

Warm-up

April 19, 2017

Solve.

$$4 \tan x + \sin 2x = 0$$

Rewrite

$$\cancel{2} \frac{4}{\cancel{2}} \left( \frac{\sin x}{\cos x} \right) + \cancel{2} \frac{\sin x \cos x}{\cancel{2}} = 0 \quad \text{GCF}$$

$$2 \sin x \left( \frac{2}{\cos x} + \frac{\cos x}{1} \right) = 0$$

$$\cancel{2} \frac{\sin x}{\cancel{2}} = 0$$

$$\frac{2}{\cos x} + \frac{\cos x}{1} = 0$$

$$\cancel{(\sin x)(\cos x)} = 0$$

~~$\sin x$~~   ~~$\cos x$~~

$$\frac{2}{\cos x} - \frac{\cos x}{1} = 0$$

$$\frac{2}{\cos x} = \frac{-\cos x}{1}$$

$$X = 0^\circ, 180^\circ$$

$$\frac{2}{-1} = \frac{-\cos^2 x}{1}$$

$$\sqrt{-2} = \sqrt{\cos^2 x}$$

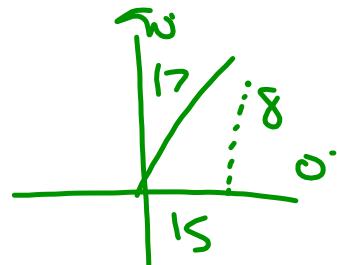
nope. no solution

- 0      1      2      3      4      *you got it!*
- I tried *struggling*.      Almost there!
- I did not try.
- Solving:
- 1. Combine like terms
  - 2. Square root both sides
  - 3. Factor
  - 4. Replace
  - 5. Square both

$$\textcircled{2} \cos \theta = \frac{15}{17}, 0^\circ < \theta < 90^\circ$$

$$\text{Find } \sin 2\theta. = 2 \sin \theta \cos \theta$$

$$\begin{aligned} &= 2 \left( \frac{4}{17} \right) \left( \frac{15}{17} \right) \\ &= \frac{240}{289} \end{aligned}$$



$$a^2 + b^2 = c^2$$

$$15^2 + b^2 = 17^2$$

$$225 + b^2 = 289$$

$$\underline{\underline{-225 -225}} \quad b^2 = 64$$

$$b = 8$$

$$\textcircled{13} \quad \csc \theta = \frac{17}{4}, \quad 0^\circ < \theta < 90^\circ$$

$$\text{Find } \tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\begin{aligned}
 & \left| \begin{array}{l} 17 \\ 8 \\ \hline 15 \end{array} \right| = \sqrt{1 - \frac{15}{17}} = \frac{2}{17} \\
 & a^2 + b^2 = c^2 \\
 & a^2 + 8^2 = 17^2 \\
 & a^2 + 64 = 289 \\
 & a^2 = 225 \\
 & a = 15
 \end{aligned}$$

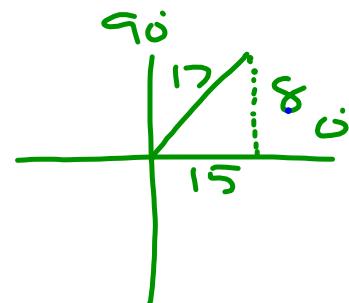
$$\begin{aligned}
 & \left| \begin{array}{l} 17 \\ 8 \\ \hline 15 \end{array} \right| = \sqrt{1 + \frac{15}{17}} = \frac{32}{17} \\
 & = \sqrt{\frac{1}{16}} = \frac{1}{4}
 \end{aligned}$$

$$\textcircled{2} \quad \cos\theta = \frac{15}{17} \text{ A}, \quad 0^\circ < \theta < 90^\circ$$

$$\text{Find } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left( \frac{8}{17} \right) \left( \frac{15}{17} \right)$$

$$= \frac{240}{289}$$



$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 15^2 + b^2 &= 17^2 \\
 225 + b^2 &= 289 \\
 -225 &\quad -225 \\
 b^2 &= 64 \\
 b &= 8
 \end{aligned}$$

