Central High Booster Club

In order to raise money for the school, the Central High School Booster Club offered spirit items prepared by members for sale at the school store and at games. They sold stuffed teddy bears dressed in school colors, tote bags and tee shirts with specially sewn and decorated school insignias. The teddy bears, tote bags, and tee shirts were purchased from wholesale suppliers and decorations were cut, sewn and painted, and attached to the items by booster club parents. The wholesale cost for each teddy bear was $6.00, each tote bag was $4.50 and each tee shirt was $5.25. Materials for the decorations cost $2.25 for the bears, $1.90 for the tote bags and $1.55 for the tee shirts. Parents estimated the time necessary to complete a bear was 15 minutes to cut out the clothes, 20 minutes to sew the outfits, and 5 minutes to dress the bears. A tote bag required 10 minutes to cut the materials, 15 minutes to sew and 10 minutes to glue the designs on the bag. Tee shirts were made using computer generated transfer designs for each sport which took 5 minutes to print out, 6 minutes to iron on the shirts, and 20 minutes to paint on extra detailing.

The booster club parents made spirit items at three different work meetings and produced 30 bears, 30 tote bags, and 45 tee shirts at the first session. Fifteen bears, 25 tote bags, and 30 tee shirts were made during the second meeting; and, 30 bears, 35 tote bags and 75 tee shirts were made at the third session. They sold the bears for $15.00 each, the tote bags for $12.00 each and the tee shirts for $13.00 each. In the first month of school, 11 bears, 16 tote bags, and 60 tee shirts were sold at the bookstore. During the same time period, Booster Club members sold 50 bears, 20 tote bags, and 100 tee shirts at the games.

The following is a **matrix**, a rectangular array of values, showing the wholesale cost of each item as well as the cost of decorations. "*wholesale*" and "*decorations*" are labels for the matrix **rows** and "*bears*", "*totes*", and "*shirts*" are labels for the matrix **columns**. The **dimensions of this matrix** called **A** are 2 rows and 3 columns and matrix **A** is referred to as a **[2 x 3] matrix**. Each number in the matrix is called an **entry**.

$$\begin{array}{c} \textit{Cost per Item} \\ A = \begin{array}{r} \\ \textit{wholesale} \\ \textit{decorations} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{bears} & \textit{totes} & \textit{shirts} \end{array} \\ \begin{bmatrix} 6.00 & 4.50 & 5.25 \\ 2.25 & 1.90 & 1.55 \end{bmatrix} \end{array} \end{array}$$

It is sometimes convenient to write matrices (plural of matrix) in a simplified format without labels for the rows and columns. Matrix A can be written as an array.

$A = \begin{bmatrix} 6.00 & 4.50 & 5.25 \\ 2.25 & 1.90 & 1.55 \end{bmatrix}$ where the values can be identified as $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$.

In this system, the entry $\boldsymbol{a_{22}}$ = 1.90, which is the cost of decorations for tote bags.

1. Write and label matrices for the information given on the Central High School Booster Club's spirit project.

a. Let matrix B show the information given on the time necessary to complete each task for each item. Label the **rows** of the matrix *cut/print*, *sew/iron*, and *dress/decorate*. Label the **columns** *bears, totes, shirts.*

b. Find matrix C to show the numbers of bears, totes, and shirts produced at each of the three meetings. Label the **rows** of the matrix *1st* , *2nd* , and *3rd* . Label the **columns** *bears, totes, shirts*

c. Matrix D should contain the information on items sold at the bookstore and at the game. Label the **rows** of the matrix *bears, totes, shirts*. Label the **columns** *bookstore, games*

d. Let matrix E show the selling prices of the three items. Label the **row** of the matrix *selling price*. Label the **columns** *bears, totes, shirts.*

2. Matrices are called **square matrices** when the number of rows equals the number of columns. A matrix with only one row or only one column is called a **row matrix** or a **column matrix**. Are any of the matrices from problem #1 square matrices or row matrices or column matrices? If so, identify them and state their dimensions.

Since matrices are arrays containing sets of discrete data with dimensions, they have a particular set of rules, or algebra, governing operations such as addition, subtraction, and multiplication. In order to **add two matrices**, the matrices must have the same dimensions. And, if the matrices have row and column labels, these labels must also match. Consider the following problem and matrices.

Several local companies wish to donate spirit items which can be sold along with the items made by the Booster Club at games help raise money for Central High School. J J's Sporting Goods store donates 110 caps and 110 pennants in September and 125 caps and 75 pennants in October. Friendly Fred's Food store donates 105 caps and 125 pennants in September and 110 caps and 100 pennants in October. How many items are available each month from both sources?

To add two matrices, add corresponding entries.

Let $J = \begin{array}{r} \\ caps \\ pennants \end{array} \begin{array}{c} \begin{array}{cc} Sept & Oct \end{array} \\ \begin{bmatrix} 110 & 125 \\ 110 & 75 \end{bmatrix} \end{array}$ and $F = \begin{array}{r} \\ caps \\ pennants \end{array} \begin{array}{c} \begin{array}{cc} Sept & Oct \end{array} \\ \begin{bmatrix} 105 & 110 \\ 125 & 100 \end{bmatrix} \end{array}$

then $J + F = \begin{array}{r} \\ caps \\ pennants \end{array} \begin{array}{c} \begin{array}{cc} Sept & Oct \end{array} \\ \begin{bmatrix} 110+105 & 125+110 \\ 110+125 & 75+100 \end{bmatrix} \end{array}$ so

$$J + F = \begin{array}{r} \\ caps \\ pennants \end{array} \begin{array}{c} \begin{array}{cc} Sept & Oct \end{array} \\ \begin{bmatrix} 215 & 235 \\ 235 & 175 \end{bmatrix} \end{array}$$

Subtraction is handled like addition by subtracting corresponding entries.

