

Warm-up

April 11, 2017

Find the exact value.

$$\sin(-255^\circ)$$

$$\sin(60 - 315) =$$

$$\sin(60) \cos(315) - \cos(60)$$

$$\sin(315)$$

$$\left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right) \left(\frac{-\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{-\sqrt{2}}{4}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$



$$\cos(345^\circ)$$

$$\begin{aligned}\cos(300+45) &= \cos^{300} \cos^{45} - \sin^{300} \sin^{45} \\ &= \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} - \frac{-\sqrt{6}}{4} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

$$\sin(w + w) = \sin w \cos w + \cos w \sin w$$

$$\sin(2w) = 2 \sin w \cos w$$

$$\cos(w + w) = \cos w \cos w - \sin w \sin w$$

$$\cos(2w) = \cos^2 w - \sin^2 w$$

$(\cos(w))^2$

$$\sin(r+r) = \sin(r)\cos(r) + \cos(r)\sin(r)$$
$$\sin(2r) = 2\sin(r)\cos(r)$$

Double Angle Identity

Double Angle Formulas:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 1 - 2 \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{2 \tan A}{1 - \tan^2 A}$$

coefficient

exponent

$$\textcircled{2} \quad \tan(60^\circ)$$

$$\tan(2 \cdot 30) = \frac{\sin(2 \cdot 30)}{\cos(2 \cdot 30)}$$

$$= \frac{2 \sin(30) \cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = \frac{\frac{2\sqrt{3}}{4} \left(\frac{\sqrt{3}}{2}\right)}{\frac{3}{4} - \frac{1}{4}} = \frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}}{\frac{2}{4}} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{4} \cdot \frac{2}{1} = \frac{3}{2}$$

$$\textcircled{1} \sin(120)$$

$$\sin(2 \cdot 60) = 2 \sin(60) \cos(60)$$

$$= 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right)$$

$$= \frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$\cos 2A = \underline{\underline{\cos^2 A}} - \sin^2 A$$

Pyth. ID. $\cos^2 A + \sin^2 A = 1$

$$\begin{array}{r} \cos^2 A + \sin^2 A = 1 \\ - \sin^2 A \quad - \sin^2 A \\ \hline \underline{\underline{\cos^2 A}} = 1 - \sin^2 A \end{array}$$

Substitute

$$1 - \sin^2 A - \sin^2 A$$

$$1 - 2\sin^2 A$$

$$\tan(60) = \tan(2 \cdot 30)$$

$$= \frac{\sin(2 \cdot 30)}{\cos(2 \cdot 30)}$$

$$= \frac{2 \sin(30) \cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{2\sqrt{3}}{4}}{\frac{3}{4} - \frac{1}{4}} = \frac{\sqrt{3}}{\frac{2}{4} - \frac{1}{2}}$$

$$\sqrt{3}$$

$$\sin(120^\circ)$$

$$\sin(2 \cdot 60^\circ) = 2 \sin 60 \cos 60$$

$$2 \cdot \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right)$$

$$\frac{2\sqrt{3}}{4}$$

$$\frac{\sqrt{3}}{2}$$

$$\cos^2 A - \sin^2 A$$

Pyth ID $\cos^2 A + \sin^2 A = 1$
 $-\sin^2 A - \sin^2 A$

$$\cos^2 A = \underline{1 - \sin^2 A}$$

$$1 - \sin^2 A - \sin^2 A$$

$$1 - 2\sin^2 A$$

$$\sin(40 - 315)$$

$$\sin 40 \cos 315 - \cos 40 \sin 315$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\sin(L + L) = \sin L \cos L + \cos L \sin L$$
$$\sin(2L) = 2 \sin L \cos L$$

Double Sine Identity

$$\sin(600)$$

$$\sin(2 \cdot 300) = 2 \sin(300) \cos(300)$$

$$2 \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{2}\right)$$

$$\frac{-2\sqrt{3}}{4}$$

$$= -\frac{\sqrt{3}}{2}$$

Half Angle Formulas:

