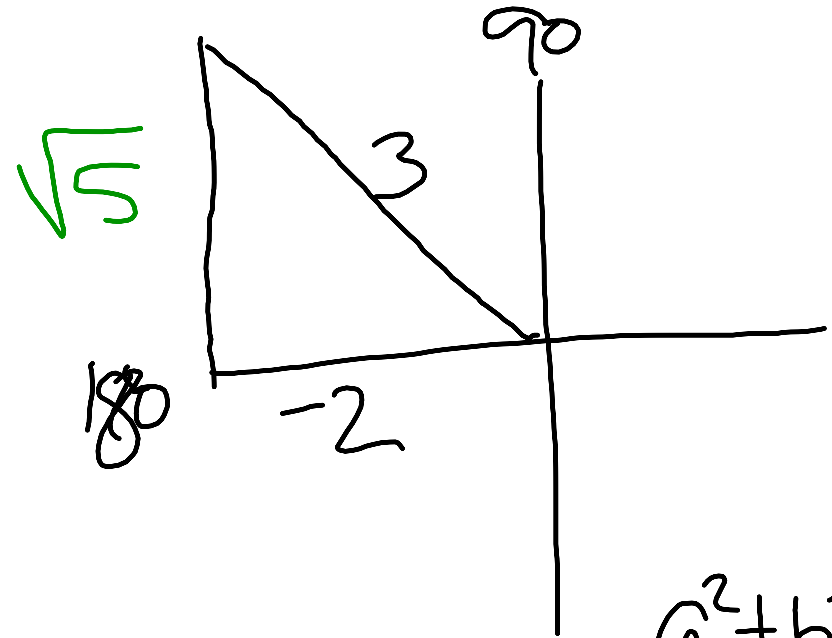
Warm-up

April 17, 2017

1. $\cos\theta = -\frac{2}{3}$ and $90^\circ < \theta < 180^\circ$

Find $\sin 2\theta$.

Find $\sin\frac{\theta}{2}$.



$$a^2+b^2=c^2$$
$$a^2+-2^2=3^2$$
$$a^2+4=9$$
$$-4\quad -4$$
$$a^2=5$$
$$a=\sqrt{5}$$

$$\sin 2\theta = 2\sin\cos\theta$$
$$=2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$$
$$\frac{-4\sqrt{5}}{9}$$

4

$$\frac{\cos x \cdot \cos x}{\cos x(1+\sin x)} + \frac{(1+\sin x)(1+\sin x)}{\cos x(1+\sin x)} = 2\sec x$$

Add fractions

	1	$+\sin x$
1	1	$\sin x$
$+\sin x$	$\sin x$	$\sin^2 x$

$1+2\sin x+\sin^2 x$

Pyth. ID.

GCF

Simplify

Rewrite

$$\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)}$$

$$\frac{\overset{2}{\cancel{1+1}} + 2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}}$$

$$2\left(\frac{1}{\cos x}\right)$$

$$2\sec x = 2\sec x$$

✓

5

$$\frac{(1+\sin x)\cos x}{(1+\sin x)(1-\sin x)} - \frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)} = 2\tan x$$

Subtract fractions

	1	$+\sin x$
1	1	$\cancel{\sin x}$
$-\sin x$	$\cancel{-\sin x}$	$-\sin^2 x$

$1-\sin^2 x$

$$\frac{\cancel{\cos x} + \cos x\sin x - \cancel{\cos x} + \cos x\sin x}{1-\sin^2 x}$$

Simplify

$$\frac{2\cos x\sin x}{(1-\sin^2 x)}$$

Pyth. ID.

$\cos^2 x + \cancel{\sin^2 x} - \cancel{\sin^2 x}$

$$\frac{2\cancel{\cos x}\sin x}{\cos^{\cancel{2}} x}$$

Simplify

$$\frac{2\sin x}{\cos x}$$

Rewrite

$2\tan x = 2\tan x$ ✓

⑥ $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

Rewrite $\rightarrow \dfrac{\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{1}\right)}{\sin x \cdot \left(\frac{\sin x}{\cos x}\right) + \left(\frac{\cos x}{\sin x}\right)\cdot \cos x}$

Add frac $\rightarrow \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$\dfrac{1}{\cos x \sin x}$

Divide $\rightarrow \dfrac{\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cos x \cancel{\sin x}}{1}}{\left(\frac{1}{\cos x \sin x}\right)}$

$\cos^2 x = \cos^2 x$ ✓

⑧ $\frac{\tan^2 x}{(\tan^2 x + 1)} = \sin^2 x$

Pyth. ID.