$$\textcircled{6} \quad \cos \theta = \frac{4\sqrt{19}}{19} \text{ A} \quad \text{H}, \quad \frac{3\pi}{2} < \theta < 2\pi$$

$$\text{Find } \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \frac{270^\circ}{2} < \frac{\theta}{2} < \frac{360^\circ}{2}$$

$$\text{+} \quad \sqrt{\frac{1 - \frac{4\sqrt{19}}{19}}{2}}$$

$$\begin{array}{c} \theta \\ \frac{\theta}{2} \\ \text{T}^+ \end{array} \quad \begin{array}{c} S^+ \\ A^+ \\ C^+ \end{array}$$

$$135^\circ < \frac{\theta}{2} < 180^\circ \quad \begin{array}{c} 4\sqrt{19} \\ | \\ \sqrt{57} \end{array} \quad 360^\circ$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (4\sqrt{19})^2 + b^2 &= 19^2 \\ 304 + b^2 &= 361 \\ b^2 &= 57 \end{aligned}$$

$$b = \sqrt{57}$$

$$\textcircled{1} \quad \cos\theta = -\frac{4}{5} \text{ A H}, \quad \pi < \theta < \frac{3\pi}{2}$$

$$\text{Find } \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$$

$$\frac{16}{25} - \frac{9}{25}$$

$$\frac{7}{25}$$

$$\begin{array}{c} -4 \\ \hline -3 \end{array} \quad \begin{array}{c} \cancel{\sqrt{5}} \\ \cancel{\sqrt{5}} \end{array}$$

$\pi$        $\frac{3\pi}{2}$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (-4)^2 + b^2 &= 5^2 \\ 16 + b^2 &= 25 \\ \cancel{16} &\quad \cancel{+b^2} \\ 5^2 &= 9 \\ b &= 3 \end{aligned}$$

$$\textcircled{3} \quad \cos \theta = \frac{24}{25} \text{ A } \frac{1}{2} \cdot \frac{3\pi}{2} < \frac{\theta}{2} < \frac{2\pi}{2}$$

$$\text{Find } \cos \frac{\theta}{2}. = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

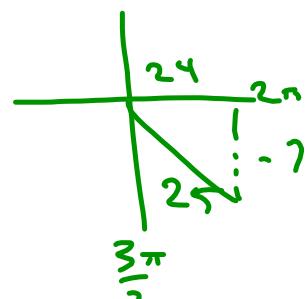
II

$$= \pm \sqrt{\frac{1 + \frac{24}{25}}{2}}$$

$$= \pm \sqrt{\frac{49}{50}}$$

$$\in \frac{7}{5\sqrt{2}} = \pm \frac{7\sqrt{2}}{10}$$

G



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 24^2 + b^2 &= 25^2 \\ 576 + b^2 &= 625 \\ -576 & \\ b^2 &= 49 \end{aligned}$$

$$b = 7$$

**Proof**

$$\begin{aligned}
 \sin x + \cos x &= \frac{\cot x + 1}{\csc x} \\
 &= (\cot x + 1) \cdot \csc x \\
 &= (\cot x + 1) \cdot \frac{1}{\csc x} \\
 &= (\cot x + 1) \cdot \sin x \\
 &= \cot x \sin x + \sin x \\
 &= \frac{\cos x}{\sin x} \sin x + \sin x \\
 &= \cos x + \sin x
 \end{aligned}$$

**Reason**

Given

Q. Dividing

multiply by reciprocal

rewrite (reciprocal ID)

Distribute

rewrite (quotient ID)

Simplify

Proof	Reason
$\begin{aligned} \sin x + \cos x &= \frac{\cot x + 1}{\csc x} \\ &= (\cot x + 1) \div \csc x \\ &= (\cot x + 1) \cdot \frac{1}{\csc x} \\ &= (\cot x + 1) \cdot \sin x \\ &= \cot x \sin x + \sin x \\ &= \frac{\cos x}{\sin x} \cancel{\sin x} + \sin x \\ &= \cos x + \sin x \end{aligned}$ 	<p>Given</p> <p>Divide</p> <p>multiply by reciprocal</p> <p>rewrite (reciprocal)</p> <p>Distribute</p> <p>rewrite (quotient I D)</p> <p>simplify</p>

2. Proof

$$\frac{1}{\sec x - \tan x} = \sec x + \tan x$$

$$\frac{1}{(\sec x - \tan x)} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} =$$

$$\frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} =$$

$$\frac{\sec x + \tan x}{1 + \tan^2 x - \tan^2 x} =$$

$$\sec x + \tan x =$$

Reason

Given

L.

multiply by conjugate  
multiply

Pythagorean ID.