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\sin \alpha}{1 + \cos \alpha} \quad \text{for } \cos \alpha \neq -1$$

$$\begin{aligned} \sin(15^\circ) \\ \sin\left(\frac{30}{2}\right) &= \pm \sqrt{\frac{1 - \cos(30)}{2}} \\ &= \sqrt{\frac{1 - \left(\frac{\sqrt{3}}{2}\right)}{2}} \end{aligned}$$

$$\textcircled{8} \cos(30^\circ)$$

$$8. \cos(30^\circ)$$

$$\cos\left(\frac{60}{2}\right) = \frac{+}{-} \sqrt{\frac{1 + \cos 60}{2}}$$

$$= \sqrt{\frac{2 \cdot 1 + \left(\frac{1}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{3}{2} \cdot \frac{1}{2}}{2}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \boxed{\frac{\sqrt{3}}{2}}$$

$$\sin u = \frac{3}{5} \begin{matrix} O \\ H \end{matrix}, \quad \frac{\pi}{2} \leq u \leq \pi$$

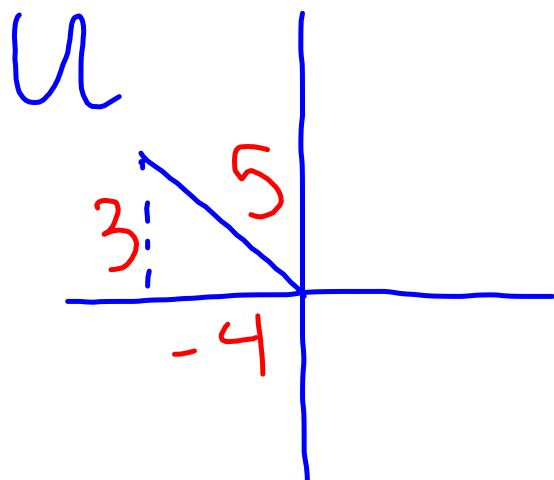
$$\tan v = \frac{5}{12} \begin{matrix} O \\ A \end{matrix}, \quad \pi \leq v \leq \frac{3\pi}{2}$$

$$\textcircled{a} \sin(u+v)$$

$$\sin u \cos v + \cos u \sin v$$

$$\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$$

$$\frac{-36}{65} + \frac{20}{65} = \boxed{\frac{-16}{65}}$$



$$a^2 + b^2 = c^2$$

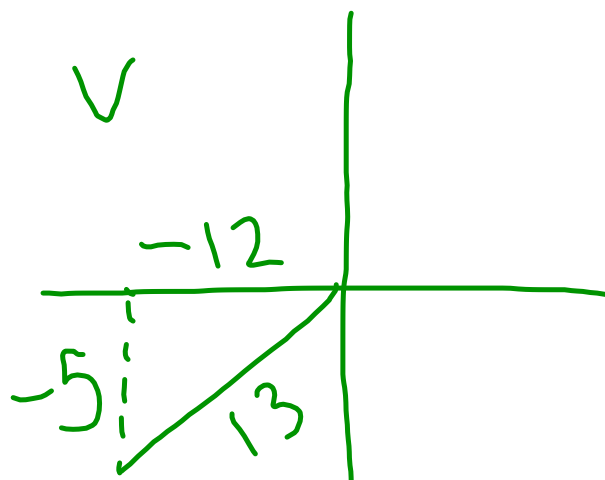
$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$\begin{array}{r} -9 \\ \hline \end{array} \quad \begin{array}{r} -9 \\ \hline \end{array}$$

$$b^2 = 16$$

$$b = 4$$



$$a^2 + b^2 = c^2$$

$$25 + 144 = c^2$$

$$\sqrt{c^2} = \sqrt{169}$$

$$c = 13$$

⑧

$$\cos(30)$$

$$\cos\left(\frac{60}{2}\right) = \sqrt{\frac{1 + \cos(60)}{2}}$$

$$= \sqrt{\frac{\frac{2}{2} + \frac{1}{2}}{2}}$$

$$= \sqrt{\frac{\left(\frac{3}{2}\right) \cdot \frac{1}{2}}{\left(\frac{2}{2}\right) \cdot 2}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

$$\textcircled{2} \quad \tan(60^\circ)$$

$$\tan(2 \cdot 30^\circ) = \frac{2 \sin(30) \cos(30)}{\cos^2(30) - \sin^2(30)}$$

$$= \frac{2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{2\sqrt{3}}{4}}{\frac{3}{4} - \frac{1}{4}}$$

$$= \frac{\cancel{\left(\frac{\sqrt{3}}{2}\right)} \cdot \cancel{2}}{\cancel{2} \left(\frac{1}{2}\right)} = \sqrt{3}$$

$$\sin(120^\circ)$$

$$\sin(2 \cdot 60^\circ) = 2 \sin(60^\circ) \cos(60^\circ)$$

$$= 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right)$$

$$\frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$\underline{\cos^2 A - \sin^2 A}$$

