Proving Triangles Congruent

So far, we have answered questions about triangles that we have been told are congruent. **But what if we are not told whether or not they are the same?** There are a few ways that we can show that the triangles MUST be the same. These ways are called **theorems** or **postulates**. If we have enough information to show that one of the theorems or postulates is reprented, **we can PROVE that two triangles are congruent.**

Congruence Postulate #1

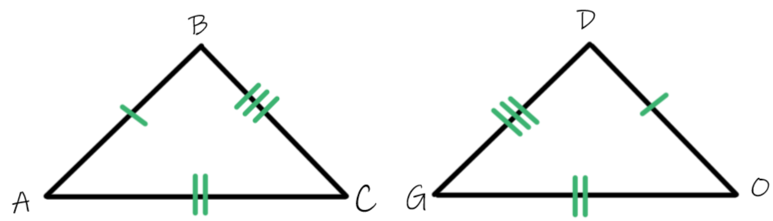
Side Side Side (SSS): If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

Example 1:

a. From the diagram we see:

$\overline{AB} \cong \overline{OD}$ and

$\overline{BC} \cong \overline{DG}$ and $\overline{CA} \cong \overline{OG}$



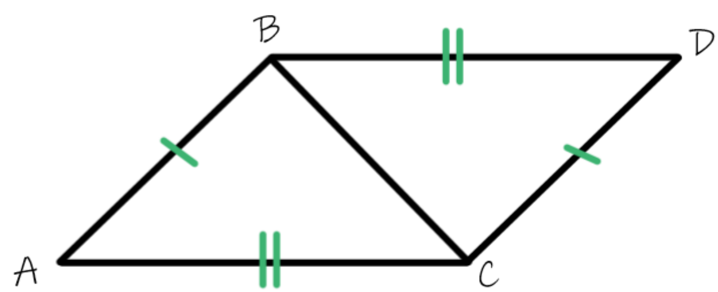
Therefore... $\triangle ABC \cong \triangle ODG$ by SSS

b. Let's take a look:

From the diagram, we know that

$\overline{AB} \cong$ ______ and $\overline{AC} \cong$ ______

But that's only 2 sides, and we need three.



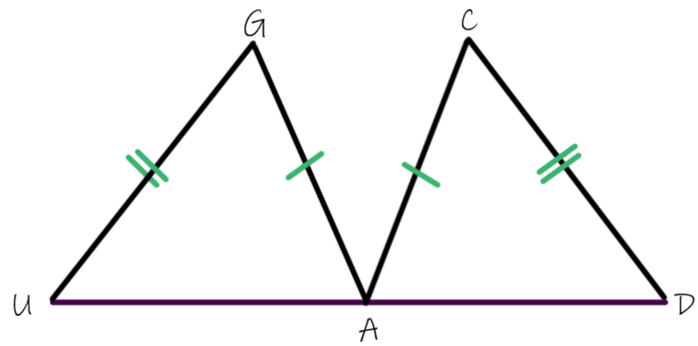
What do you notice about the third side of each triangle? ____

Anytime we add something to a diagram, we **must** have a property or justification!

When two triangles are sharing a side length we can use the __________ property to show that it is congruent to itself! Therefore: ____________

Now we can prove: $\triangle ABC \cong$ ______ by the __________ postulate.

c. Another property we'll see with side lengths...



Given: **A is the midpoint of** $\overline{DU}$, Can we prove that $\triangle UGA \cong \triangle DCA$?

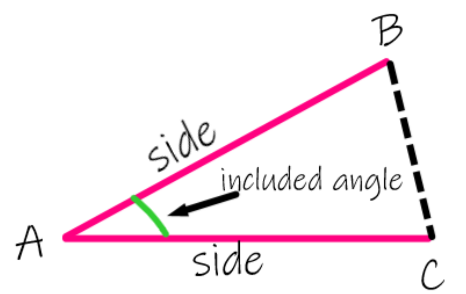
So... we know that: $\overline{UG} \cong$ _____ and $\overline{GA} \cong$ _____. That's only two sides, so we are going to need another side length. The **given** information in the problem says that **A** is the **midpoint** of $\overline{DU}$.

Define Midpoint: ____

Based on this definition, know we know __________ $\cong$ ____________.

Therefore $\triangle UGA \cong \triangle DCA$ by ____________.