Simplify

## 2. Proof

$$\frac{1}{\sec x - \tan x} = \sec x + \tan x$$

$$\frac{1}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} =$$

$$\frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} =$$

$$\frac{\sec x + \tan x}{1 + \tan^2 x - \tan^2 x} =$$

$$\sec x + \tan x =$$

## Reason

Given

multiply by conjugate

multiply

Pythagorean ID

Simplify

$$\textcircled{1} \quad \tan^2 x - 1 = 0$$

Rewrite

$$\frac{2\tan x}{1-\tan^2 x} - 1 = 0$$

$$(1-\tan^2 x) \frac{2+\tan x}{1-\tan^2 x} = 1(1-\tan^2 x) \quad \begin{matrix} \text{cross multiply} \\ 2+\text{ring}, 3+\text{arrow} \end{matrix}$$

$$\begin{matrix} 2\tan x = 1-\tan^2 x \\ -1+\tan^2 x \end{matrix} \quad \begin{matrix} -1-\tan^2 x \\ \hline \end{matrix}$$

$$\tan^2 x + 2\tan x - 1 = 0$$

AC method

$$x^2 + 2x - 1$$

a · c

- 1

2

b

A hand-drawn diagram showing a red parabola opening upwards. The equation  $x^2 + 2x - 1$  is written above it. The vertex of the parabola is labeled '2'. The left side of the parabola is labeled 'a · c' with a small arrow pointing to the curve, and the right side is labeled 'b'.

$$\textcircled{2} \quad \sin^2 x - 2\sin x - 3 = 0 \quad \begin{matrix} \text{2 trig} \\ \text{3 terms} \end{matrix}$$

$$(\sin x - 3)(\sin x + 1) = 0 \quad \text{AC}$$

$$\begin{array}{r} \sin x - 3 = 0 \\ +3 +3 \\ \hline (\sin x) = 3 \end{array}$$

~~$\sin$~~   ~~$\sin$~~

$x =$   
no solution

$$\begin{array}{r} \sin x + 1 = 0 \\ -1 -1 \\ \hline (\sin x) = -1 \end{array}$$

~~$\sin$~~   ~~$\sin$~~

$$x = 270^\circ$$

$$\begin{array}{c} a \cdot c \\ -3 \\ \times \quad \quad \quad 1 \\ -3 \\ \quad \quad \quad -2 \\ \hline b \end{array}$$

( $x - 3$ ) ( $x + \frac{1}{3}$ )

Divide by  $x - 3$

You  
can do  $2,4 \rightarrow 6-14$

(2)

$$\sin^2 x - 2\sin x - 3 = 0$$

<sup>2 trig  
3 turns</sup>

AC

$$(\sin x - 3)(\sin x + 1) = 0$$

$$\begin{array}{r} \sin x - 3 = 0 \\ +3 +3 \\ \hline \end{array}$$

$$\cancel{\sin x} = 3$$

$$\begin{array}{r} x \\ \cancel{\sin^{-1}} \quad \cancel{\sin^{-1}} \\ x = \end{array}$$

$$\begin{array}{r} \sin x + 1 = 0 \\ -1 -1 \\ \hline \end{array}$$

$$\cancel{\sin x} = -1$$

$$x = 270^\circ$$

$$1x^2 - 2x - 3 = 0$$

$\begin{array}{r} a : 1 \\ b : -3 \\ c : -3 \end{array}$

-3      -2      1

↓

*Divide by a*

$$(x - 3)(x + 1)$$

(4)

