Warm-up

April 11, 2017

Find the exact value.

$\sin(-255^\circ)$

$$\sin(60 - 315) =$$

$$\sin(60)\cos(315) - \cos(60)\sin(315)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{-\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{-\sqrt{2}}{4}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos(345^\circ)$$

$$\cos(300+45) = \cos^{300}\cos^{45} - \sin^{300}\sin^{45}$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{-\sqrt{6}}{4}$$

$$\boxed{= \frac{\sqrt{2}+\sqrt{6}}{4}}$$

$$\sin(w+w) = \sin w \cos w + \cos w \sin w$$

$$\sin(2w) = 2\sin(w)\cos(w)$$

$$\cos(w+w) = \cos w \cos w - \sin w \sin w$$

$$\cos(2w) = \cos^2 w - \sin^2 w$$

$$(\cos(w))^2$$

$$\sin(r+r) = \sin(r)\cos(r) + \overset{\sin(r)\cos(r)}{\cos(r)\sin(r)}$$

$$\sin(2r) = 2\sin(r)\cos(r)$$

Double Angle Identity

Double Angle Formulas:

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$
$$= 2\cos^2 A - 1$$

coefficient

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{2\tan A}{1-\tan^2 A}$$

exponent

(16) $\sin\theta = \frac{1}{3}$, $0° < \theta < 90°$

Find $\cos 2\theta = 1 - 2\sin^2\theta$

$1 - 2\left(\frac{1}{3}\right)^2$

$1 - 2\left(\frac{1}{9}\right)$

$\frac{1}{1} - \frac{2}{9}$

$\frac{9}{9} - \frac{2}{9}$

$\frac{7}{9}$

16 $\sin\theta = \frac{1}{3}$, $0^\circ < \theta < 90^\circ$

Find $\cos 2\theta = 1 - 2\sin^2\theta$

$1 - 2\left(\frac{1}{3}\right)^2$

$1 - 2\left(\frac{1}{9}\right)$

$\frac{1}{1} - \frac{2}{9}$

common denom.

$\frac{9}{9} - \frac{2}{9}$

$\frac{7}{9}$

16 $\sin\theta = \frac{1}{3}$, $0^\circ < \theta < 90^\circ$

$0 < 2\theta < 180^\circ$

$2(0^\circ) < 2\theta < 2(90^\circ)$

Find $\cos 2\theta$.

$= 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{1}{3}\right)^2$

$1 - 2\left(\frac{1}{9}\right)$

common denominator $\frac{9}{9} - \frac{2}{9}$

$\cos 2\theta = \frac{7}{9}$

② $\tan(60^\circ)$

$$\tan(2\cdot 30^\circ) = \frac{\sin(2\cdot 30^\circ)}{\cos(2\cdot 30^\circ)}$$

$$= \frac{2\sin(30)\cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = \frac{\frac{2\sqrt{3}}{4}}{\frac{2}{4}} = \frac{\left(\frac{\sqrt{3}}{\cancel{2}}\right)\frac{2}{1}}{\left(\frac{1}{2}\right)\,2}$$

$$\frac{3}{4} - \frac{1}{4}$$

$$= \sqrt{3}$$

1

$\sin(120^\circ)$

$\sin(2 \cdot 60^\circ) = 2\sin(60)\cos(60)$

$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

$$\cos 2A = \cos^2 A - \sin^2 A$$

Pyth. ID. $\cos^2 A + \sin^2 A = 1$

$$-\sin^2 A \qquad -\sin^2 A$$

$$\cos^2 A = 1 - \sin^2 A$$

Substitute

$$1 - \sin^2 A - \sin^2 A$$

$$1 - 2\sin^2 A$$

$$\tan(60) = \tan(2\cdot 30)$$

$$= \frac{\sin(2\cdot 30)}{\cos(2\cdot 30)}$$

$$= \frac{2\sin(30)\cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{2\sqrt{3}}{4}}{\frac{3}{4} - \frac{1}{4}} = \frac{\frac{\sqrt{3}}{\cancel{2}}\ \frac{\cancel{2}}{1}}{\frac{\cancel{2}}{\cancel{4}}\ \frac{1}{2}}$$

$$\sqrt{3}$$

$\sin(120°)$

$\sin(2\cdot 60°) = 2\sin 60 \cos 60$

$2\cdot\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{2\sqrt{3}}{4}$

$\boxed{\frac{\sqrt{3}}{2}}$

$\cos^2 A - \sin^2 A$

Pyth ID $\cos^2 A + \sin^2 A = 1$

$-\sin^2 A \quad -\sin^2 A$

$\cos^2 A = \underline{1 - \sin^2 A}$

$1 - \sin^2 A - \sin^2 A$

$1 - 2\sin^2 A$

$$\sin(60-315)$$

$$\sin 60 \cos 315 - \cos 60 \sin 315$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} \quad \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin(L+L) = \sin L \cos L + \cos L \sin L$$

$\sin L \cos L$

$$\sin(2L) = 2\sin L \cos L$$

Double Sine Identity

$$\sin(600)$$

$$\sin(2\cdot 300) = 2\sin(300)\cos(300)$$

$$2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{-2\sqrt{3}}{4}$$

$$= -\frac{\sqrt{3}}{2}$$

Half Angle Formulas:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} \quad \text{for } \cos\alpha \neq -1$$

#19

$$\frac{3\sqrt{10}}{10}$$

(15) $\sin\theta = \frac{-7\ \text{o}}{25\ \text{H}}$, $\frac{270}{2} < \frac{\theta}{2} < \frac{360}{2}$

$135 < \frac{\theta}{2} < 180$

Find $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

S^{+} A^{+}

$\frac{\theta}{2}$ $T^{\pm}$ C^{+}

$$= -\sqrt{\frac{1 + \frac{24}{25}}{2}}$$

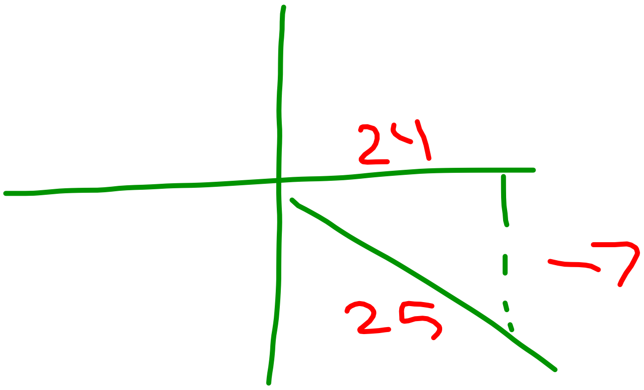
$$= -\sqrt{\frac{\frac{49}{25}\cdot\frac{1}{2}}{\left(\frac{2}{1}\right)}}$$

$$= -\frac{\sqrt{49}}{\sqrt{50}} = \frac{-7}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}}{10}$$

