Warm-up

April 19, 2017

Solve.

$4\tan x + \sin 2x = 0$ Rewrite

$2\,4\left(\frac{\sin x}{\cos x}\right) + 2\sin x\cos x = 0$ GCF

$2\sin x\left(\frac{2}{\cos x} + \frac{\cos x}{1}\right) = 0$

$\frac{2\sin x}{2} = \frac{0}{2}$

$(\sin x)(0)$

$\sin^{-1}$ $\sin^{-1}$

$x = 0°, 180°$

$\frac{2}{\cos x} + \frac{\cos x}{1} = 0$

$-\frac{\cos x}{1} \quad -\frac{\cos x}{1}$

$\frac{2}{\cos x} = \frac{-\cos x}{1}$

$\frac{2}{-1} = \frac{-\cos^2 x}{-1}$

$\sqrt{-2} = \sqrt{\cos^2 x}$

nope. no solution

0 1 2 3 4

I did not try.

I tried...

Struggling.

Almost there!

You got it!

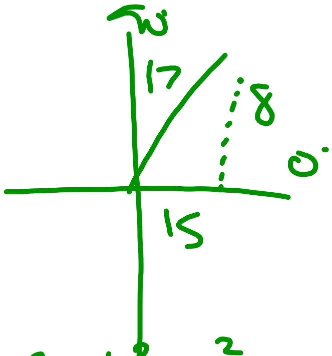
Solving:
1. Combine like terms
2. Square root both sides
3. Factor
4. Replace
5. Square both

② $\cos\theta = \frac{15}{17}\frac{A}{H}$, $0° < \theta < 90°$

Find $\sin 2\theta$. $= 2\sin\theta\cos\theta$

$= 2\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)$

$= \frac{240}{289}$



$a^2 + b^2 = c^2$

$15^2 + b^2 = 17^2$

$225 + b^2 = 289$

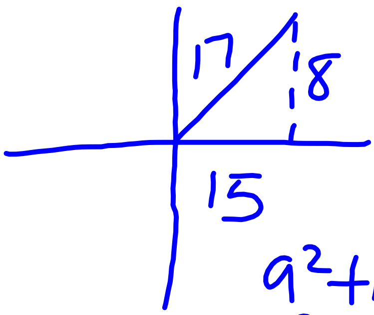
$-225 \qquad -225$

$b^2 = 64$

$b = 8$

(13) $\csc\theta = \frac{17}{8}$, $0° < \theta < 90°$

Find $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$



$$= \sqrt{\frac{1-\frac{15}{17}}{1+\frac{15}{17}}} = \sqrt{\frac{\frac{2}{17}}{\frac{32}{17}}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$a^2+b^2=c^2$

$a^2+8^2=17^2$

$a^2+64=289$

$-64 = -64$

$\sqrt{a^2}=\sqrt{225}$

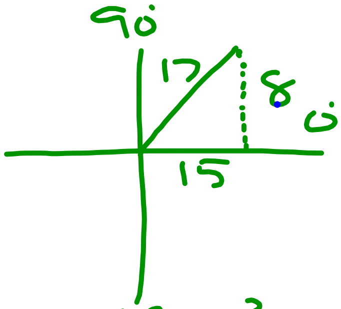
$a = 15$

(2) $\cos\theta = \frac{15}{17}\frac{A}{H}$, $0° < \theta < 90°$

Find $\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)$

$= \frac{240}{289}$



$a^2 + b^2 = c^2$

$15^2 + b^2 = 17^2$

$225 + b^2 = 289$

$-225 \quad -225$

$b^2 = 64$

$b = 8$

(6) $\cos\theta = \frac{4\sqrt{19}}{19}$ A H , $\cancel{\frac{3\pi}{2} < \theta < 2\pi}$

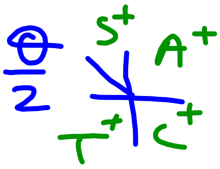
Find $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

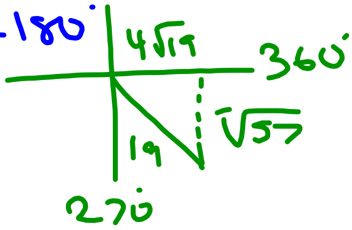
$\frac{270°}{2} < \frac{\theta}{2} < \frac{360°}{2}$

$135 < \frac{\theta}{2} < 180°$

$= + \sqrt{\frac{1-\frac{4\sqrt{19}}{19}}{2}}$

$\frac{\theta}{2}$





$a^2+b^2=c^2$

$(4\sqrt{19})^2+b^2=19^2$

$16 \cdot 19$

$304+b^2=361$

$-304 \quad -304$

$b^2=57$

$b=\sqrt{57}$

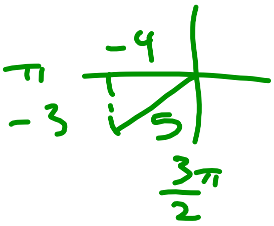
① $\cos\theta = \frac{-4}{5} \frac{A}{H}, \quad \pi < \theta < \frac{3\pi}{2}$