1 trig

$$\cos x = 3 \cos x + 1$$

$$\begin{array}{r} -\cos x \\ -\cos x \\ \hline \end{array}$$

$$2 \cos x + 1 = 0$$

$$\begin{array}{r} -1 \\ -1 \\ \hline \end{array}$$

$$\frac{2 \cos x}{2} = \frac{-1}{2}$$

$$\cancel{\cos^{-1}}(\cos x) = \left( \frac{-1}{2} \right)$$

$$\cos$$

$$X = 120^\circ, 240^\circ$$

(12)

$$\sqrt{3} \cot x \sin x + 2 \cos^2 x = 0$$

$$\sqrt{3} \left( \frac{\cos x}{\sin x} \right) \left( \frac{\sin x}{1} \right) + 2 \cos^2 x = 0$$

$$\frac{\sqrt{3} \cos x}{\cos x} + \frac{2 \cos^2 x}{\cos x} = 0 \quad \text{GCF}$$

$$\cos x (\sqrt{3} + 2 \cos x) = 0$$

$$\begin{cases} \cos x = 0 \\ \sqrt{3} + 2 \cos x = 0 \end{cases}$$

$$\frac{\sqrt{3} + 2 \cos x = 0}{-\sqrt{3}}$$

$$x = 90^\circ, 270^\circ$$

$$\frac{2 \cos x}{2} = \frac{-\sqrt{3}}{2}$$

$$\cos x = \frac{-\sqrt{3}}{2}$$

$$x = 150^\circ, 210^\circ$$

$$\textcircled{2} \quad \sin^2 x - 2\sin x - 3 = 0 \quad \begin{matrix} \text{2 trig} \\ \text{3 terms} \end{matrix}$$

$$(\sin x - 3)(\sin x + 1) = 0 \quad \text{AC}$$

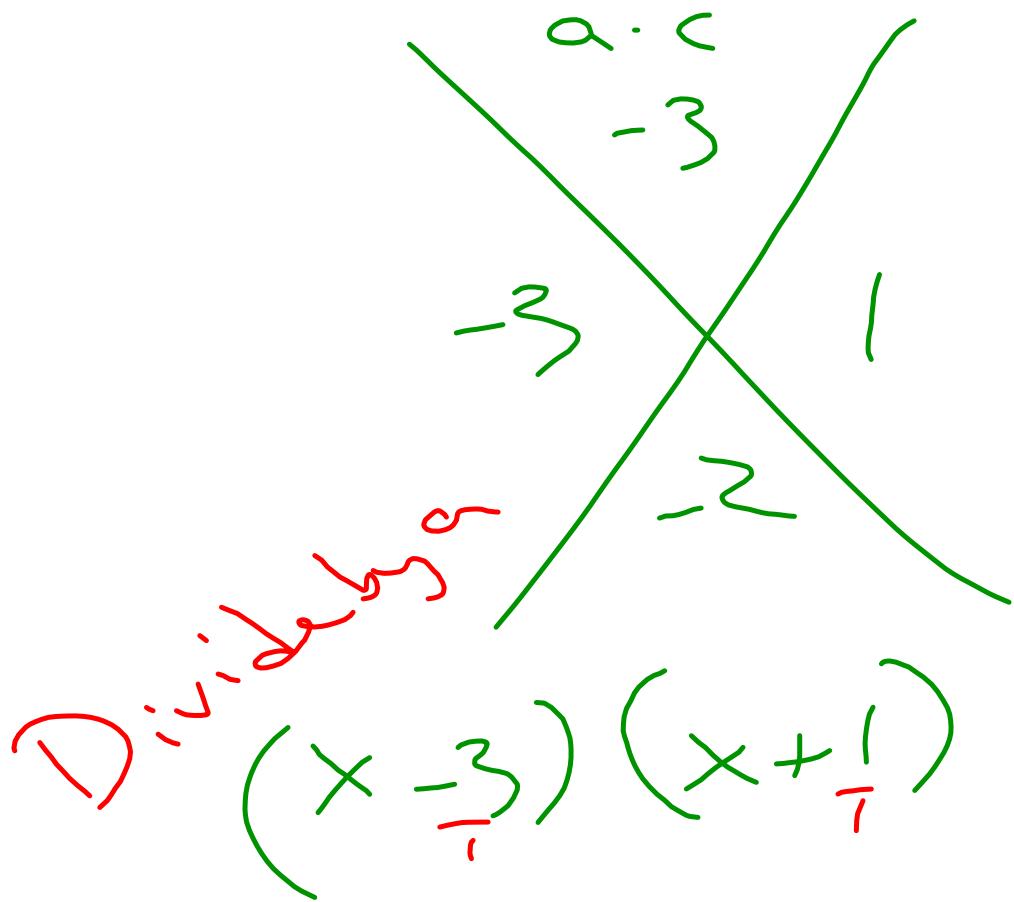
$$\begin{array}{rcl} \sin x - 3 & = & 0 \\ +3 +3 & & \\ \hline (\sin x) & = & 3 \\ \cancel{\sin} & & \cancel{\sin} \end{array}$$