PythID $\rightarrow \cos^2 A + \sin^2 A = 1$

$$\begin{array}{r} \cos^2 A + \sin^2 A = 1 \\ - \sin^2 A \quad - \sin^2 A \\ \hline \cos^2 A = 1 - \sin^2 A \end{array}$$

Substitute

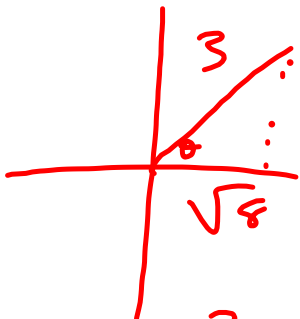
$$\underline{1 - \sin^2 A} - \sin^2 A$$

$$\underline{1 - 2 \sin^2 A}$$

$$1 - 2 \sin^2 A$$

(16) $\sin \theta = \frac{1}{3}$, $0 < \theta < \frac{\pi}{2}$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$



$$a^2 + b^2 = c^2$$

$$a^2 + 1^2 = 3^2$$

$$a^2 + 1 = 9$$

$$\sqrt{a^2} = \sqrt{8}$$

$$a = \sqrt{8}$$

$$= 1 - 2\left(\frac{1}{3}\right)^2$$

$$= 1 - 2\left(\frac{1}{9}\right)$$

$$= \frac{9-2}{9}$$

$$= \frac{7}{9}$$

$$\frac{7}{9}$$

⑤

$$\cos\left(\frac{4\pi}{3}\right)$$

$$\cos(240^\circ)$$

$$\cos(2(120))$$

$$\begin{aligned} &= \cos^2(120) - \sin^2(120) \\ &= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} - \frac{3}{4} \\ &= \frac{-2}{4} = \left(-\frac{1}{2}\right) \end{aligned}$$

Double Angles

①

$$\sin(120)$$

Half of what's given

$$\sin(2(60))$$

$$= 2 \sin(60) \cos(60)$$

$$\frac{2}{1} \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right)$$

$$\frac{2 \sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\cos^2 \theta - \sin^2 \theta$$

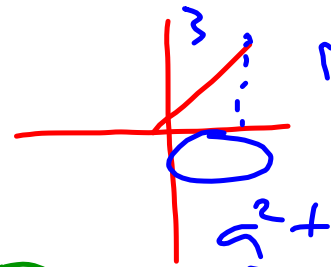
$$\begin{array}{l} \cos^2 + \sin^2 = 1 \\ - \sin^2 \quad - \sin^2 \\ \hline \cos^2 \theta = \underline{1 - \sin^2 \theta} \end{array}$$

$$1 - \sin^2 \theta - \sin^2 \theta$$

$$1 - 2\sin^2 \theta$$

16) $\sin \theta = \frac{1}{3}$, $0^\circ < \theta < 90^\circ$
I

Find $\cos(2\theta)$



$$= 1 - 2 \sin^2 \theta$$

$$= 1 - 2 \left(\frac{1}{3} \right)^2$$

$$= 1 - 2 \left(\frac{1}{9} \right)$$

$$= \frac{9}{9} - \frac{2}{9}$$

$$a^2 + b^2 = c^2$$

$$a^2 + 1^2 = 3^2$$

$$a^2 + 1 = 9$$

$$a^2 = 8$$

$$a = \sqrt{8}$$

$$a = 2\sqrt{2}$$

$\frac{7}{9}$

$$\textcircled{3} \quad \cos\left(\frac{4\pi}{3}\right)$$

$$\cos(240)$$

$$\cos(2(120))$$

$$= \cos^2(120) - \sin^2(120)$$

$$\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\frac{1}{4} - \frac{3}{4}$$

$$\frac{-2}{4} = \textcircled{\frac{-1}{2}}$$

$$\textcircled{1} \sin(120^\circ)$$

Double Angle
↪ use half of given angle

$$\sin(2(60))$$

$$= 2 \sin(60) \cos(60)$$

$$\frac{2}{1} \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right)$$

$$\frac{2 \cdot \sqrt{3}}{4 \cdot 2} = \frac{\sqrt{3}}{2}$$

$$\textcircled{16} \quad \sin \theta = \frac{1}{3}, \quad 0 < \theta < 90^\circ$$