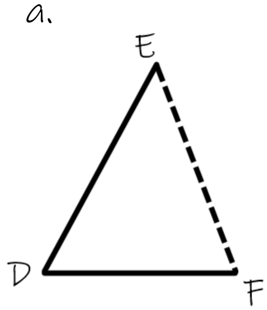
Our next postulate with involve using some angles, so we need to understand some vocabulary first. When two sides of a triangle meet, they form an angle. The angle where two sides meet is called their "**included**" angle.



- In the diagram to the right, angle A is the included angle of sides _______ and ________.

You try: Identify the included angle of the solid sides of each triangle.

2. a.



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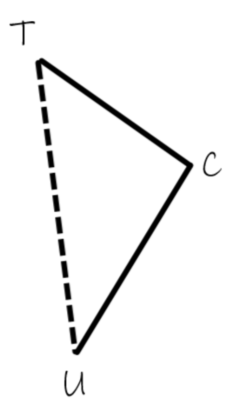
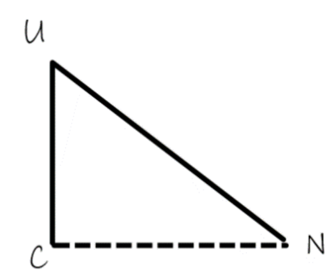
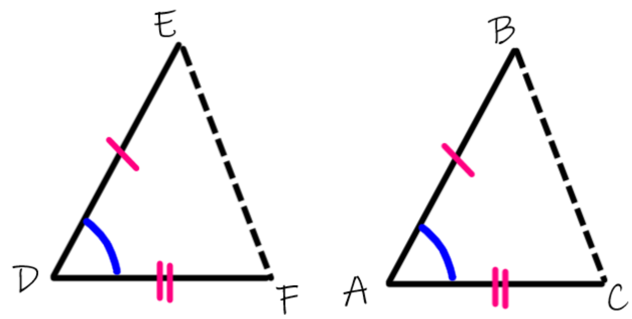


c.



______________________ ______________________ ______________________

Let's Take a closer look:



- From the diagram we can see that $\overline{DE} \cong \overline{AB}$ and $\overline{DF} \cong \overline{AC}$
- We can also see that the **Included angle** between the sides is marked as congruent.
- Anytime you have **two fixed distances** (your solid sides) bound by the **same angle** (the included angle) the distances it takes to connect those endpoints (E to F) or (B to C) will always be the same!

This means anytime you see two **congruent sides** with **congruent included angles** you know that the two triangles MUST be congruent.

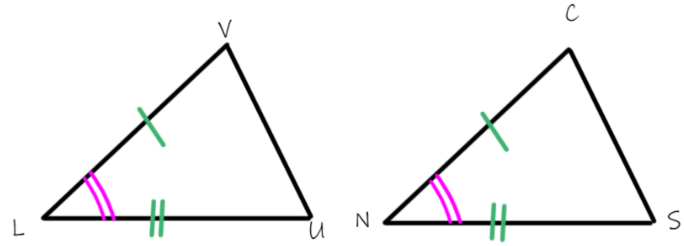
When we prove triangles congruent this way, we are using our second postulate:

Congruence Postulate #2

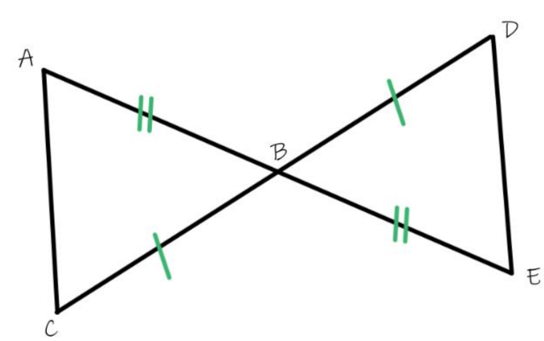
Side Angle Side (SAS): It two sides and the included angle of one triangle are congruent two sides and the included angle of another triangle, the two triangles will be congruent.