$\frac{\tan^2 x}{\sec^2 x}$

Rewrite

$\frac{\left(\frac{\sin^2 x}{\cancel{\cos^2 x}}\right) \cdot \frac{\cancel{\cos^2 x}}{1}}{\left(\frac{1}{\cos^2 x}\right)}$

Divide

$\sin^2 x = \sin^2 x$

✓

16

$$\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3+\sin x}{1-\sin x}$$

$$\frac{(\sin x + 3)(\sin x + 1)}{\cos^2 x}$$

Factor

Pyth. ID.

$$\cos^2\theta + \sin^2\theta = 1$$

$$-\sin^2 \quad -\sin^2$$

$$\cos^2 = 1 - \sin^2$$

$$\frac{(\sin x + 3)\cancel{(\sin x + 1)}}{(1-\sin^2 x)}$$

$$\frac{}{\cancel{(1+\sin x)}(1-\sin x)}$$

factor

Simplify

$$\frac{\sin x + 3}{1-\sin x} = \frac{3+\sin x}{1-\sin x}$$

✓

$1 - \sin^2 x$

$1 - x^2$ Diff. of Squares

$1 \quad 1 \qquad x \quad x$

$(1+x)(1-x)$

$(1+\sin x)(1-\sin x)$

$1x^2 + 4x + 3$

$ax^2 + bx + c$

Big X

$a \cdot c$

3

3 1

4

b

$(x+3)(x+1)$

(4) $\dfrac{\cos x}{\cos x}\cdot\dfrac{\cos x}{(1+\sin x)} + \dfrac{1+\sin x}{(\cos x)}\cdot\dfrac{(1+\sin x)}{(1+\sin x)} = 2\sec x$

	1	$+\sin x$
1	1	$\sin x$
$+\sin x$	$\sin x$	$\sin^2 x$

$1 + 2\sin x + \sin^2 x$

Add Fractions

Pyth. ID.

GCF

Simplify

Rewrite

$$\frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)}$$

$$\frac{\overset{2}{\cancel{1+1}} + 2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2(\cancel{1+\sin x})}{\cos x(\cancel{1+\sin x})}$$

$$\frac{2}{\cos x} = 2\cdot\frac{1}{\cos x}$$

$$2\sec x = 2\sec x$$

✓

$$\frac{2\cancel{\cos x}\sin x}{\cancel{\cos x}\cos x}$$

5) $\frac{(1+\sin x)\cos x}{(1+\sin x)(1-\sin x)} - \frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)} = 2\tan x$ Add fract.

	1	$-\sin x$
1	1	$\cancel{-\sin x}$
$+\sin x$	$\cancel{\sin x}$	$-\sin^2 x$

$1-\sin^2 x$

$$\frac{\cancel{\cos x} + \cos x \sin x - \cancel{\cos x} + \cos x \sin x}{1-\sin^2 x}$$ Simplify

$$\frac{2\cos x \sin x}{\cancel{1}-\sin^2 x}$$

$\cos^2 + \cancel{\sin^2} - \cancel{\sin^2 x}$

Pyth. ID

Simplify

$$\frac{2\cancel{\cos x}\sin x}{\cos^{\cancel{2}} x} = 2\left(\frac{\sin x}{\cos x}\right)$$ Rewrite

$2\tan x = 2\tan x$ ✓

⑥ $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

Rewrite

$$\frac{\left(\frac{1}{\sin x}\right)\frac{\cos x}{1}}{\left(\frac{\sin x}{\cos x}\right)\frac{\cdot \sin}{\cdot \sin} + \left(\frac{\cos x}{\sin x}\right)\frac{\cdot \cos}{\cdot \cos}}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$\frac{1}{\cos x \sin x}$$

$$\frac{\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cos x \cancel{\sin x}}{1}}{\left(\frac{1}{\cos x \sin x}\right)}$$

Divide

$\cos^2 x = \cos^2 x$

✓

8 $\dfrac{\tan^2 x}{(\tan^2 x + 1)} = \sin^2 x$

Pyth. ID.

Rewrite

Divide

$\dfrac{\tan^2 x}{\sec^2 x}$

$\dfrac{\left(\dfrac{\sin^2 x}{\cancel{\cos^2 x}}\right) \cdot \dfrac{\cancel{\cos^2 x}}{1}}{\left(\dfrac{1}{\cos^2 x}\right)}$

$\sin^2 x = \sin^2 x$ ✓

16

$$\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3+\sin x}{1-\sin x}$$

Factor

$$\frac{(\sin x+3)(\sin x+1)}{\cancel{\cos^2 x}}$$

$1-\sin^2\theta$

Pyth. ID.