Find $\cos 2\theta$

$$= 1 - 2 \sin^2 \theta$$

$$= 1 - 2 (\sin \theta)^2$$

$$= 1 - 2 \left(\frac{1}{3}\right)^2$$

$$= 1 - 2 \left(\frac{1}{9}\right)$$

$$\frac{9}{9} \cdot \frac{1}{1} - \frac{2}{9}$$

$$\frac{9-2}{9} = \frac{7}{9}$$

three

③

$$\cos\left(\frac{4\pi}{3}\right) \cdot 240$$

$$\cos\left(2\left(\frac{2\pi}{3}\right)\right) \cdot 2(120)$$

$$\cos(2(120))$$

$$= (\cos(120))^2 - (\sin(120))^2$$

$$\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\frac{1}{4} - \frac{3}{4}$$

$$\frac{1-3}{4} = \left(-\frac{1}{2}\right)$$

① Double angle *half it*

$$\sin(120^\circ)$$
$$\sin(2(60)) = 120$$
$$= 2 \sin(60) \cos(60)$$
$$= 2 \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{2} \right)$$
$$= \frac{2 \sqrt{3}}{4}$$
$$= \frac{\sqrt{3}}{2}$$

$$\sin u = \frac{3}{5}, \quad 0 < u < \frac{\pi}{2} \quad \text{SUM/DIFF Identities}$$

$$\cos v = -\frac{7}{10}, \quad \pi < v < \frac{3\pi}{2}$$

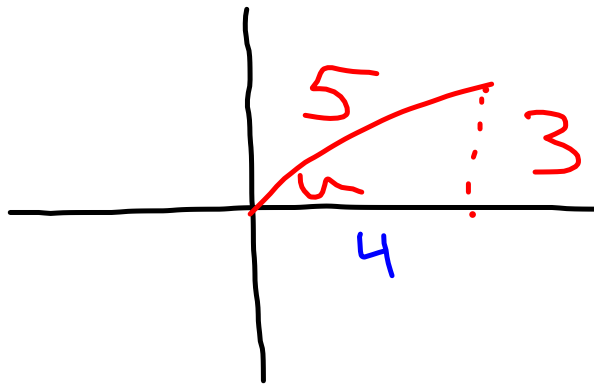
Find sin(u+v).

$$= \sin u \cos v + \cos u \sin v$$

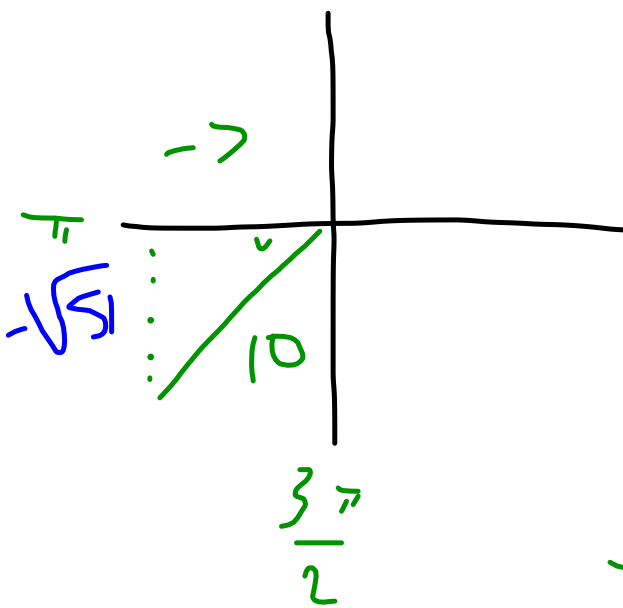
$$\left(\frac{3}{5}\right) \left(-\frac{7}{10}\right) + \left(\frac{4}{5}\right) \left(-\frac{\sqrt{51}}{10}\right)$$