3. **a.** From the diagram we see: $\overline{LV} \cong \overline{NC}$ and $\overline{LU} \cong \overline{NS}$ and the included angles: $\angle L \cong \angle N$

Therefore... $\triangle LVU \cong \triangle NCS$ by SAS



b. Lets Take a Look:



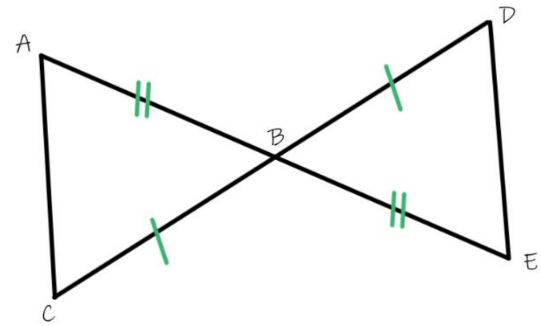
From the diagram, we know that

$\overline{AB} \cong$ ______ and $\overline{BC} \cong$ ______

But that's only 2 sides. We either need another side for SSS or the included angle for SAS.

Remember, we MUST have a property or justification to add anything to our diagram.... Do you notice anything about the included angles that we know from previous lessons? ____________________________

3b Continued:

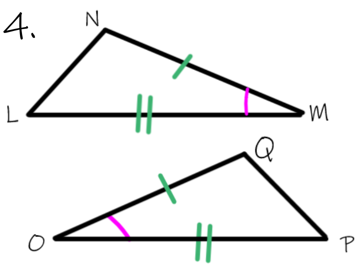


Any time you see __________ __________ you can add a marking for them into your diagram!

Now we know that ∠__________ ≅ ∠____________.
Since these are the included angles, we can now say that △ABC ≅ ______ by the ________ Postulate.

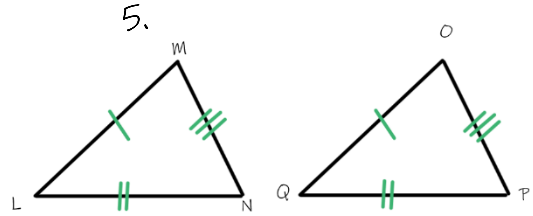
You Try: Decide which congruence postulate can be used for each pair of triangles below. If they are congruent, write a congruence statement. If neither postulate can be used, put an "X" in each blank.

4.



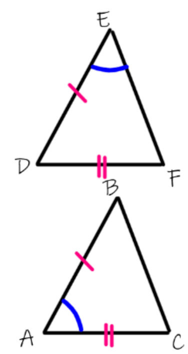
△LMN ≅ ______ by ______

5.



△LMN ≅ ______ by ______

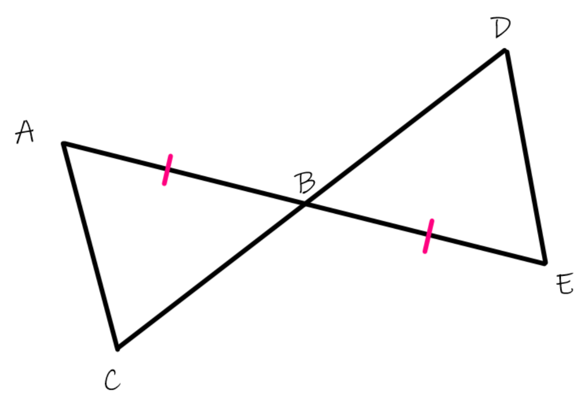
6.



△DEF ≅ ______ by ______

Challenge Problem! Putting it all together 😊

7. Given: **B** is the **Midpoint** of $\overline{CD}$.



From the diagram we know.... $\overline{AB}$ ≅ _____

- What can we mark because of the midpoint?

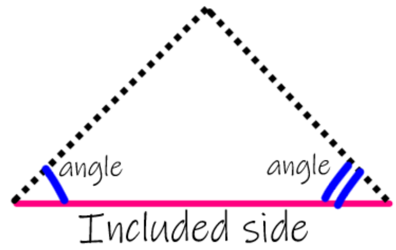
  _________ ≅ _________

- We can mark _________ ≅ _________ because they are ________ __________.

Therefore: △ABC ≅△ _______ by ________

Vocabulary to help us with our next postulate:

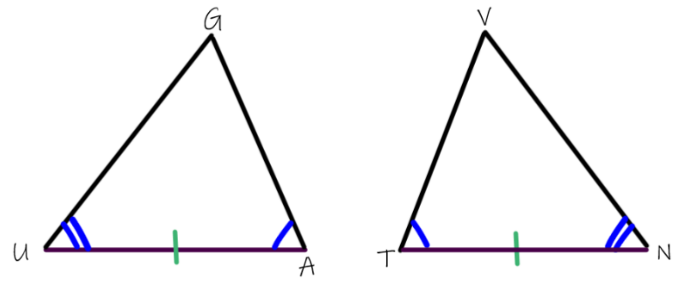
The side length that is between two angles is called the **included side.**