θ

$$\cos\theta = \frac{A}{H} = \frac{24}{25}$$



$$a^2 + b^2 = c^2$$

$$(-7)^2 + b^2 = 25^2$$

$$49 + b^2 = 625$$

$$-49 \qquad -49$$

$$\sqrt{b^2} = \sqrt{576}$$

$$b = 24$$

20

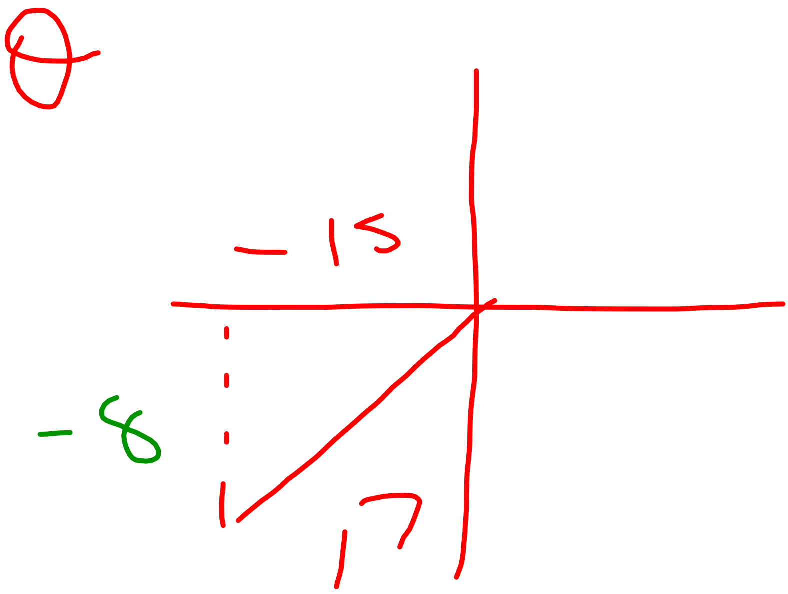
$\cos\theta = \frac{-15}{17}$ A H $180° < \theta < 270°$

Find $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

$$= \frac{\frac{-8}{17}}{\frac{1\cdot 17}{1\cdot 17} + \frac{-15}{17}}$$

$$\frac{\frac{-8}{17}\cdot\frac{17}{2}}{\left(\frac{2}{17}\right)}$$

$= -4$



$$a^2 + b^2 = c^2$$

$$225 + b^2 = 289$$

$$-225 \qquad -225$$

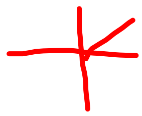
$$b^2 = 64$$

$$b = 8$$

(18) $\cos\theta = \frac{2\sqrt{5}}{5}$, $\frac{0^\circ}{2} < \frac{\theta}{2} < \frac{90^\circ}{2}$

$0 < \frac{\theta}{2} < 45^\circ$

Find $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\frac{\theta}{2}$ 

$$= \sqrt{\frac{\frac{1}{1}\cdot\frac{5}{5} - \frac{2\sqrt{5}}{5}}{2}}$$

$$= \sqrt{\frac{\frac{5}{5} - \frac{2\sqrt{5}}{5}}{2}}$$

$$\sqrt{\frac{\frac{5-2\sqrt{3}}{5}\cdot\frac{1}{2}}{\left(\frac{2}{1}\right)}} = \boxed{\sqrt{\frac{5-2\sqrt{5}}{10}}}$$

(15) $\sin\theta = \frac{-7}{25 H}$, $\frac{270°}{2} < \frac{\theta}{2} < \frac{360°}{2}$ $\quad 135 < \frac{\theta}{2} < 180$

Find $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\frac{\theta}{2}$ $\quad S^{+}$ A^{+} T^{+} C^{+}

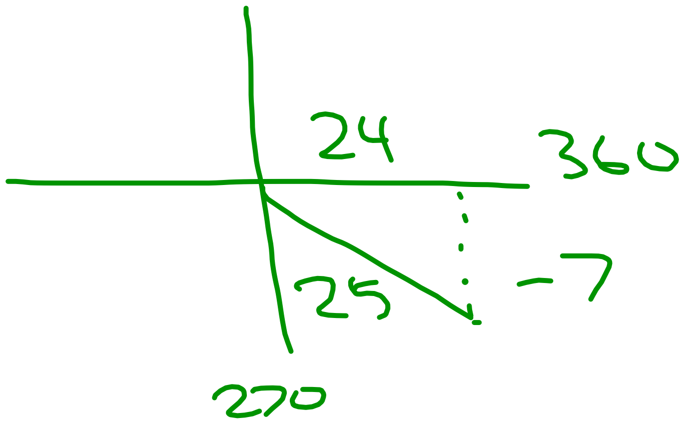
$= -\sqrt{\frac{\frac{1 \cdot 25}{1 \cdot 25}+\frac{24}{25}}{2}}$

$\frac{\frac{49}{25}}{\left(\frac{2}{1}\right)} \quad \frac{49}{25}\cdot\frac{1}{2} = \frac{49}{50}$

$-\sqrt{\frac{49}{50}} = \frac{-7}{\sqrt{50}} = \frac{-7}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= -\frac{7\sqrt{2}}{10}$

θ



$a^2 + b^2 = c^2$

$(-7)^2 + b^2 = 25^2$

$49 + b^2 = 625$

$-49 \qquad -49$

$\sqrt{b^2} = \sqrt{576}$

$b = 24$

20 $\cos\theta = \frac{-15}{17}$, $\frac{180^\circ}{2} < \frac{\theta}{2} < \frac{270^\circ}{2}$

$90^\circ < \frac{\theta}{2} < 135^\circ$

Find $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

II

S^{+} A^{+}

T^{+} C^{+}

$$= -\sqrt{\frac{1 - \frac{-15}{17}}{1 + \frac{-15}{17}}}$$

$$= -\sqrt{\frac{\frac{17}{17} + \frac{15}{17}}{\frac{17}{17} - \frac{15}{17}}}$$

$$= -\sqrt{\frac{\frac{32}{17}}{\frac{2}{17}}} = -\sqrt{16}$$

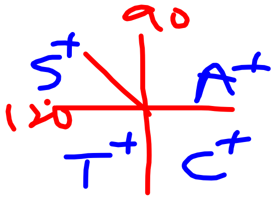
$$= -4$$

15

$135 < \frac{\theta}{2} < 180$

$\sin\theta = \frac{7}{25}\,\frac{O}{H}, \quad \frac{270^\circ}{2} < \frac{\theta}{2} < \frac{360^\circ}{2}$

