Name: _____

Ellipses

In Exercises 1–6, match the equation with its graph.
[The graphs are labeled (a), (b), (c), (d), (e), and (f).]

(a)

(b)

(c)

(d)

(e)

(f)
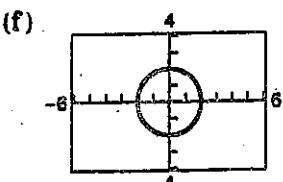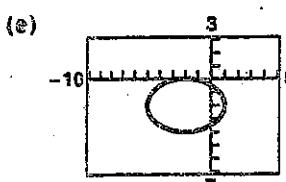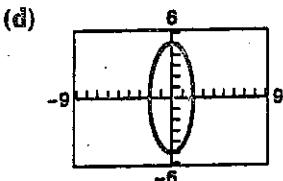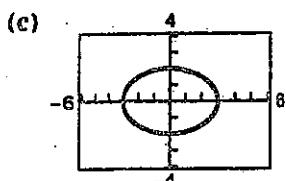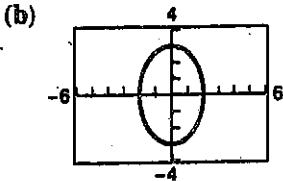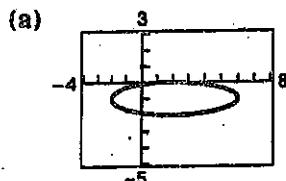
1. $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$

2. $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

3. $\dfrac{x^2}{4}+\dfrac{y^2}{25}=1$

4. $\dfrac{y^2}{4}+\dfrac{x^2}{4}=1$

5. $\dfrac{(x-2)^2}{16}+(y+1)^2=1$

6. $\dfrac{(x+2)^2}{9}+\dfrac{(y+2)^2}{4}=1$

find the center, vertices, foci of the ellipse, and sketch its graph.

7. $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$

9. $\dfrac{x^2}{5}+\dfrac{y^2}{9}=1$

11. $\dfrac{(x+3)^2}{16}+\dfrac{(y-5)^2}{25}=1$

12. $\dfrac{(x-4)^2}{12}+\dfrac{(y+3)^2}{16}=1$

13. $\dfrac{(x+5)^2}{9/4}+(y-1)^2=1$

14. $(x+2)^2+\dfrac{(y+4)^2}{1/4}=1$

15. $9x^2+4y^2+36x-24y+36=0$

16. $9x^2+4y^2-54x+40y+37=0$

17. $x^2+5y^2-8x-30y-39=0$

find the standard form of the equation of the ellipse with its center at the origin.

27.
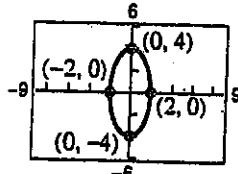
28.
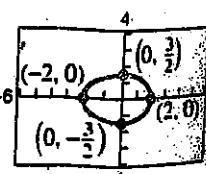
29. Vertices: $(\pm 6, 0)$; Foci: $(\pm 2, 0)$

30. Vertices: $(0, \pm 8)$; Foci: $(0, \pm 4)$

find the standard form of the equation of the specified ellipse.

35.
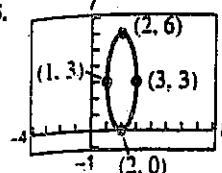
36.
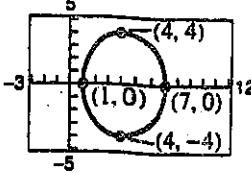
37.
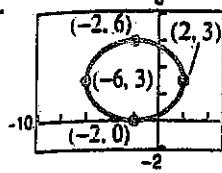
38.
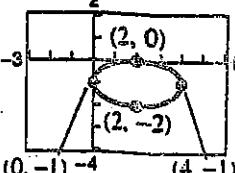
39. Vertices: $(0, 4)$, $(4, 4)$; Minor axis of length 2

40. Foci: $(0, 0)$, $(4, 0)$; Major axis of length 8

41. Foci: $(0, 0)$, $(0, 8)$; Major axis of length 16

42. Center: $(2, -1)$; Vertex: $\left(2, \tfrac{1}{2}\right)$; Minor axis of length 2

43. Vertices: $(3, 1)$, $(3, 9)$; Minor axis of length 6

49. *Fireplace Arch* A fireplace arch is to be built in the shape of a semiellipse. The opening is to have a height of 2 feet at the center and a width of 6 feet along the base. The contractor draws the outline of the ellipse using the method described on page 705. Give the required positions of the tacks and the length of the string.
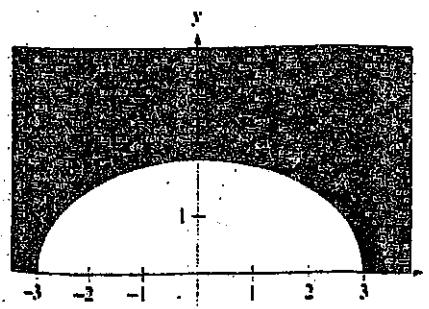
Hyperbolas

In Exercises 1–4, match the equation with its graph.
[The graphs are labeled (a), (b), (c), and (d).]