$$\frac{-21}{50} + \frac{-4\sqrt{51}}{50}$$

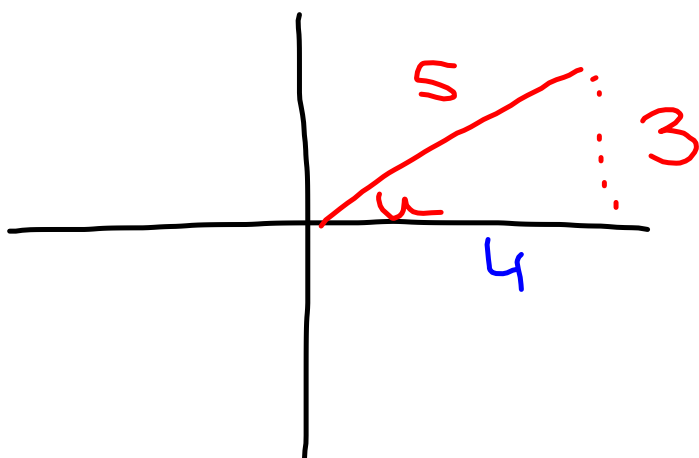
$$\frac{-21 - 4\sqrt{51}}{50}$$



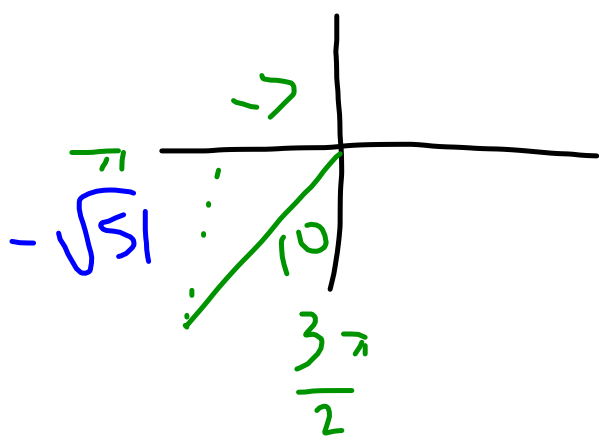
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 a^2 + 3^2 &= 5^2 \\
 a^2 + 9 &= 25 \\
 -9 &\quad -9 \\
 \hline
 \sqrt{a^2} &= \sqrt{16} \\
 a &= 4
 \end{aligned}$$



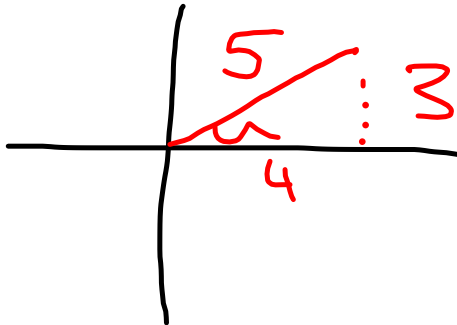
$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (-7)^2 + b^2 &= 10^2 \\
 49 + b^2 &= 100 \\
 -49 &\quad -49 \\
 \hline
 \sqrt{b^2} &= \sqrt{51} \\
 b &= \sqrt{51}
 \end{aligned}$$



$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 3^2 + b^2 &= 5^2 \\
 9 + b^2 &= 25 \\
 -9 &\quad -9 \\
 \hline
 \sqrt{b^2} &= \sqrt{16} \\
 b &= 4
 \end{aligned}$$



$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 (-7)^2 + b^2 &= 10^2 \\
 b &= \sqrt{51}
 \end{aligned}$$



$$a^2 + b^2 = c^2$$

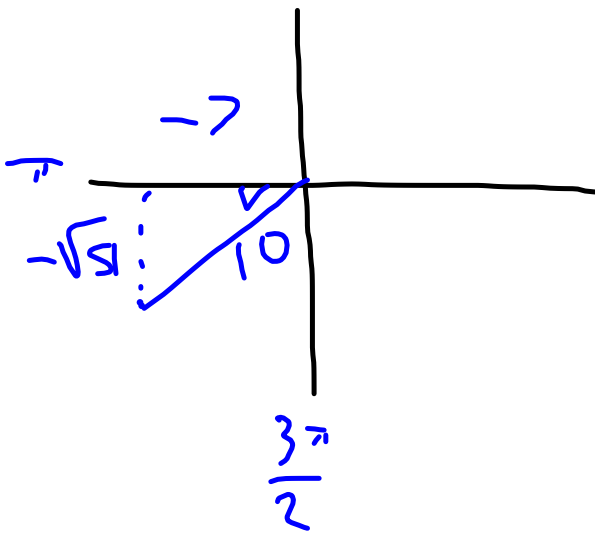
$$(3)^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$\begin{array}{r} -9 \quad -9 \\ \hline \end{array}$$