Find $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$



$\frac{\theta}{2}$

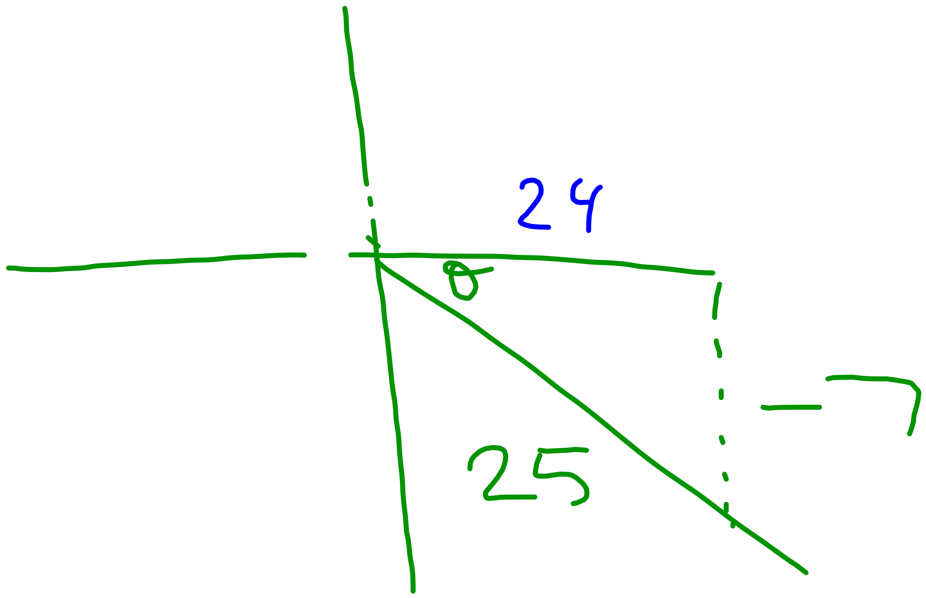
$$= -\sqrt{\frac{1+\frac{24}{25}}{2}}$$

$$= -\sqrt{\frac{\frac{25}{25}+\frac{24}{25}}{2}}$$

$$= -\sqrt{\frac{\left(\frac{49}{25}\right)}{\left(\frac{2}{1}\right)}} \cdot \frac{1}{2} = -\sqrt{\frac{49}{50}}$$

$$= -\frac{\sqrt{49}}{\sqrt{50}} = \frac{-7}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \frac{-7\sqrt{2}}{10}$$

θ



$$a^2 + b^2 = c^2$$
$$(-7)^2 + b^2 = 25^2$$
$$49 + b^2 = 625$$
$$-49 \qquad -49$$
$$\sqrt{b^2} = \sqrt{576}$$
$$b = 24$$

8

$\cos(30^\circ)$

8. $\cos(30°)$

$$\cos\left(\frac{60}{2}\right) = \pm\sqrt{\frac{1+\cos 60}{2}}$$

$$= \sqrt{\frac{\frac{2\cdot 1}{2\cdot 1}+\left(\frac{1}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{3}{2}\cdot\frac{1}{2}}{\frac{2}{1}}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \boxed{\frac{\sqrt{3}}{2}}$$

$\sin u = \frac{3}{5}\,\frac{O}{H}, \quad \frac{\pi}{2} \leq u \leq \pi$

$\tan v = \frac{5}{12}\,\frac{O}{A} \quad \pi \leq v \leq \frac{3\pi}{2}$

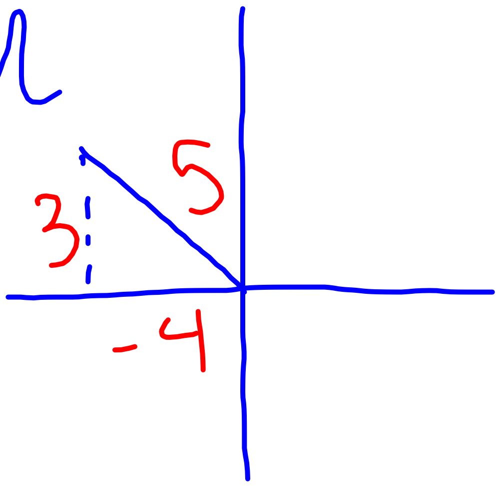
a) $\sin(u+v)$

$\sin u \cos v + \cos u \sin v$

$\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

$\frac{-36}{65} + \frac{20}{65} = \boxed{\frac{-16}{65}}$

u



$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

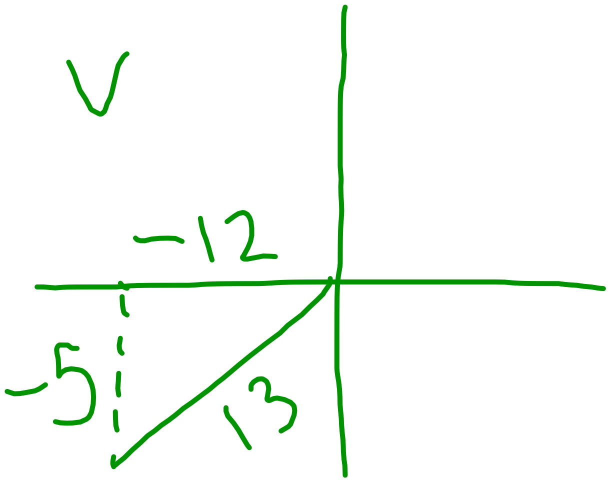
$9 + b^2 = 25$

$-9 \qquad -9$

$b^2 = 16$

$b = 4$

v



$a^2 + b^2 = c^2$

$25 + 144 = c^2$

$\sqrt{c^2} = \sqrt{169}$

$c = 13$

8

$\cos(30)$

$$\cos\left(\frac{60^\circ}{2}\right) = \pm\sqrt{\frac{1+\cos(60^\circ)}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}1 + \frac{1}{2}}{2}}$$

$$= \sqrt{\frac{\left(\frac{3}{2}\right)\cdot\frac{1}{2}}{\left(\frac{2}{1}\right)}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

② $\tan(60^\circ)$

$\tan(2\cdot 30^\circ)$

$$= \frac{2\sin(30)\cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{2\sqrt{3}}{4}}{\frac{3}{4} - \frac{1}{4}}$$

$$= \frac{\left(\frac{\sqrt{3}}{\cancel{2}}\right)\cdot\frac{\cancel{2}}{1}}{\frac{\cancel{2}}{\cancel{4}}\left(\frac{1}{2}\right)} = \sqrt{3}$$

$$\sin(120^\circ)$$

$$\sin(2 \cdot 60^\circ) = 2\sin(60^\circ)\cos(60^\circ)$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$\cos^2 A - \sin^2 A$$