$\cos^2\theta + \sin^2\theta = 1$

$-\sin^2\theta \quad -\sin^2\theta$

$\cos^2\theta = 1-\sin^2\theta$

$$\frac{(\sin x+3)\cancel{(\sin x+1)}}{\cancel{1-\sin^2 x}}$$

$\cancel{(1+\sin x)}(1-\sin x)$

Factor

Simplify

$$\frac{\sin x+3}{1-\sin x} = \frac{3+\sin x}{1-\sin x}$$

✓

$1-\sin^2 x$

Pretend:

$1 - x^2$ Diff. of Squares

$1 \cdot 1 \quad x \cdot x$

$(1+x)(1-x)$

$(1+\sin x)(1-\sin x)$

$\underset{a}{1}x^2 + \underset{b}{4}x + \underset{c}{3}$ $\quad a \cdot c$

BIG X

3

3 1

4

b

$(x+3)(x+1)$

Solve.

$$3x^2 - 2 = 10$$
$$+2 \qquad +2$$

$$\frac{3x^2}{3} = \frac{12}{3}$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$3\sec^2 x - 2 = 10$$
$$\quad\quad +2 \quad +2$$

$$\frac{\cancel{3}\sec^2 x}{\cancel{3}} = \frac{12}{3}$$

$$\sec^2 x = 4$$

$$\sqrt{(\sec x)^2} = \sqrt{4}$$

$$(\sec x) = (\pm 2)$$

$$\sec^{-1} \qquad \sec^{-1}$$

$$\cancel{\sec}(\cancel{\cos} x) = (\pm \tfrac{1}{2})$$

$$\cos^{-1} \qquad \cos^{-1}$$

$$x = 60°, 120°, 240°, 300°$$

Solve.

$$3\tan^2 x - 26 = 1$$

$$+26 \quad +26$$

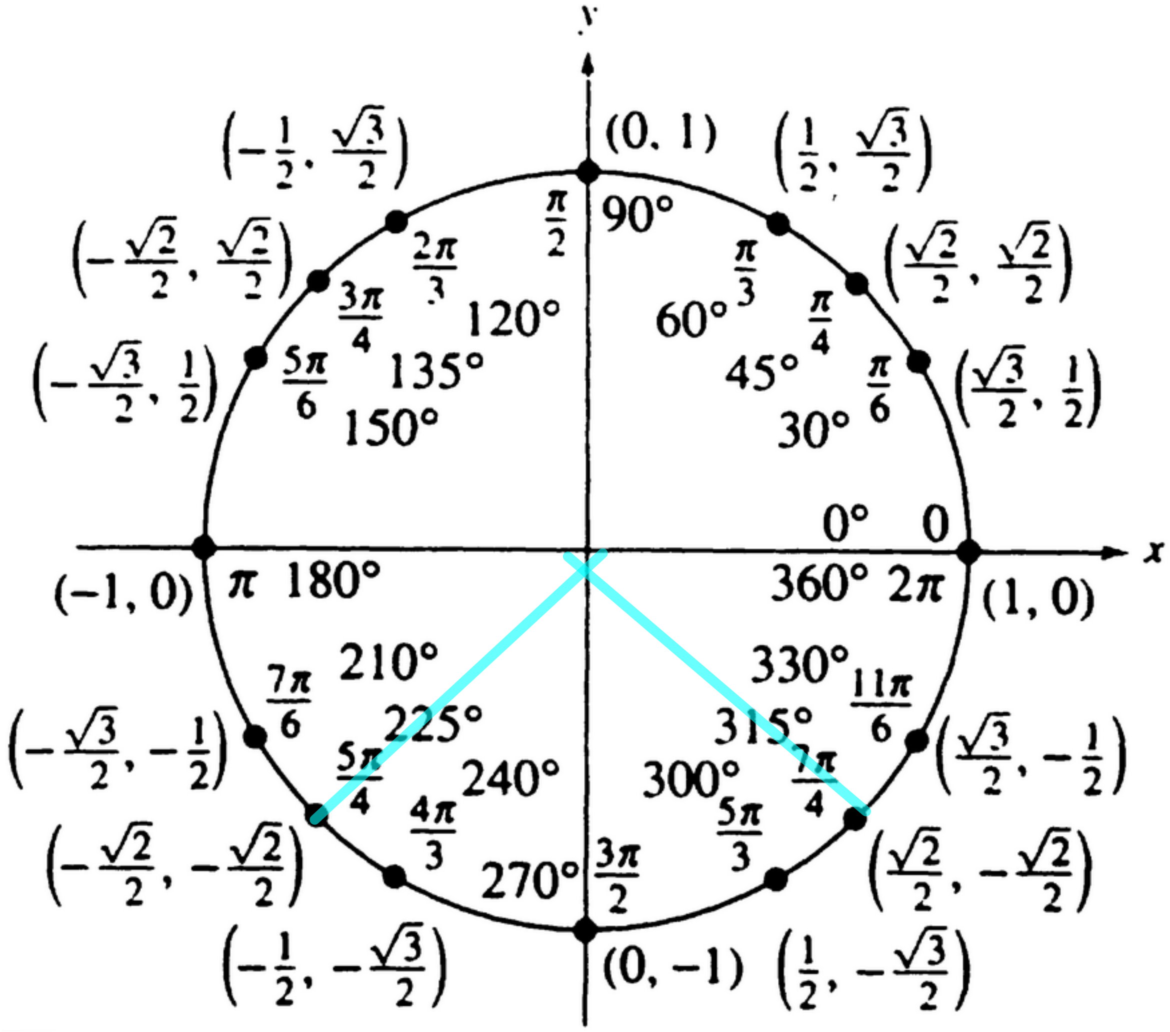
$$\frac{\cancel{3}\tan^2 x}{\cancel{3}} = \frac{27}{3}$$