Find $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$$

$$\frac{16}{25} - \frac{9}{25}$$

$$\frac{7}{25}$$



$a^2+b^2=c^2$

$(-4)^2+b^2=5^2$

$16+b^2=25$

$-16 \quad -16$

$b^2=9$

$b=3$

③ $\cos\theta = \frac{24}{25}\frac{A}{H}$, $\frac{1}{2}\cdot\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

Find $\cos\frac{\theta}{2}$. $= \pm\sqrt{\frac{1+\cos\theta}{2}}$

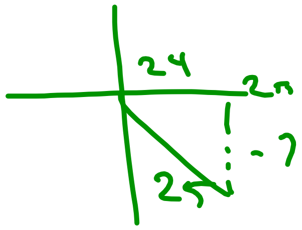
II

$$\boxed{= \pm\sqrt{\frac{\frac{1}{1}+\frac{24}{25}}{2}}}$$

$$\pm\sqrt{\frac{49}{50}}$$

$$\pm\frac{7}{5\sqrt{2}} = \pm\frac{7\sqrt{2}}{10}$$

θ



$\frac{3\pi}{2}$

$a^2+b^2=c^2$

$24^2+b^2=25^2$

$576+b^2=625$

$-576 \qquad -576$

$b^2=49$

$b=7$

Proof	Reason
$\sin x + \cos x = \dfrac{\cot x + 1}{\csc x}$	Given
$= (\cot x + 1) \div \csc x$	R. Dividing
$= (\cot x + 1) \cdot \dfrac{1}{\csc x}$	multiply by reciprocal
$= (\cot x + 1) \cdot \sin x$	rewrite (reciprocal ID)
$= \cot x \sin x + \sin x$	Distribute
$= \dfrac{\cos x}{\cancel{\sin x}} \cancel{\sin x} + \sin x$	rewrite (quotient ID)
$= \cos x + \sin x$	Simplify

Proof	Reason
$\sin x + \cos x = \dfrac{\cot x + 1}{\csc x}$	Given
$= (\cot x + 1) \div \csc x$	Divide
$= (\cot x + 1) \cdot \dfrac{1}{\csc x}$	multiply by reciprocal
$= (\cot x + 1) \cdot \sin x$	rewrite (reciprocal)
$= \cot x \sin x + \sin x$	Distribute
$= \dfrac{\cos x}{\cancel{\sin x}} \cancel{\sin x} + \sin x$	rewrite (quotient I D)
$= \cos x + \sin x$	simplify

2.

Proof	Reason
$\frac{1}{\sec x - \tan x} = \sec x + \tan x$	Given
	L.
$\frac{1}{(\sec x - \tan x)} \cdot \frac{\sec x + \tan x}{(\sec x + \tan x)} =$	multiply by conjugate
$\frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} =$	multiply
$\frac{\sec x + \tan x}{1 + \tan^2 x - \tan^2 x} =$	Pythagorean ID.
$\sec x + \tan x =$	Simplify

2.

Proof	Reason
$\frac{1}{\sec x - \tan x} = \sec x + \tan x$	Given
$\frac{1}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} =$	multiply by conjugate
$\frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} =$	multiply
$\frac{\sec x + \tan x}{1 + \tan^2 x - \tan^2 x} =$	Pythagorean ID
$\sec x + \tan x =$	Simplify