Pyth ID → $\cos^2 A + \sin^2 A = 1$

$\quad\quad -\sin^2 A \quad -\sin^2 A$

$$\cos^2 A = 1 - \sin^2 A$$

substitute

$$1 - \sin^2 A - \sin^2 A$$

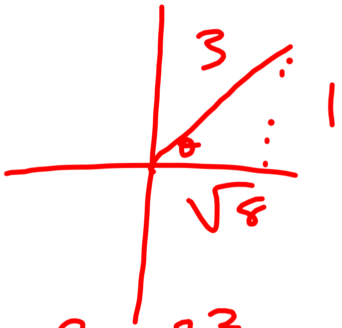
$$1 - 2\sin^2 A$$

$$1 - 2\sin^2 A$$

16 $\sin\theta = \frac{1}{3}$, $0 < \theta < 90$

I

$$\cos 2\theta = 1 - 2\sin^2\theta$$



$$= 1 - 2\left(\frac{1}{3}\right)^2$$

$$= 1 - \frac{2}{1}\left(\frac{1}{9}\right)$$

$$= \frac{9}{9}\cdot\frac{1}{1} - \frac{2}{9}$$

$$= \frac{9-2}{9} = \frac{7}{9}$$

$a^2 + b^2 = c^2$

$a^2 + 1^2 = 3^2$

$a^2 + 1 = 9$

$\quad -1 \quad -1$

$\sqrt{a^2} = \sqrt{8}$

$a = \sqrt{8}$

5

$$\cos\left(\frac{4\pi}{3}\right)$$

$$\cos(240°)$$

$$\cos(2(120))$$

$$= \cos^2(120) - \sin^2(120)$$

$$\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\frac{1}{4} - \frac{3}{4}$$

$$-\frac{2}{4} = -\frac{1}{2}$$

Double Angles

1) $\sin(120°)$ Half of what's given

$\sin(2(60))$

$= 2\sin(60)\cos(60)$

$\frac{2}{1}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{2\sqrt{3} \div 2}{4 \div 2} = \frac{\sqrt{3}}{2}$

$$\underbrace{\cos^2\theta}_{\substack{\cos^2 + \sin^2 = 1 \\ -\sin^2 \quad -\sin^2 \\ \cos^2\theta = 1 - \sin^2\theta}} - \sin^2\theta$$

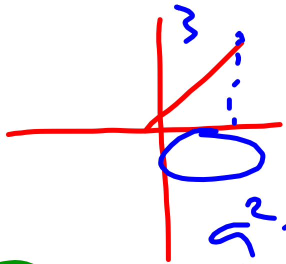
$$1 - \sin^2\theta \quad - \sin^2\theta$$

$$1 - 2\sin^2\theta$$

16) $\sin\theta = \frac{1}{3}$, $0^\circ < \theta < 90^\circ$

I

Find $\cos(2\theta)$



$a^2 + b^2 = c^2$

$a^2 + 1^2 = 3^2$

$a^2 + 1 = 9$

$-1 \quad -1$

$a^2 = 8$

$\sqrt{a^2} = \sqrt{8}$

$a = \sqrt{8}$

$a = 2\sqrt{2}$

$= 1 - 2\sin^2\theta$

$1 - 2\left(\frac{1}{3}\right)^2$

$1 - \frac{2}{1}\left(\frac{1}{9}\right)$

$\frac{9}{9} \cdot \frac{1}{1} - \frac{2}{9}$

$\frac{7}{9}$

3 $\cos\left(\frac{4\pi}{3}\right)$

$\cos(240)$

$\cos(2(120))$

$= \cos^2(120) - \sin^2(120)$

$\left(\frac{-1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} - \frac{3}{4}$

$\frac{-2}{4} = \frac{-1}{2}$

① $\sin(120°)$

Double Angle
↳ use half of given angle

$\sin(2(60))$

$= 2\sin(60)\cos(60)$

$\frac{2}{1}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{2 \div 2\sqrt{3}}{4 \div 2} = \frac{\sqrt{3}}{2}$

16 $\sin\theta = \frac{1}{3}$, $0^\circ < \theta < 90^\circ$

Find $\cos 2\theta$

$= 1 - 2\sin^2\theta$

$1 - 2(\sin\theta)^2$

$1 - 2\left(\frac{1}{3}\right)^2$

$1 - \frac{2}{1}\left(\frac{1}{9}\right)$

$\frac{9}{9}\cdot\frac{1}{1} - \frac{2}{9}$ $\frac{9-2}{9} = \frac{7}{9}$

three

3 $\cos\left(\frac{4\pi}{3}\right)$ or 240

$\cos\left(2\left(\frac{2\pi}{3}\right)\right)$ or $2(120)$

$\cos(2(120))$

$= (\cos(120))^2 - (\sin(120))^2$

$\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} - \frac{3}{4}$

$-\frac{2}{4} = -\frac{1}{2}$

1 Double angle

$\sin(120°)$ half it

$\sin(2(60)) = 120$

$= 2\sin(60)\cos(60)$

$= \frac{2}{1}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\cancel{2}^{1}\sqrt{3}}{\cancel{4}_{2}}$

$= \frac{1\sqrt{3}}{2}$

$\sin u = \frac{3}{5}\frac{O}{H}, \quad 0 < u < \frac{\pi}{2}$ I

Sum/Diff Identities

$\cos v = -\frac{7}{10}\frac{A}{H}, \quad \pi < v < \frac{3\pi}{2}$

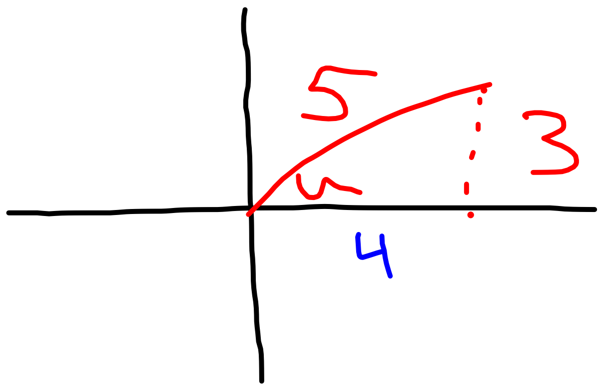
Find $\sin(u + v)$.

