Warm-Up

April 13, 2017

Is the following equation an identity? Why or why not?

$1 + \sec^2(x) = \tan^2(x)$ $-\sec^2(x)$ (2) ←

$-\sec^2(x)$

$1 = \tan^2(x) - \sec^2(x)$

$-\tan \quad -\tan$

$1 - \tan^2(x) = -\sec^2(x) \neq$ (2)

NOTES: Three Pythagorean Identities

1. $\cos^2\theta + \sin^2\theta = 1$

2. $1 + \tan^2\theta = \sec^2\theta$

3. $\cot^2\theta + 1 = \csc^2\theta$

Reciprocal ID.

$\cos\theta = \frac{1}{\sec\theta}$ | $\sec\theta = \frac{1}{\cos\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ | $\csc\theta = \frac{1}{\sin\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ | $\cot\theta = \frac{1}{\tan\theta}$

Quotient ID.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

(3) $\csc^2\theta - \cot^2\theta$

Pyth. ID. ↓

$\cot^2\theta + 1 - \cot^2\theta$

(1)

$8x + 16$

$8x + 2 \cdot 8$ Factor GCF

$8(x + 2)$

(4)

$$\frac{\sec^2\theta}{\sec^2\theta} - \cos^2\theta\frac{\sec^2\theta}{\sec^2\theta}$$

$$\sec^2\theta\left(1 - \cos^2\theta\right)$$

$$\sec^2\theta\left(\cos^2\theta + \sin^2\theta - \cos^2\theta\right)$$ Pyth. ID

$$\sec^2\theta\left(\sin^2\theta\right)$$ Rewrite

$$\frac{1}{\cos^2\theta}\left(\frac{\sin^2\theta}{1}\right) = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta$$

(2)

$$\sin\theta + \cot\theta\cos\theta$$

Rewrite

$$\sin\theta + \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{1}\right)$$

$$\frac{\sin\theta}{\sin\theta}\sin\theta + \frac{\cos^2\theta}{\sin\theta}$$

$$\frac{\sin\theta}{\sin\theta}\cdot\frac{1}{1}$$

Add fractions

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

Pyth. ID.

$$\frac{1}{\sin\theta}$$

Rewrite $= \csc\theta$

NOTES Simplifying Trig Expressions

Strategy 1: Rewriting in terms of sine and cosine

$$\frac{\tan x}{\sec x}$$

Rewrite

$$\frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)} \cdot \frac{\cos x}{1}$$

Divide

$$\sin x$$

① $\csc\theta \tan\theta$ Rewrite

$\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$

$\frac{1}{\cos\theta}$ Rewrite

$\sec\theta$

1) $\csc\theta \tan\theta$

$$\left(\frac{1}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\frac{\sin\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$4x + 16$

$4x + 4 \cdot 4$

GCF $4(x+4)$

Strategy 2: Factoring

$$\cos x - \cos x \sin^2 x$$

GCF

$$\cos x(1 - \sin^2 x)$$

Pythl

$$\cos x(\cos^2 x + \sin^2 x - \sin^2 x)$$

$$\cos x(\cos^2 x)$$

$$\cos^3 x$$

(4) $\sec^2\theta - \cos^2\theta\sec^2\theta$

$\sec^2(1-\cos^2)$

$\sec^2\theta(\cos^2\theta+\sin^2\theta-\cos^2\theta)$

$\sec^2\theta(\sin^2\theta)$

$\frac{1}{\cos^2\theta}\cdot\frac{\sin^2\theta}{1}=\frac{\sin^2\theta}{\cos^2\theta}=\tan\theta^2$

$8 - 16x$

$8 - 8 \cdot 2x$ factor GCF

$8(1 - 2x)$

4. $\sec^2 x - \cos^2 x \sec^2 x$

$\sec^2 x(1 - \cos^2 x)$

$\sec^2\theta(\cos^2\theta + \sin^2\theta - \cos^2\theta)$

$\sec^2\theta(\sin^2\theta)$

$\frac{1}{\cos^2\theta}\left(\frac{\sin\theta}{1}\right)$

$\frac{\sin^2\theta}{\cos^2\theta}$ $\tan^2\theta$

4

$$\sec^2\theta - \cos^2\theta\sec^2\theta$$

GCF

$$\sec^2\theta\left(1 - \cos^2\theta\right)$$

Pyth. ID

$$\sec^2\theta\left(\cos^2\theta + \sin^2\theta - \cos^2\theta\right)$$

$$\sec^2\theta\left(\sin^2\theta\right)$$ Rewrite

$$\frac{1}{\cos^2\theta}\left(\frac{\sin^2\theta}{1}\right)$$ Multiply

Rewrite

$$\frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta$$

Do number 8 by any method you choose.