3. Construct a matrix G with dimensions [1 x 3] corresponding to the total production **cost** per item for *bears, totes, shirts*. Use this new matrix G and matrix E from #1 (which corresponded to the **selling price** for each item) to find matrix P, the **profit** the Booster Club can expect from the sale of each bear, tote bag, and tee shirt.

Another type of matrix operation is known as scalar multiplication. A **scalar** is a single number such as 3 and matrix **scalar multiplication** is done by multiplying each entry in a matrix by the same scalar.

Let $M = \begin{bmatrix} -2 & 0 & 5 \\ 1 & -3 & 4 \end{bmatrix}$, then $3M = \begin{bmatrix} -6 & 0 & 15 \\ 3 & -9 & 12 \end{bmatrix}$.

4. Use scalar multiplication to change matrix B (problem #1) from minutes required per item to hours required per item.

Matrices can also be multiplied together. Since each matrix represents an array of data, rules for multiplying them together depend on the position of each entry. Consider the following example.

At the beginning of November a stomach virus hits Central High School. Students in the Freshman and Sophomore classes are either well, a little sick, or really sick. The following tables show Freshmen and Sophomores according to their levels of sickness and their gender.

Student Population

Categories	Male	Female
Freshmen	250	300
Sophomores	200	275

% of Sick Students

Categories	Freshmen	Sophomores
Well	20%	25%
Little Sick	50%	40%
Really Sick	30%	35%

Suppose school personnel needed to prepare a report and include the total numbers of well and sick male Freshmen and Sophomores in the school.

well Freshmen males + well Sophomore males = well males
(.2)(250) + (.25)(200) = 100

a little sick Freshmen males + a little sick Sophomore males = a little sick males
(.5)(250) + (.4)(200) = 205

really sick Freshmen males + really sick Sophomore males = really sick males
(.3)(250) + (.35)(200) = 145

Notice the positions of the values in these products. We are multiplying rows by columns to get the information we want. Translating the tables to matrices and using the **rows by columns pattern of multiplication** we get the following result

$$\begin{matrix} & F & S \\ W & .2 & .25 \\ L & .5 & .4 \\ R & .3 & .35 \end{matrix} * \begin{matrix} & M & F \\ F & 250 & 300 \\ S & 200 & 275 \end{matrix} = \begin{bmatrix} (.2)(250)+(.25)(200) & (.2)(300)+(.25)(275) \\ (.5)(250)+(.4)(200) & (.5)(300)+(.4)(275) \\ (.3)(250)+(.35)(200) & (.3)(300)+(.35)(275) \end{bmatrix} = \begin{matrix} & M & F \\ W & 100 & 128.75 \\ L & 205 & 260 \\ R & 145 & 186.25 \end{matrix}$$

$$\begin{matrix} & F & S \\ W & .2 & .25 \\ L & .5 & .4 \\ R & .3 & .35 \end{matrix} * \begin{matrix} & M & F \\ F & 250 & 300 \\ S & 200 & 275 \end{matrix} = \begin{matrix} & M & F \\ W & 100 & 128.75 \\ L & 205 & 260 \\ R & 145 & 186.25 \end{matrix}$$

[level of sickness x class] * [class x gender] = [level of sickness x gender]

[3 x 2] * [2 x 2] = [3 x 2]

This procedure illustrates the multiplication of two matrices. In order to multiply two matrices, the number of columns of the matrix on the left must equal the number of rows of the matrix on the right. Also the labels of the columns of the left matrix must be the same as the labels of the rows of the right matrix. **If the dimensions of two matrices are not appropriately matched, it is not possible to multiply them.**

5. Given the following matrices, find their products if possible.

$$L = \begin{bmatrix} 1 & 3 \\ -5 & 4 \end{bmatrix} \quad M = \begin{bmatrix} -1 & 2 & 7 & -1 \\ -5 & 7 & 3 & 5 \end{bmatrix} \quad N = \begin{bmatrix} 3 & 6 \\ -2 & 1 \\ 0 & 5 \\ -1 & 2 \\ 5 & 3 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S = \begin{matrix} & male & female \\ well & 55\% & 68\% \\ sick & 45\% & 32\% \end{matrix} \qquad C = \begin{matrix} & male & female \\ Jr & 150 & 210 \\ Sr & 100 & 50 \end{matrix}$$

a. LM

b. LN

c. LT

d. MN

e. SC [Sometimes it is necessary to exchange the rows and columns of a matrix in order to make it possible to multiply. This process is called finding the **transpose** of a matrix and is most useful with labeled matrices.]

f. Interpret $SC^{T}{}_{1,2}$

6. Using the matrices you wrote in problems #1 and #3 and matrix multiplication, find matrices to show

   a. the amount of profit made at the bookstore and at the games

   b. the amount of time (in minutes) it took to perform each task at the three work sessions.