$= \sin u \cos v + \cos u \sin v$

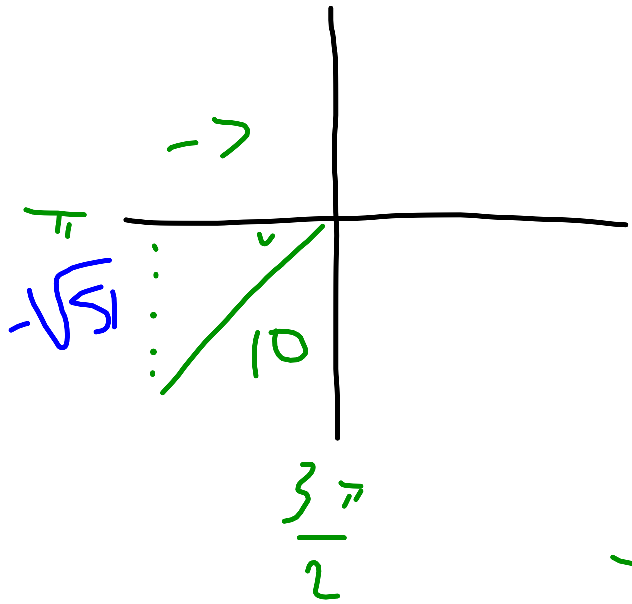
$\left(\frac{3}{5}\right)\left(\frac{-7}{10}\right) + \left(\frac{4}{5}\right)\left(\frac{-\sqrt{51}}{10}\right)$

$\frac{-21}{50} + \frac{-4\sqrt{51}}{50}$

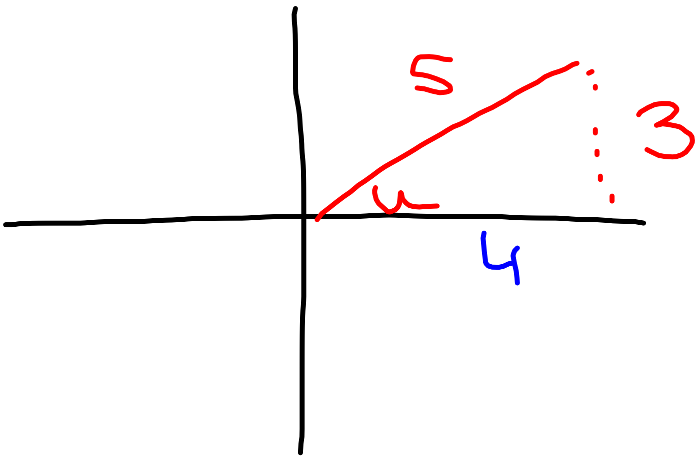
$$\frac{-21 - 4\sqrt{51}}{50}$$



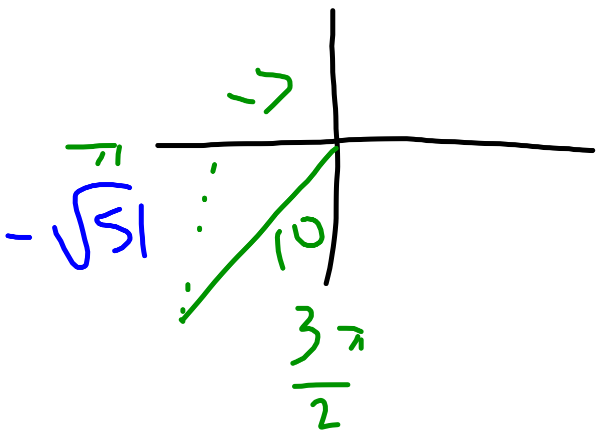
$$a^2+b^2=c^2$$
$$a^2+3^2=5^2$$
$$a^2+9=25$$
$$\quad -9 \quad -9$$
$$\sqrt{a^2}=\sqrt{16}$$
$$a=4$$



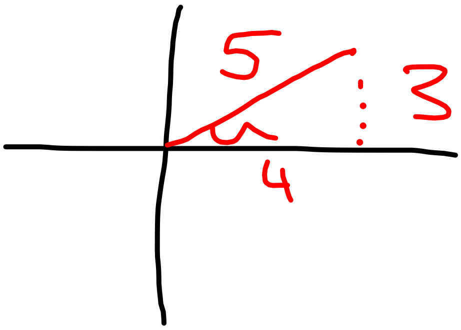
$$a^2+b^2=c^2$$
$$(-7)^2+b^2=10^2$$
$$49+b^2=100$$
$$-49 \quad -49$$
$$\sqrt{b^2}=\sqrt{51}$$
$$b=\sqrt{51}$$



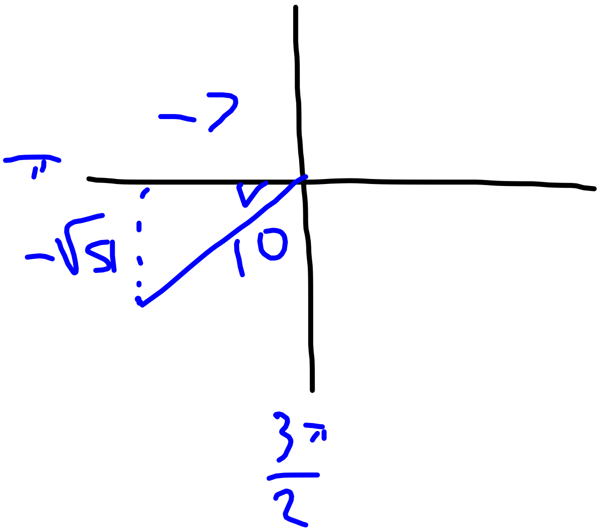
$$a^2+b^2=c^2$$
$$3^2+b^2=5^2$$
$$9+b^2=25$$
$$-9 \qquad -9$$
$$\sqrt{b^2}=\sqrt{16}$$
$$b=4$$



$$a^2+b^2=c^2$$
$$(-7)^2 \qquad 10^2$$
$$b=\sqrt{51}$$



$$a^2 + b^2 = c^2$$
$$(3)^2 + b^2 = 5^2$$
$$9 + b^2 = 25$$
$$-9 \qquad -9$$
$$\sqrt{b^2} = \sqrt{16}$$
$$b = 4$$



$$a^2 + b^2 = c^2$$
$$(-7)^2 + b^2 = 10^2$$
$$49 + b^2 = 100$$
$$-49 \qquad -49$$
$$\sqrt{b^2} = \sqrt{51}$$
$$b = \sqrt{51}$$

9A $\sin(u+v)$

$\sin u = \frac{3}{5} \frac{O}{H}$ $\frac{\pi}{2} < u < \pi$

$\tan v = \frac{5}{12} \frac{O}{A}$ $\pi < v < \frac{3\pi}{2}$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$

