \cong \triangle$ ____ by ____

b. Given: AB || CD.



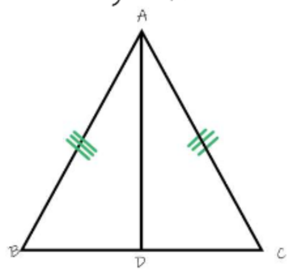
$\triangle ABE \cong \triangle$ ____ by ____

c. Given: C is the midpoint of AE.



$\triangle ABC \cong \triangle$ ____ by ____

D. Given $\overline{AD}$ is bisecting $\angle BAC$.



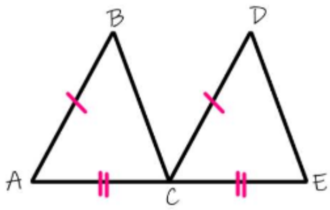
$\triangle BAD \cong \triangle$ ____ by ____

Challenge Section, TEST PREP:

5. What **additional** information is needed to prove....

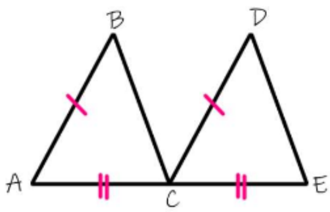
a. $\triangle ABC \cong \triangle CDE$ **by** SSS?

If ____ is congruent to ____ then that would meet the criteria for SSS.



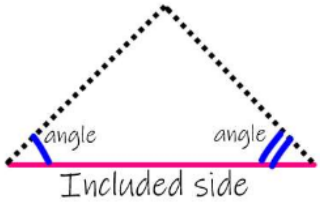
b. $\triangle ABC \cong \triangle CDE$ **by** SAS?

If ____ is congruent to ____ then that would meet the criteria for SAS.



Vocabulary to help us with our next postulate:

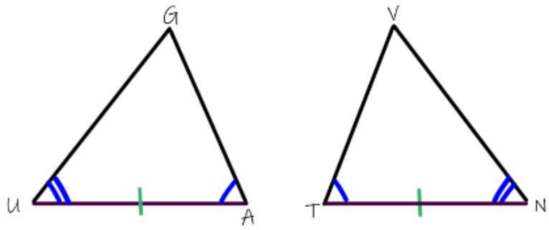
The side length that is between two angles is called the **included side.**



Postulate #3

Angle Side Angle (ASA)-If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

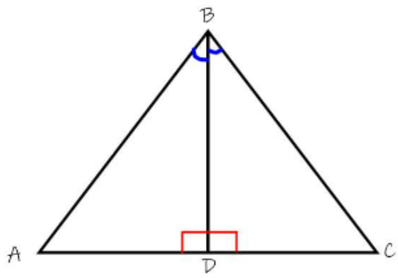
8. Example:



From the diagram we see: $\angle U \cong \angle N$ and $\angle A \cong \angle T$ and the included sides $\overline{UA} \cong \overline{NT}$

Therefore... $\triangle UGA \cong \triangle NVT$ by ASA

9. . **Lets Take a Look:**



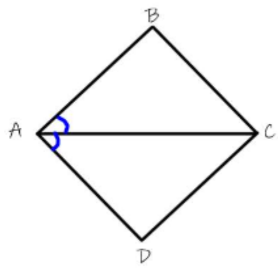
From the diagram, we know that

$\angle ABD \cong \angle$ ______ and $\angle ADB \cong$ ______

But that's only two angles. We the included sides to be congruent for ASA.

Remember, we **MUST** have a property or justification to add anything to our diagram...Do you see anything we are allowed to mark?

Since ______ by the ______ Therefore $\triangle ABD \cong \triangle$ ______ by ______



10. Another Property we might see dealing with angles:

Given: $\overline{AC}$ is an **angle bisector** for $\angle BCD$.