$$\tan^2 x = 9$$

$$\sqrt{(\tan x)^2} = \sqrt{9}$$

$$\cancel{\tan}^{-1}(\tan x) = \tan^{-1}(\pm 3)$$

$$x = 71.56°, 108.44°, 251.56°, 288.44°$$


y
x
(0, 1)
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
(1, 0)
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, −1)
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
(−1, 0)
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0° 0
360° 2π
$\frac{2\pi}{3}$ 120°
$\frac{3\pi}{4}$ 135°
$\frac{5\pi}{6}$ 150°
π 180°
$\frac{7\pi}{6}$ 210°
$\frac{5\pi}{4}$ 225°
$\frac{4\pi}{3}$ 240°
270° $\frac{3\pi}{2}$
300° $\frac{5\pi}{3}$
315° $\frac{7\pi}{4}$
330° $\frac{11\pi}{6}$

Solve by combining like terms

$\sin x + \sqrt{2} = -\sin x$

1 trig funct.
nothing is squared

$+\sin x \qquad +\sin x$

$2\sin x + \sqrt{2} = 0$

$-\sqrt{2} \quad -\sqrt{2}$

$\frac{2\sin x}{2} = \frac{-\sqrt{2}}{2}$

$\cancel{\sin^{-1}}(\sin x) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$x = 225°, 315°$

Solve with square roots

$$3\tan^2 x - 1 = 0$$

You try!

1. $3\sec^2 x - 4 = 0$

$+4 \quad +4$

$\frac{3\sec^2 x}{3} = \frac{4}{3}$

$\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}$

$\sec x = \frac{2}{\sqrt{3}} \rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

$x = 30, 150, 210, 330$

Solve by factoring

$$\csc^4 x - 4\csc^2 x = 0$$

You try!

2. $2\sin^{\cancel{4}\,2} x - \sin^{\cancel{2}\,0} x = 0$

$\sin^2 x \left(2\sin^2 x - 1\right) = 0$

$\sqrt{\sin^2 x} = \sqrt{0}$

$\sin^{-1}(\sin x = 0)\sin^{-1}$

$\boxed{x = 0°, 180°}$

$2\sin^2 x - 1 = 0$
$\quad\;\; +1 \quad +1$

$\frac{2\sin^2 x}{2} = \frac{1}{2}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{\sqrt{1}}{\sqrt{2}} = \frac{\pm 1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{\pm\sqrt{2}}{2}\right)$

$\boxed{x = 45°, 135°, 225°, 315°}$

Quadratic type

$$2\sin^2 x - \sin x - 1 = 0$$

a b c

$2m^2 - m - 1$

2 trig terms
3 terms altogether
0 on da right

$a \neq 1$ AC method

$a \cdot c$

-2

-2 1

-1

b

$+$

$\left(\sin x - \frac{2}{2}\right)\left(\sin x + \frac{1}{2}\right) = 0$

$\sin x - 1 = 0$

$+1 \quad +1$

$(\sin x = 1)$

$\sin^{-1} \quad \sin^{-1}$

$x = 90°$

$\sin x + \frac{1}{2} = 0$

$-\frac{1}{2} \quad -\frac{1}{2}$

$(\sin x) = \left(-\frac{1}{2}\right)$

$\sin^{-1} \quad \sin^{-1}$

$x = 210°, 330°$

You try!

3. $2\cot^4 x - \cot^2 x - 15 = 0$