Postulate #3

Angle Side Angle (ASA)- If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

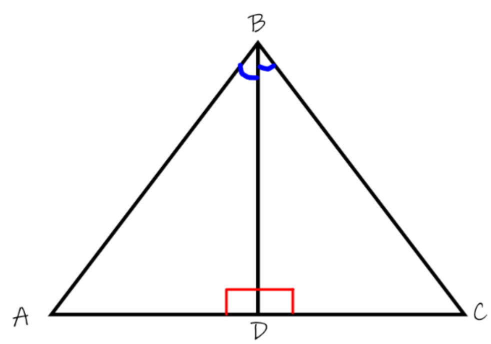
8. Example:



From the diagram we see: $\angle U \cong \angle N$ and $\angle A \cong \angle T$ and the included sides $\overline{UA} \cong \overline{NT}$

Therefore... $\triangle UGA \cong \triangle NVT$ by ASA

9. . **Lets Take a Look:**



From the diagram, we know that

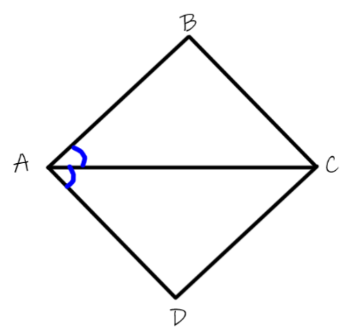
$\angle ABC \cong \angle$ ______ and $\angle ADB \cong$ ______

But that's only two angles. We the included sides to be congruent for ASA.

Remember, we **MUST** have a property or justification to add anything to our diagram...Do you see anything we are allowed to mark?

Since ________ $\cong$ _________ by the ________ _________,

Therefore $\triangle ABD \cong \triangle$ ________ by _________.



10. Another Property we might see dealing with angles:

Given: $\overline{AC}$ is an **angle bisector** for $\angle BCD$.

Can you prove the two triangles are congruent? We know....

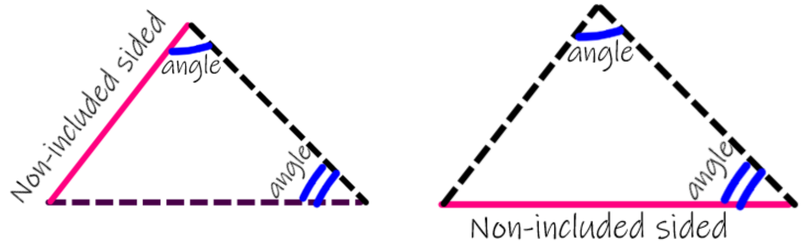
$\angle BAC \cong \angle$ ____ from the diagram, and ________ $\cong$ __________ by the reflexive Property.

Define Angle Bisector: ____

Therefore, $\angle BCA \cong \angle$ ____ and that means that $\triangle ABC \cong \triangle$ _______ by ________

Vocabulary for our next postulate:

A side length that is not directly in between to angles is called a **non-included** side.

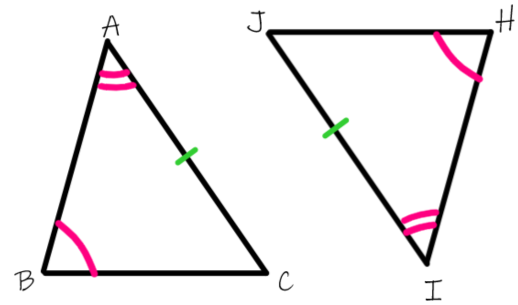


Theorem #4

Angle Angle Side(AAS): If two angles and a non-included side of one triangle are congruent the two angles and the corresponding non-included side of another triangle, the two triangles are congruent.

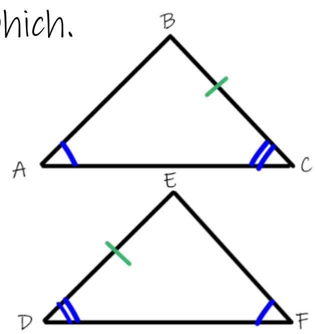
Example:

11. From the diagram we see: $\angle A \cong \angle I$ and $\angle B \cong \angle H$ and the corresponding **non-included** sides $\overline{AC} \cong \overline{IJ}$ Therefore... $\triangle ABC \cong \triangle IJH$ by AAS



12. Two of the examples below are examples of AAS, one is an example of ASA. Decide which is which.

a.