$$\begin{array}{rcl} \sin x + 1 & = & 0 \\ -1 -1 & & \\ \hline \sin x & = & -1 \\ \cancel{\sin} & & \cancel{\sin} \end{array}$$

$$x = \text{no solution}$$

$$x = 270^\circ$$

$$1x^2 - 2x - 3 = 0$$



⑩  $3\sin^2 x - \cos^2 x = 0$

$3(1 - \cos^2 x) - \cos^2 x = 0$

$$\cos \theta = -\frac{4}{5} \quad \Rightarrow \quad \begin{matrix} \text{I} \\ \text{II} \end{matrix} \quad \theta < \frac{3\pi}{2}$$

Find  $\cos 2\theta$ .

$$\begin{aligned}\cos 2\theta &= 2\cos^2 \theta - 1 && \begin{matrix} -3 \\ -1 \end{matrix} : \cancel{\begin{array}{c} \sqrt{a^2 + b^2} \\ \sqrt{4^2 + 5^2} \\ = \sqrt{16 + 25} \\ = \sqrt{41} \\ b = \sqrt{41} \end{array}} \\ &= 2 \left( \frac{-4}{5} \right)^2 - 1 && \begin{matrix} a^2 + b^2 \\ 16 + 25 \\ = 41 \end{matrix} \\ &= 2 \left( \frac{16}{25} \right) - 1 && b = 3 \\ &= \frac{32}{25} - 1 \cdot \frac{25}{25} && b = 3\end{aligned}$$

$$\frac{32}{25} - \frac{25}{25}$$

$$\begin{aligned}
 (\sin x)^2 &= (\cos x)^2 \\
 \sin^2 x &= \cos^2 x \quad \text{Pyth. ID.} \\
 \sin^2 x + \cos^2 x &= 1 \\
 -\cos^2 x - \cos^2 x \\
 \hline
 \sin^2 x &= \underline{1 - \cos^2 x} \\
 1 - \cos^2 x &= \frac{\cos^2 x}{\cos^2 x + \cos^2 x} \quad \text{Like terms} \\
 + \cos^2 x & \\
 \hline
 1 &= 2 \cos^2 x
 \end{aligned}$$

$$\cos \theta = -\frac{4}{5} \quad \pi < \theta < \frac{3\pi}{2}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$2\left(-\frac{4}{5}\right)^2 - 1$$

$$\frac{2}{1} \left(\frac{16}{25}\right) - 1$$

$$\frac{32}{25} - \frac{1}{1} \cdot .25$$

$$\frac{32 - 25}{25} = \frac{7}{25}$$

$$\cos \theta = -\frac{4}{5} \quad \pi < \theta < \frac{3\pi}{2}$$

$$\cos 2\theta = \cos^2 \theta - \underline{\sin^2 \theta}$$

Circular diagram showing a point on the third quadrant of a unit circle.

$$= \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$\boxed{\frac{7}{25}}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (-4)^2 + b^2 &= 5^2 \\ 16 + b^2 &= 25 \\ b^2 &= 9 \\ b &= 3 \end{aligned}$$