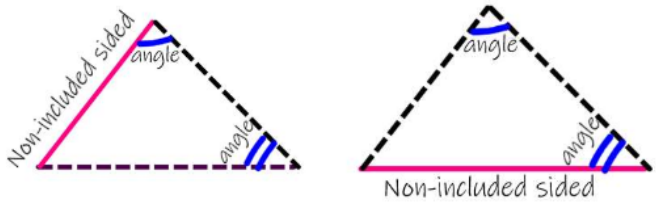
Can you prove the two triangles are congruent? We know....

$\angle BAC \cong \angle$ ______ from the diagram, and ______ by the reflexive Property.

Define Angle Bisector: ______ Therefore, $\angle BCA \cong \angle$ ______ and that means that $\triangle ABC \cong \triangle$ ______ by ______

Vocabulary for our next postulate:

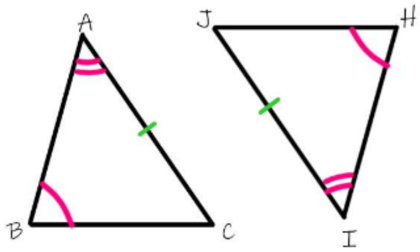
A side length that is not directly in between to angles is called a **non-included** side.



Theorem #4

Angle Angle Side(AAS): If two angles and a non-included side of one triangle are congruent the two angles and the corresponding non-included side of another triangle, the two triangles are congruent.

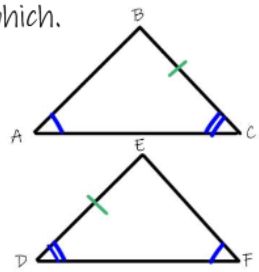
Example:



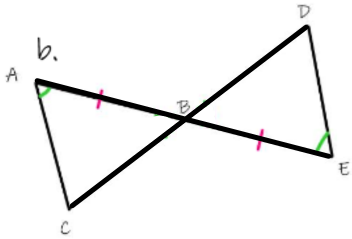
11. From the diagram we see: $\angle A \cong \angle I$ and $\angle B \cong \angle H$ and the corresponding **non-included** sides $\overline{AC} \cong \overline{IJ}$ Therefore... $\triangle ABC \cong \triangle IHJ$ by AAS

12. Two of the examples below are examples of AAS, one is an example of ASA. Decide which is which.

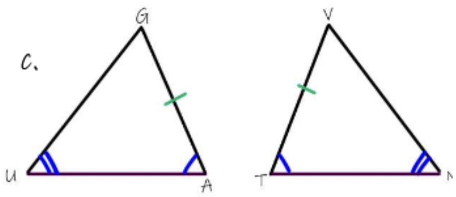
a.



b.

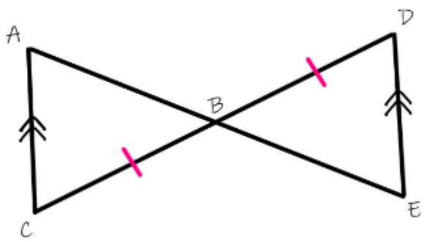


c.



$\triangle ABC \cong$ ____ by ____ $\triangle ABC \cong$ ____ by ____ $\triangle UGA \cong$ ____ by ____

13. Another property we may see with angles...



Given: $\overline{AC} \parallel \overline{DE}$, prove the two triangles congruent.

We know.... $\overline{CB} \cong \overline{DB}$ from the diagram, and ____ $\cong$ ____ because they are vertical angles.

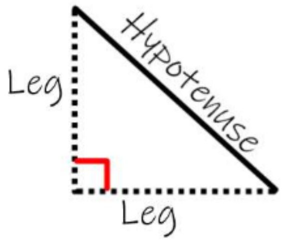
Since $\overline{AC} \parallel \overline{DE}$, what kind of angles are $\angle A$ and $\angle E$?