$-\frac{36}{65} + \frac{20}{65}$

$\sin(u+v) = -\frac{16}{65}$

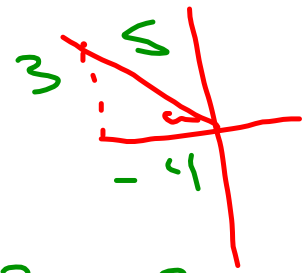
$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)$$

$$\frac{48}{65} - \frac{-15}{65}$$

$$\frac{63}{65}$$

u



$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

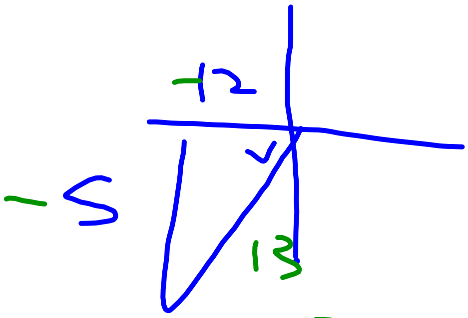
$9 + b^2 = 25$

$-9 \qquad -9$

$\sqrt{b^2} = \sqrt{16}$

$b = 4$

v



$a^2 + b^2 = c^2$

$5^2 + 12^2$

$169 = c^2$

$c = 13$

$\sin\theta = \frac{7}{-25}$ $270 < \theta < 360$

$\sin(u+v)$

$\sin u = \frac{3}{5}$ $\frac{\pi}{2} < u < \pi$ II

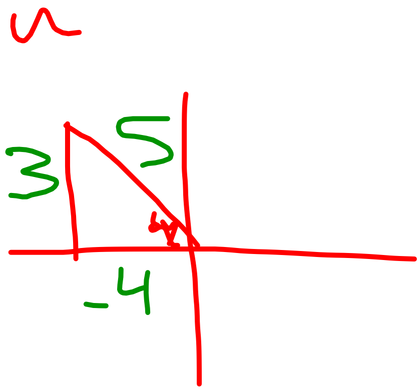
$\tan v = \frac{5}{12}$ $\pi < v < \frac{3\pi}{2}$ A

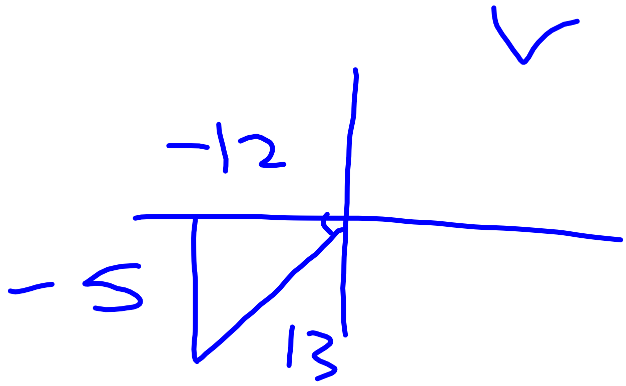
$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$$

$$\frac{-36}{65} + \frac{20}{65}$$

$$\frac{-16}{65}$$





$a^2 + b^2 = c^2$

$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

$-9 \quad -9$

$\sqrt{b^2} = \sqrt{16}$

$b = 4$

$a^2 + b^2 = c^2$

$(-5)^2 + (-12)^2 = c^2$

$25 + 144$

$169 = c^2$

$c = 13$

Warm-up

April 12, 2017

Find $\cos(u-v)$ if $\cot u = \frac{-7}{5}$ $\frac{A}{O}$, $\frac{\pi}{2} < u < \pi$

and $\sin v = \frac{-7}{15}$ $\frac{O}{H}$, $\frac{3\pi}{2} < v < 2\pi$

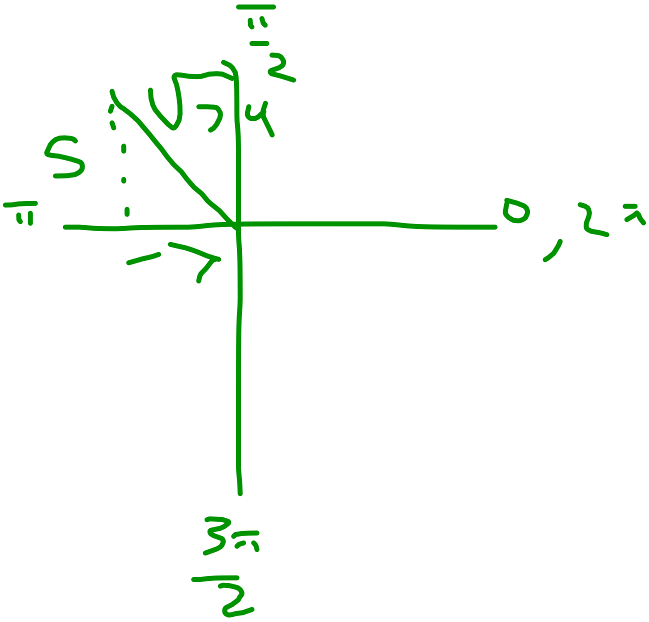
$\cos(u-v) = \cos u \cos v + \sin u \sin v$

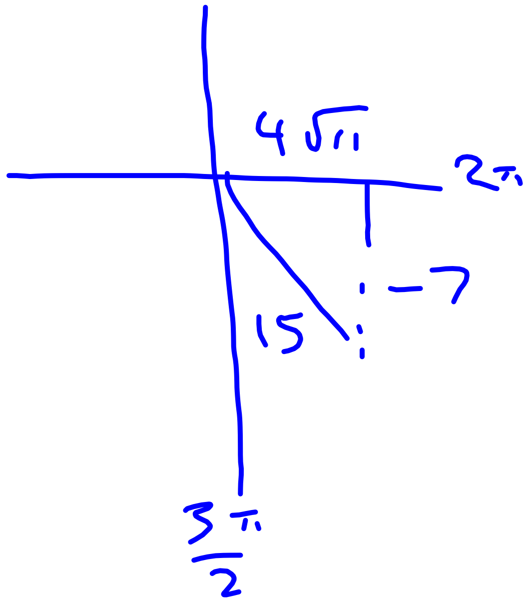
$\left(\frac{-7}{\sqrt{74}}\right)\left(\frac{4\sqrt{11}}{15}\right) + \left(\frac{5}{\sqrt{74}}\right)\left(\frac{-7}{15}\right)$