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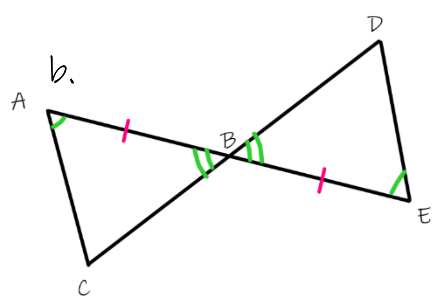
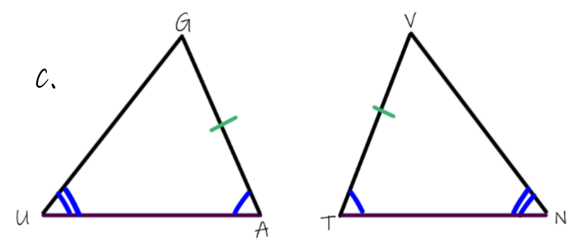


c.



$\triangle ABC \cong$ ______ by ______

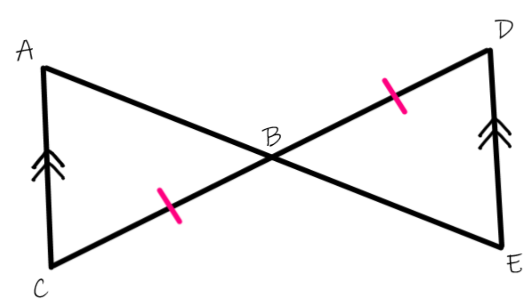
$\triangle ABC \cong$ ______ by ______

$\triangle UGA \cong$ ______ by ______

13. Another property we may see with angles...

Given: $\overline{AC} \parallel \overline{DE}$, prove the two triangles congruent.

We know.... $\overline{CB} \cong \overline{DB}$ from the diagram, and ________ $\cong$ __________ because they are vertical angles. Since $\overline{AC} \parallel \overline{DE}$, what kind of angles are $\angle A$ and $\angle E$? ____________________________

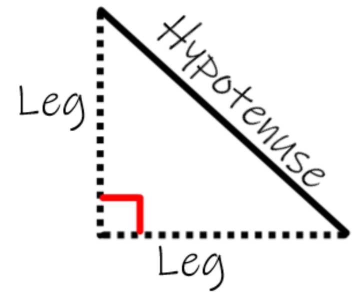


Therefore, $\angle A \cong \angle$ ___ and that means that $\triangle ABC \cong \triangle$ ______ by ________.

$\angle C$ and $\angle D$ are also ________ _________, so there is more than one correct way to do this one. 😊

Recall: Right Triangles

In a Right triangle, the side lengths that form the right angle are called the _______ of the triangle, and the side opposite the right angle is called the ___________.

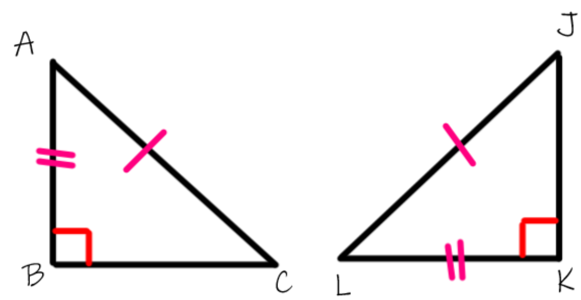


- Right triangles have many special properties! We have a triangle congruence theorem that works ONLY for right triangles!
- All our other postulates and theorems work for right triangles too! Right triangles just have an extra on that is special just for them.

Hypotenuse Leg (HL): If the hypotenuse and one leg in a right triangle are congruent to the hypotenuse and one leg of another right triangle, the two triangles are congruent.