____

Therefore, $\angle A \cong \angle$ ____ and that means that $\triangle ABC \cong \triangle$ ____ by ____ $\angle C$ and $\angle D$ are also ____ so there is more than one correct way to do this one. 😊

Recall: Right Triangles

In a Right triangle, the side lengths that form the right angle are called the ______ of the triangle, and the side opposite the right angle is called the ______



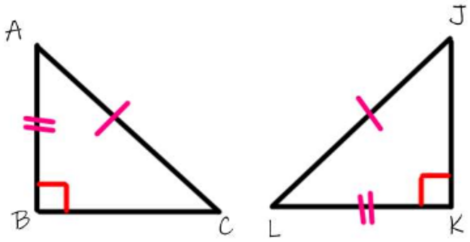
- Right triangles have many special properties! We have a triangle congruence theorem that works ONLY for right triangles!
- All our other postulates and theorems work for right triangles too! Right triangles just have an extra on that is special just for them.

Hypotenuse Leg(HL): If the hypotenuse and one leg in a right triangle are congruent to the hypotenuse and one leg of another right triangle, the two triangles are congruent.

Example:

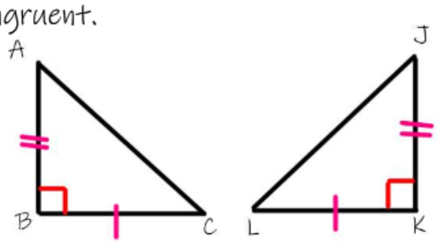
14. From the diagram we see: $\angle B$ and $\angle K$ are both right angles, making these right triangles. $\overline{AB} \cong \overline{LK}$. These are ______ of the right triangles. $\overline{AC} \cong \overline{LK}$ These segments are the ______ of the right triangles.

Therefore... $\triangle ABC \cong \triangle LKJ$ by HL



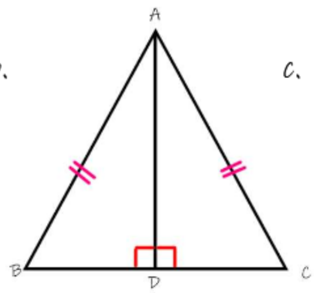
15. **You Try!** Determine which postulate or theorem you can use to prove the triangles congruent.

a.



______

b.



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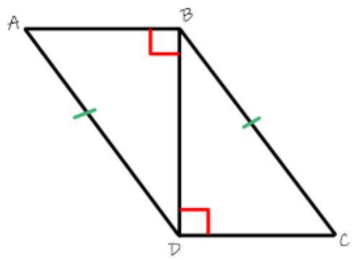
c.

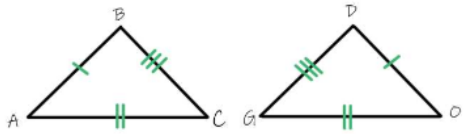
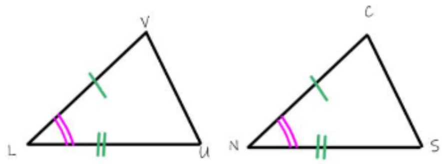
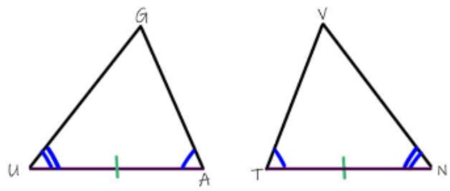
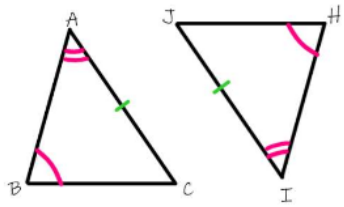
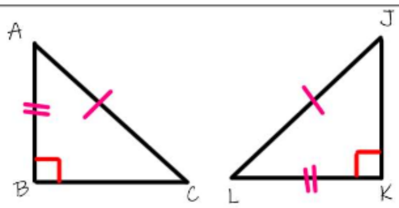


______

d. **True or False:** HL is the only method to prove that two right triangles are congruent.

______

Congruent Triangles Summary

Side Side Side (SSS):	 $\triangle ABC \cong \triangle ODG$
Side Angle Side (SAS):	 $\triangle LVU \cong \triangle NCS$
Angle Side Angle (ASA) -	 $\triangle UGA \cong \triangle NVT$
Angle Angle Side(AAS):	 $\triangle ABC \cong \triangle IHJ$
Hypotenuse Leg(HL):	 $\triangle ABC \cong \triangle LKJ$