(a)

(b)

(c)

(d)
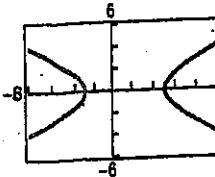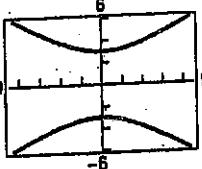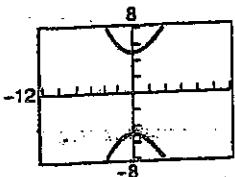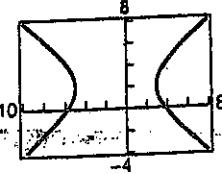
1. $\dfrac{y^2}{9} - \dfrac{x^2}{25} = 1$

2. $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1$

3. $\dfrac{(x-1)^2}{16} - \dfrac{y^2}{4} = 1$

4. $\dfrac{(x+1)^2}{16} - \dfrac{(y-2)^2}{9} = 1$

find the center, vertices, foci, and asymptotes of the hyperbola, and sketch its graph, using the asymptotes as an aid.

6. $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

8. $\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

10. $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

11. $\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{1} = 1$

12. $\dfrac{(x+3)^2}{144} - \dfrac{(y-2)^2}{25} = 1$

13. $(y+6)^2 - (x-2)^2 = 1$

14. $\dfrac{(y-1)^2}{1/4} - \dfrac{(x+3)^2}{1/16} = 1$

5. $9x^2 - y^2 - 36x - 6y + 18 = 0$

6. $x^2 - 9y^2 + 36y - 72 = 0$

7. $x^2 - 9y^2 + 2x - 54y - 80 = 0$

find the standard form of the equation of the specified hyperbola.

23. Vertices: $(0, \pm 2)$; Foci: $(0, \pm 4)$

26. Vertices: $(0, \pm 3)$; Asymptotes: $y = \pm 3x$

27. Foci: $(0, \pm 8)$; Asymptotes: $y = \pm 4x$

31. Vertices: $(4, 1)$, $(4, 9)$; Foci: $(4, 0)$, $(4, 10)$

32. Vertices: $(-2, 1)$, $(2, 1)$; Foci: $(-3, 1)$, $(3, 1)$

33. Vertices: $(2, 3)$, $(2, -3)$;
    Passes through the point $(0, 5)$

34. Vertices: $(-2, 1)$, $(2, 1)$;
    Passes through the point $(5, 4)$

35. Vertices: $(0, 4)$, $(0, 0)$;

43. *Hyperbolic Mirror* A hyperbolic mirror (used in some telescopes) has the property that a light ray directed at a focus will be reflected to the other focus. The focus of a hyperbolic mirror has coordinates $(24, 0)$. Find the vertex of the mirror if its mount has coordinates $(24, 24)$.
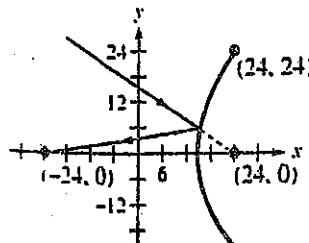
In Exercises 44–51, classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

44. $x^2 + y^2 - 6x + 4y + 9 = 0$

45. $x^2 + 4y^2 - 6x + 16y + 21 = 0$

46. $4x^2 - y^2 - 4x - 3 = 0$

47. $y^2 - 4y - 4x = 0$

48. $4x^2 + 3y^2 + 8x - 24y + 51 = 0$

49. $4y^2 - 2x^2 - 4y - 8x - 15 = 0$

50. $25x^2 - 10x - 200y - 119 = 0$

51. $4x^2 + 4y^2 - 16y + 15 = 0$

52. $4x^2 - y^2 + 8x - 6y + 4 = 0$

53. $2x^2 + 2y^2 - 8x + 12y + 2 = 0$