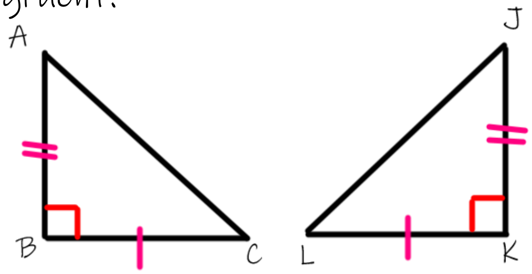
Example:

14. From the diagram we see: $\angle B$ and $\angle K$ are both right angles, making these right triangles. $\overline{AB} \cong \overline{LK}$. These are the ______ of the right triangles. $\overline{AC} \cong \overline{LK}$ These segments are the ___________ of the right triangles. Therefore... $\triangle ABC \cong \triangle LKJ$ by HL

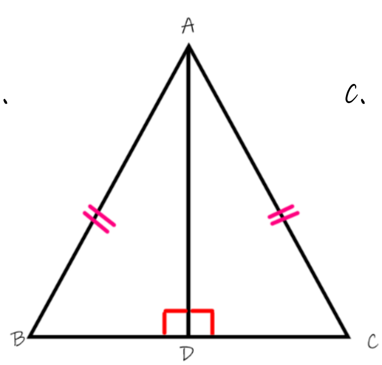


15. You Try! Determine which postulate or theorem you can use to prove the triangles congruent.

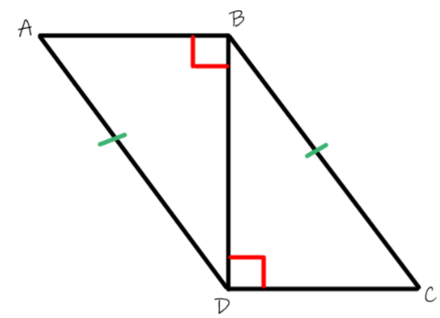
a.



b.

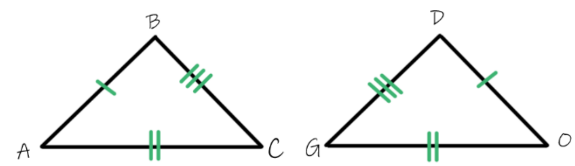
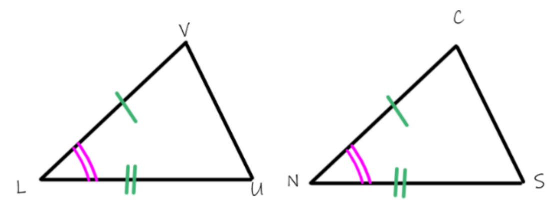
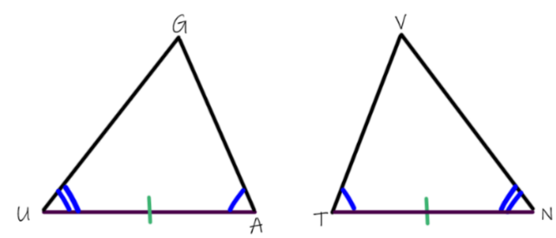
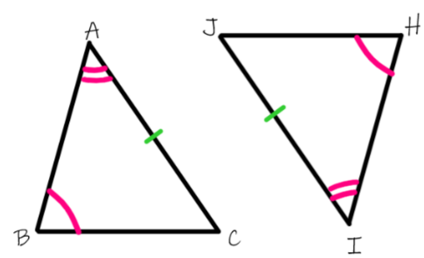
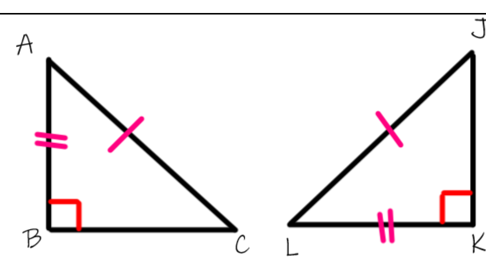


c.



d. True or False: HL is the only method to prove that two right triangles are congruent.

Congruent Triangles Summary

<u>Side Side Side (SSS):</u> If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.	 $\triangle ABC \cong$ ________
<u>Side Angle Side (SAS):</u> If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, the two triangles will be congruent.	 $\triangle LVU \cong$ ________
<u>Angle Side Angle (ASA)-</u> If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent	 $\triangle UGA \cong$ ________
<u>Angle Angle Side(AAS):</u> If two angles and a non-included side of one triangle are congruent the two angles and the corresponding non-included side of another triangle, the two triangles are congruent.	 $\triangle ABC \cong$ ________
<u>Hypotenuse Leg(HL):</u> If the hypotenuse and one leg in a right triangle are congruent to the hypotenuse and one leg of another right triangle, the two triangles are congruent.	 $\triangle ABC \cong$ ________