8) $\sec\theta - \sin\theta\tan\theta$

$\frac{1}{\cos\theta} - \frac{\sin\theta}{1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$ Subtract fraction

$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$ Pyth. ID.

$\frac{\cos^2\theta + \sin^2\theta - \sin^2\theta}{\cos\theta}$

$\frac{\cos\theta \cdot \cos\theta}{\cos\theta} = \cos\theta$

$5x + 10$

$5(x + 2)$

$\sin^2\theta + \sin^2\theta \tan^2\theta$

$\sin^2\theta(1 + \tan^2\theta)$

$\sin^2\theta \sec^2\theta$

$\sin^2\theta \frac{1}{\cos^2\theta}$

$\tan^2\theta$

11. $(\csc\theta + \cot\theta)(1-\cos\theta)$

$\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)(1-\cos\theta)$

$\left(\frac{1+\cos\theta}{\sin\theta}\right)(1-\cos\theta)$

$\frac{(1+\cos\theta)(1-\cos\theta)}{\sin\theta}$

$\frac{1+\cos\theta-\cos\theta-\cos^2\theta}{\sin\theta}$

$\frac{1-\cos^2\theta}{\sin\theta}$ $\frac{\sin^2\theta}{\sin\theta}$

$\sin\theta$

Do number 5.

$(\sin^2\theta + \cos^2\theta) + \tan^2\theta$

$1 + \tan^2\theta$

$\sec^2\theta$

Strategy 3: Getting a common denominator

Add/subtract fractions

$$\sin x + \cos x \cot x$$

$$\frac{\sin x}{1} + \frac{\cos x}{1}\left(\frac{\cos x}{\sin x}\right)$$

Rewrite

Simplify

$$\frac{\sin x}{\sin x}\cdot\frac{\sin x}{1} + \frac{\cos^2 x}{\sin x}$$

Add Fractions

$$\frac{\sin^2 x + \cos^2 x}{\sin x}$$

Pyth. ID.

$$\frac{1}{\sin x}$$

Rewrite

$$\csc x$$

(8) $\sec\theta - \sin\theta\tan\theta$ Rewrite

$\frac{1}{\cos\theta} - \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)$ Simplify

$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$ Subt. Fractions

$\frac{1 - \sin^2\theta}{\cos\theta}$ Pyth. ID

$\frac{\cos^2\theta + \sin^2\theta - \sin^2\theta}{\cos\theta}$

$\frac{\cos^2\theta}{\cos\theta} = \frac{\cos\theta\cos\theta}{\cos\theta} = \cos\theta$

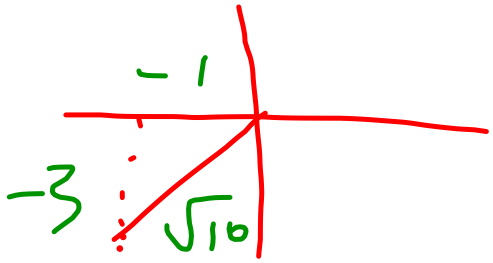
NOTES Evaluating Trig Expressions

Using reciprocals and the Pythagorean identities, we can evaluate all of the trig functions given one and the quadrant θ is in.