$\frac{-28\sqrt{11}}{15\sqrt{74}} + \frac{-35}{15\sqrt{74}}$

$\frac{-28\sqrt{11} - 35}{15\sqrt{74}}$

u





$a^2+b^2=c^2$

$5^2+(-7)^2=c^2$

$25+49$

$\sqrt{74} \quad \pm\sqrt{c^2}$

$\sqrt{74}=c$

$a^2+b^2=c^2$

$(-7)^2+b^2=15^2$

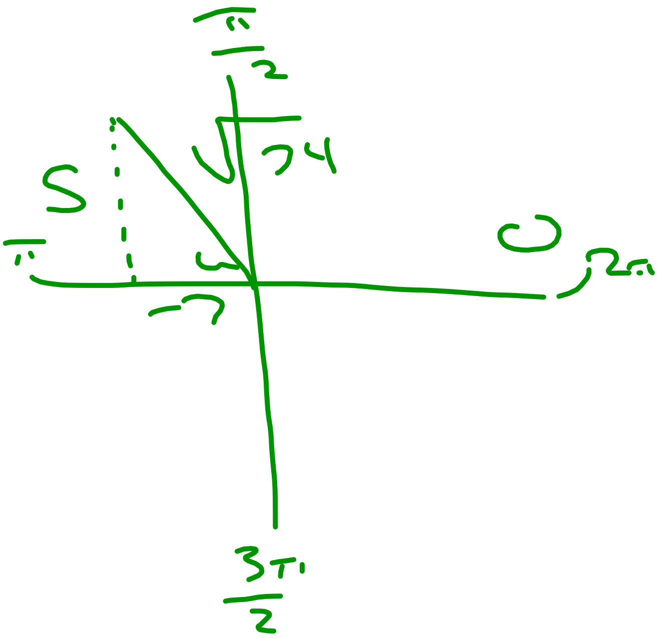
$49+b^2=225$

$-49 \qquad -49$

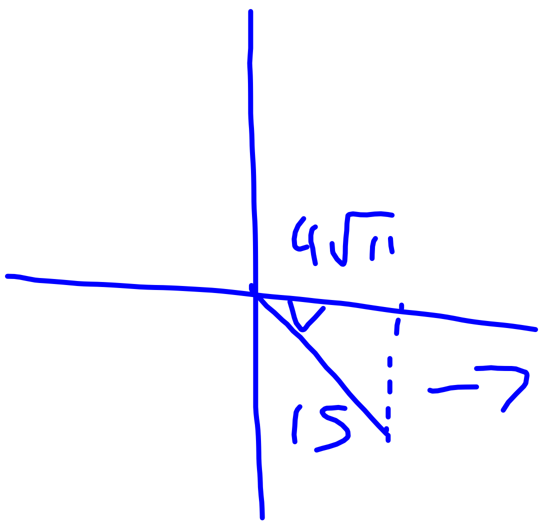
$\sqrt{b^2}=\sqrt{176}$

$b=4\sqrt{11}$

u



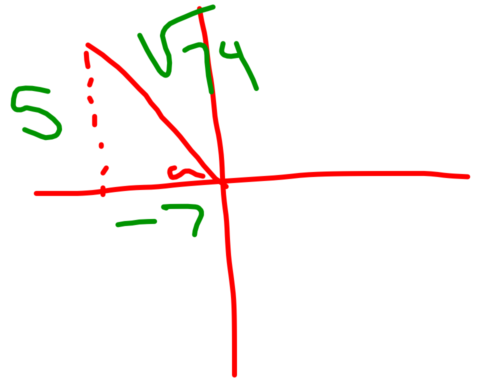
v



$$a^2+b^2=c^2$$
$$5^2+(-7)^2=c^2$$
$$25+49$$
$$\sqrt{74}=\sqrt{c^2}$$
$$c=\sqrt{74}$$

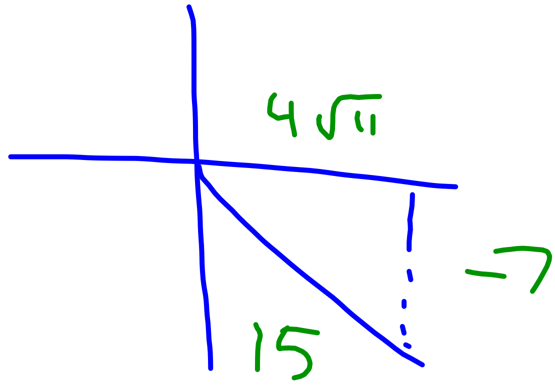
$$a^2+b^2=c^2$$
$$(-7)^2+b^2=15^2$$
$$49+b^2=225$$
$$-49 \qquad -49$$
$$\sqrt{b^2}=\sqrt{176}$$
$$b=4\sqrt{11}$$

u



$$a^2+b^2=c^2$$
$$5^2+(-7)^2=c^2$$
$$25+49$$
$$\sqrt{74}=\sqrt{c^2}$$
$$c=\sqrt{74}$$

v



$$a^2+b^2=c^2$$
$$(-7)^2+b^2=15^2$$
$$49+b^2=225$$
$$-49 \qquad -49$$
$$\sqrt{b^2}=\sqrt{176}$$
$$b=4\sqrt{11}$$

$\sin u = -\frac{4}{7}$ O H $\pi < u < \frac{3\pi}{2}$

9c $\cos(u+v)$

$\cos v = \frac{3}{8}$, A H $0 < v < \frac{\pi}{2}$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

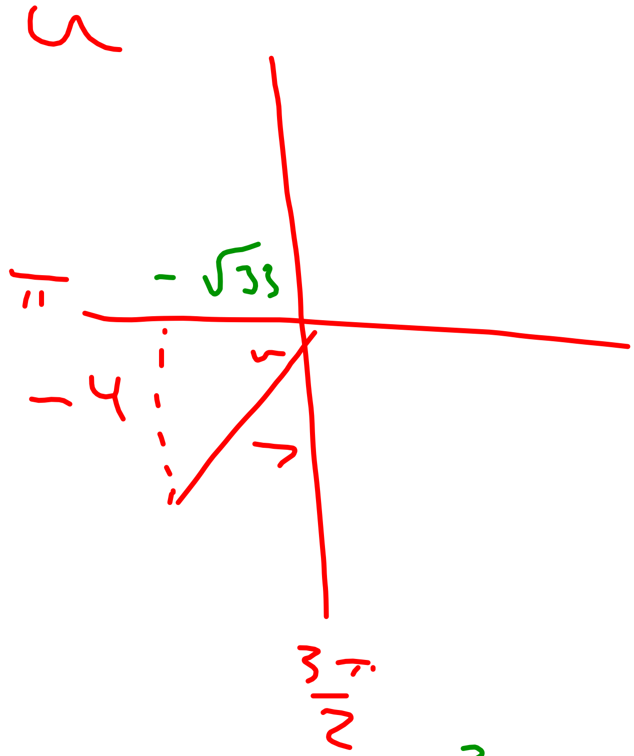
$$\left(\frac{-\sqrt{33}}{7}\right)\left(\frac{3}{8}\right) - \left(\frac{-4}{7}\right)\left(\frac{\sqrt{55}}{8}\right)$$