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$



$$a^2 + b^2 = c^2$$

$$(-7)^2 + b^2 = 10^2$$

$$49 + b^2 = 100$$

$$\begin{array}{r} -49 \quad -49 \\ \hline \end{array}$$

$$\sqrt{b^2} = \sqrt{51}$$

$$b = \sqrt{51}$$

$$\textcircled{9A} \sin(u+v)$$

$$\sin u = \frac{3}{5} \quad 0 < u < \frac{\pi}{2}$$

$$\tan v = \frac{5}{12} \quad \frac{\pi}{2} < v < \frac{3\pi}{2}$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$$

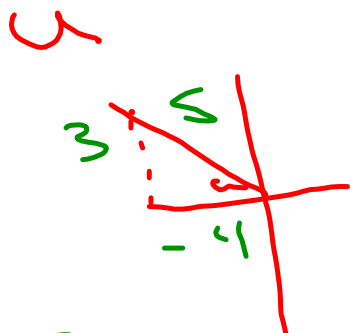
$$-\frac{36}{65} + \frac{20}{65}$$

$$\sin(u+v) = -\frac{16}{65}$$

$$\cos(u+v) = \cos^u \cos^v - \sin^u \sin^v$$
$$\left(\frac{-4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right)$$

$$\frac{48}{65} - \frac{-15}{65}$$

$$\frac{63}{65}$$



$$a^2 + b^2 = c^2$$

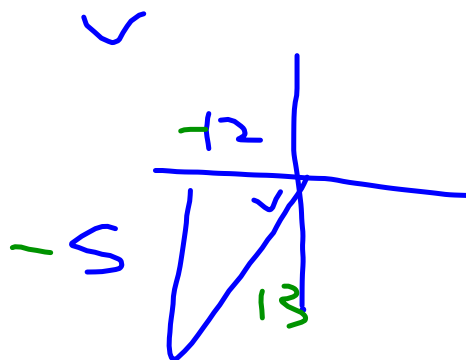
$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$-9 \quad -9$$

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$



$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$169 = c^2$$

$$c = 13$$

$$\sin \theta = \frac{7}{-25}$$

$$270 < \theta < 360$$

$$\sin(u+v)$$

$$\sin u = \frac{3}{5} \quad \frac{\pi}{2} < u < \pi$$

$$\tan v = \frac{5}{12} \quad \pi < v < \frac{3\pi}{2}$$

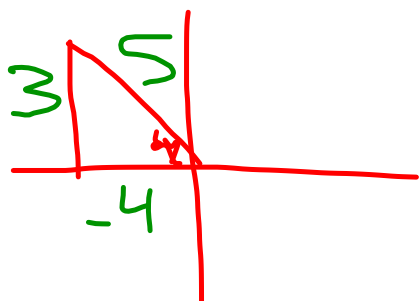
$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$$

$$-\frac{36}{65} + \frac{20}{65}$$

$$\frac{-16}{65}$$

u



$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 5^2$$

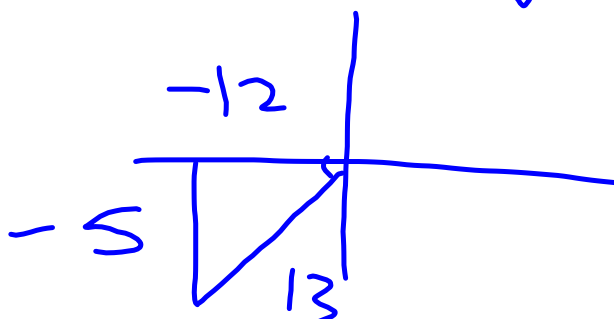
$$9 + b^2 = 25$$

$$\begin{array}{r} -9 \quad -9 \\ \hline \end{array}$$

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$

v



$$a^2 + b^2 = c^2$$

$$(-5)^2 + (-12)^2 = c^2$$

$$25 + 144$$

$$169 = c^2$$

$$c = 13$$