(9) Find $\sin\theta$ & $\sec\theta$

$\tan\theta = \frac{3}{-1} \frac{O}{A}$ $\tan\theta$ is pos

$\cos\theta < 0$ $\cos\theta$ is neg



$a^2 + b^2 = c^2$

$(-1)^2 + (-3)^2 = c^2$

$1 + 9$

$\sqrt{10} = \sqrt{c^2}$

$\sin\theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\sec\theta = \frac{H}{A}$

$= \frac{\sqrt{10}}{-1}$

$\therefore -\sqrt{10}$

$\tan\theta = 3$

$\cos\theta < 0$

$1 + \tan^2\theta = \sec^2\theta$

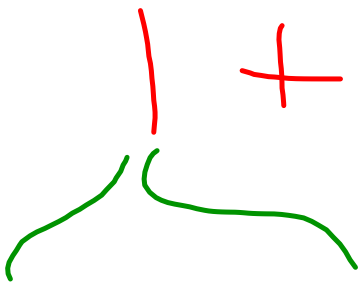
$1 + (3)^2$

$1 + 9$

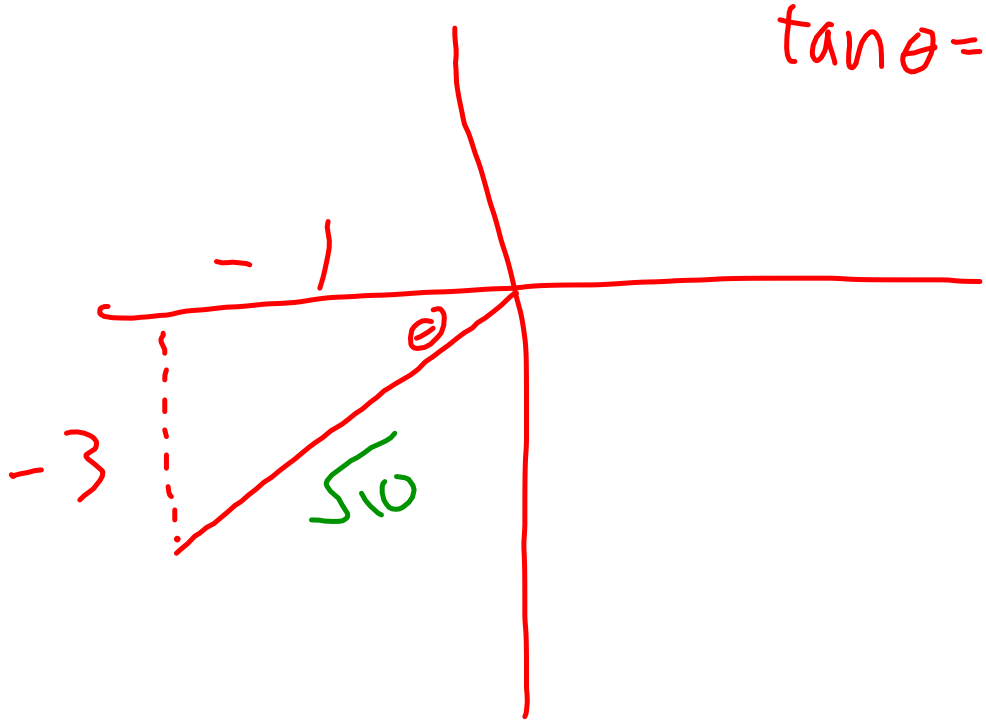
$\sqrt{10} = \sqrt{\sec^2\theta}$

$\sec\theta = -\sqrt{10}$

$1 + \cos^2\theta =$



$\tan\theta = \frac{3}{1}$



$a^2 + b^2 = c^2$

$(-1)^2 + (-3)^2 = c^2$

$1 + 9 = c^2$

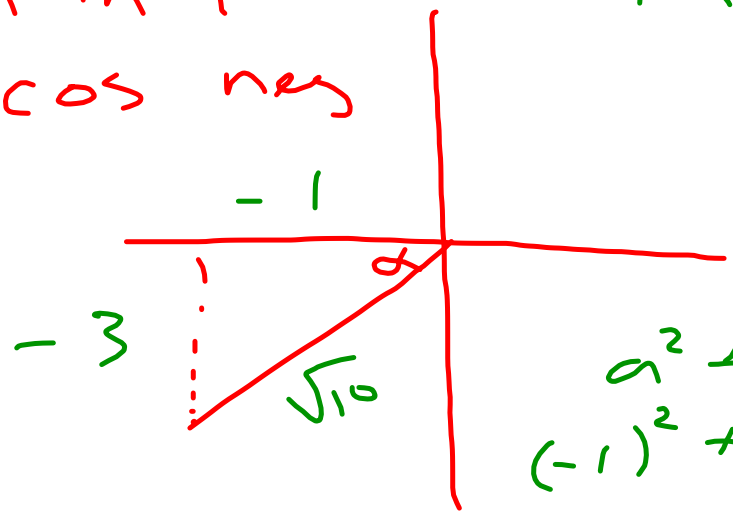
$c = \sqrt{10}$

Get out your phones and go to:
kahoot.it

tan pos

cos neg

$\tan\theta = \frac{3}{1}$



$a^2 + b^2 = c^2$

$(-1)^2 + (-3)^2$

$1 + 9$

$\sqrt{10} = \sqrt{c^2}$

$\sqrt{10} = c$

$\sec\theta = \frac{H}{A} = \frac{\sqrt{10}}{-1}$

$= -\sqrt{10}$

$\sin\theta = \frac{o}{H} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

Strategy 4: Splitting one fraction into two

$$\frac{\sec x - \cos x}{\sec x}$$

Split

$$\frac{\sec x}{\sec x} - \frac{\cos x}{\sec x}$$

Simplify

$$1 - \frac{\cos x}{\sec x}$$

Rewrite

$$1 - \frac{\cos x}{\left(\frac{1}{\cos x}\right)} \cdot \frac{\cos x}{1}$$

Divide

$$1 - \cos^2 x$$

Pyth. ID.

$$\cos^2 x + \sin^2 x - \cos^2 x$$

$$\sin^2 x$$

(10) $\dfrac{\sin\theta}{\cos\theta\tan\theta}$

Rewrite

$\dfrac{\sin\theta}{\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)}$

Simplify

$\dfrac{\sin\theta}{\sin\theta}$

1

12. $(\tan^2 x - \sec^2 x)(\sin^2 x + \cos^2 x)$

$(\tan^2 x - \sec^2 x)(1)$

$(\tan^2 x - (1 + \tan^2 x))(1)$