① $\tan 2x - 1 = 0$

Rewrite

$$\frac{2\tan x}{1-\tan^2 x} - 1 = 0$$
$$\quad +1 \qquad +1$$

$$(1-\cancel{\tan^2 x})\,\frac{2\tan x}{1-\cancel{\tan^2 x}} = 1(1-\tan^2 x)$$

cross multiply

2 trig, 3 terms

$$2\tan x = 1-\tan^2 x$$
$$-1+\tan^2 x \qquad -1+\tan^2 x$$

$$\tan^2 x + 2\tan x - 1 = 0$$

AC method

$x^2 + 2x - 1$

$a \cdot c$

-1

2

b

② $\sin^2 x - 2\sin x - 3 = 0$ 2 trig 3 terms

$(\sin x - 3)(\sin x + 1) = 0$ AC

$\sin x - 3 = 0$
$+3 \quad +3$

$(\sin x) = (3)$
$\sin^{-1} \quad \sin^{-1}$

$x =$ no solution

$\sin x + 1 = 0$
$-1 \quad -1$

$(\sin x) = (-1)$
$\sin^{-1} \quad \sin^{-1}$

$x = 270^\circ$

a · c

-3

-3 1

-2

b

$\left(x - \frac{3}{1}\right)\left(x + \frac{1}{1}\right)$

Divide by a

you
can do 2, 4, 6–14

2

2 trig
3 terms

$$\sin^2 x - 2\sin x - 3 = 0$$

AC

$$(\sin x - 3)(\sin x + 1) = 0$$

$$\sin x - 3 = 0$$
$$+3 \quad +3$$

$$(\sin x) = (3)$$
$$\sin^{-1} \quad \sin^{-1}$$

$x =$
no solution

$$\sin x + 1 = 0$$
$$-1 \quad -1$$

$$(\sin x) = (-1)$$
$$\sin^{-1} \quad \sin^{-1}$$

$$x = 270°$$

$$1x^2 - 2x - 3 = 0$$

$a \cdot c$

$1 \cdot -3$

-3

$-3 \quad 1$

-2

b

Divide by a

$$\left(x - \frac{3}{1}\right)\left(x + \frac{1}{1}\right)$$

4

1 trig

$\cos x = 3\cos x + 1$

$-\cos x \quad -\cos x$

$2\cos x + 1 = 0$

$-1 \quad -1$

$\frac{2\cos x}{2} = \frac{-1}{2}$

$\cos^{-1}(\cos x) = \cos^{-1}\left(-\frac{1}{2}\right)$

$x = 120^\circ, 240^\circ$

(12) $\sqrt{3}\cot x\sin x + 2\cos^2 x = 0$

$$\sqrt{3}\left(\frac{\cos x}{\cancel{\sin x}}\right)\left(\frac{\cancel{\sin x}}{1}\right) + 2\cos^2 x = 0$$

GCF

$$\frac{\sqrt{3}\cancel{\cos x}}{\cos x} + \frac{2\cos^{\cancel{2}} x}{\cancel{\cos x}} = 0$$

$$\cos x\left(\sqrt{3} + 2\cos x\right) = 0$$

$(\cos x = 0)$
$\cos^{-1}$ $\cos^{-1}$

$$x = 90°, 270°$$

$$\sqrt{3} + 2\cos x = 0$$
$$-\sqrt{3} \qquad -\sqrt{3}$$

$$\frac{2\cos x}{2} = \frac{-\sqrt{3}}{2}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$x = 150°, 210°$$

(2) $\sin^2 x - 2\sin x - 3 = 0$ 2 trig 3 terms

$(\sin x - 3)(\sin x + 1) = 0$ AC

$\sin x - 3 = 0$
$+3 \quad +3$

$(\cancel{\sin} x) = (3)$
$\cancel{\sin^{-1}} \quad \sin^{-1}$

$x =$ no solution

$\sin x + 1 = 0$
$-1 \quad -1$

$(\cancel{\sin} x) = (-1)$
$\cancel{\sin^{-1}} \quad \sin^{-1}$

$x = 270^\circ$

$$1x^2 - 2x - 3 = 0$$

$a \cdot c$

-3

-3 1

-2

Divide by a

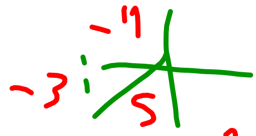
$$(x - \tfrac{3}{1})(x + \tfrac{1}{1})$$

(10) $3\sin^2 x - \cos^2 x = 0$

$3(1-\cos^2 x) - \cos^2 x = 0$

$\cos\theta = \frac{-4}{5}$ $\frac{A}{H}$ $\quad \pi < \theta < \frac{3\pi}{2}$ III

Find $\cos 2\theta$.



$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{-4}{5}\right)^2 - 1$

$\frac{2}{1}\left(\frac{16}{25}\right) - 1$

$\frac{32}{25} - \frac{1 \cdot 25}{1 \cdot 25}$

$\frac{7}{25}$

$a^2 + b^2 = c^2$

$(-4)^2 + b^2 = 5^2$

$16 + b^2 = 25$

$b^2 = 9$

$b = 3$

$$(\sin x)^2 = (\cos x)^2$$

$$\sin^2 x = \cos^2 x$$

Pyth. ID.

$$\sin^2 x + \cos^2 x = 1$$
$$-\cos^2 x \quad -\cos^2 x$$
$$\sin^2 x = 1 - \cos^2 x$$

$$1 - \cos^2 x = \cos^2 x$$
$$+\cos^2 x \quad +\cos^2 x$$
$$1 = 2\cos^2 x$$

Like terms

$\cos\theta = \frac{-4}{5}$ $\pi < \theta < \frac{3\pi}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\left(\frac{-4}{5}\right)^2 - 1$

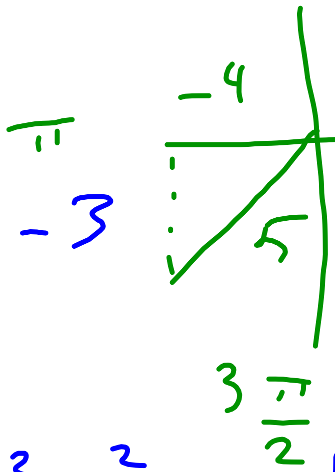
$\frac{2}{1}\left(\frac{16}{25}\right) - 1$

$\frac{32}{25} - \frac{1}{1}\frac{\cdot 25}{\cdot 25}$

$\frac{32-25}{25} = \frac{7}{25}$

$\cos\theta = \frac{-4}{5}$ $\frac{A}{H}$ $\pi < \theta < \frac{3\pi}{2}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$



$a^2 + b^2 = c^2$

$(-4)^2 + b^2 = 5^2$

$16 + b^2 = 25$

$-16 \quad -16$

$\sqrt{b^2} = \sqrt{9}$

$b = 3$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$