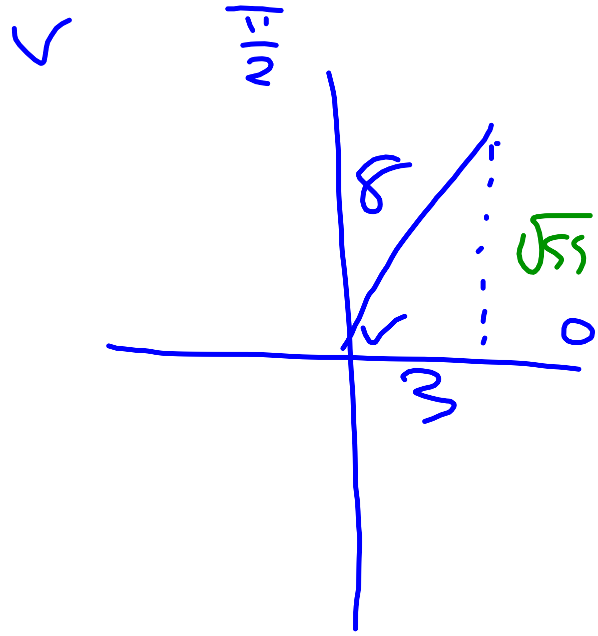
$$\frac{-3\sqrt{33}}{56} - \frac{-4\sqrt{55}}{56}$$

$$\frac{-3\sqrt{33}+4\sqrt{55}}{56}$$

OR

$$\frac{4\sqrt{55}-3\sqrt{33}}{56}$$





$$a^2+b^2=c^2$$
$$(-4)^2+b^2=7^2$$
$$16+b^2=49$$
$$-16 \qquad -16$$
$$\sqrt{b^2}=\pm\sqrt{33}$$
$$b=\sqrt{33}$$

$$a^2+b^2=c^2$$
$$3^2+b^2=8^2$$
$$9+b^2=64$$
$$-9 \qquad -9$$
$$b^2=55$$
$$b=\sqrt{55}$$

9d

$$\sin(u-v)$$

$$\sqrt{33\cdot 55}$$

$$11\cdot 3 \quad 11\cdot 5$$

$$11\sqrt{15}$$

$$=\sin u\cos v-\cos u\sin v$$

$$\left(\frac{-4}{7}\right)\left(\frac{3}{8}\right)-\left(\frac{-\sqrt{33}}{7}\right)\left(\frac{\sqrt{55}}{8}\right)$$

$$\frac{-12}{56}-\frac{-11\sqrt{15}}{56}$$

$$\frac{-12+11\sqrt{15}}{56}$$

9c.

$\sin u = \frac{-4}{7}$ O H $\quad \pi < u < \frac{3\pi}{2}$

$\cos v = \frac{3}{8}$ A H $\quad 0 < v < \frac{\pi}{2}$

$\cos(u+v)$

$= \cos u \cos v - \sin u \sin v$

$= \left(\frac{-\sqrt{33}}{7}\right)\left(\frac{3}{8}\right) - \left(\frac{-4}{7}\right)\left(\frac{\sqrt{55}}{8}\right)$

$\frac{-3\sqrt{33}}{56} - \frac{-4\sqrt{55}}{56}$

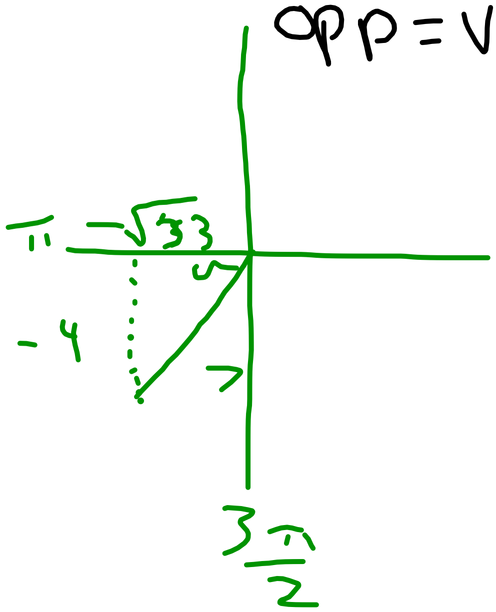
$$\boxed{\frac{-3\sqrt{33}+4\sqrt{55}}{56}}$$

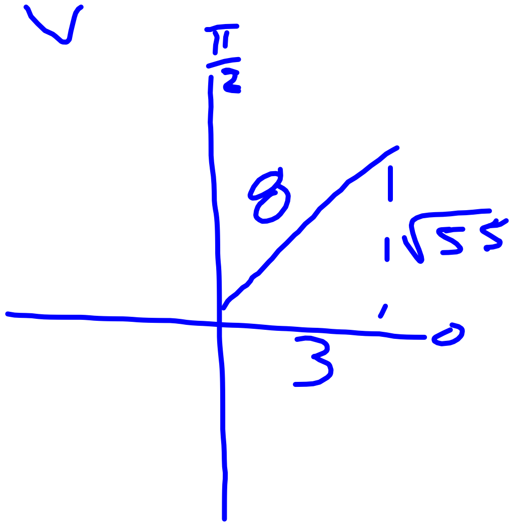
u

adj = hor.

opp = vert.

v





$$(-4)^2+b^2=7^2$$
$$16+b^2=49$$
$$-16 \qquad -16$$
$$\sqrt{b^2}=\sqrt{33}$$
$$\boxed{b=\sqrt{33}}$$

$$3^2+b^2=8^2$$
$$9+b^2=64$$
$$-9 \qquad -9$$
$$\sqrt{b^2}=\sqrt{55}$$
$$b=\sqrt{55}$$

9D

$\sin(u-v)$

$\sqrt{33 \cdot 55}$

$11 \cdot 3 \quad 11 \cdot 5$

$\sin u \cos v - \cos u \sin v$

$$\left(\frac{-4}{7}\right)\left(\frac{3}{4}\right) - \left(\frac{-\sqrt{33}}{7}\right)\left(\frac{\sqrt{55}}{8}\right)$$

$$\frac{-12}{56} + \frac{11\sqrt{15}}{56}$$

$$\frac{-12 + 11\sqrt{15}}{56}$$