$\tan^2 x - 1 - \tan^2 x \ (1)$

$(-1)(1)$

-1

(5) $\sin^2\theta + \cos^2\theta + \tan^2\theta$

Pyth. ID.

$1 + \tan^2\theta$

Pyth. ID.

$\sec^2\theta$

(8) $\sec\theta - \sin\theta\tan\theta$

Rewrite

$$\frac{1}{\cos\theta} - \left(\frac{\sin\theta}{1}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$$

Simplify

$$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$$

Add/subt. fraction

$$\frac{1 - \sin^2\theta}{\cos\theta}$$

Pyth. ID.

$$\frac{\cos^2\theta + \sin^2\theta - \sin^2\theta}{\cos\theta}$$

$$\frac{\cos^2\theta}{\cos\theta} \qquad \frac{\cos\theta\cos\theta}{\cos\theta}$$

$$\cos\theta$$

What to ask.

1. Do I have anything squared and added? or subtracted

   If yes, Pythagorean Identities

2. Do they have anything in common?

   If yes, factor by GCF

3. Rewrite in terms of sine and cosine.

8. $\sec\theta - \sin\theta\tan\theta$

$\frac{1}{\cos\theta} - \left(\frac{\sin\theta}{1}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$ rewrite

$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$ multiply

$\frac{1-\sin^2\theta}{\cos\theta}$ add fractions

$\frac{\cos^2\theta + \cancel{\sin^2\theta} - \cancel{\sin^2\theta}}{\cos\theta}$ pythID

$\frac{\cos^2\theta}{\cancel{\cos\theta}} = \cos\theta$ simp

⑪ $(\csc\theta + \cot\theta)(1-\cos\theta)$

Rewrite

$\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)(1-\cos\theta)$

Simplify

$\left(\frac{1+\cos\theta}{\sin\theta}\right)\left(\frac{1-\cos\theta}{1}\right)$

multiply

	1	$+\cos\theta$
1	1	$\cos\theta$
$-\cos\theta$	$-\cos\theta$	$-\cos^2\theta$

$1-\cos^2\theta$

$\frac{1-\cos^2\theta}{\sin\theta}$

Pyth. ID.

$\frac{\cos^2\theta + \sin^2\theta - \cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\sin\theta}$

$\sin\theta$