Transformation Rules

<u>Translation-</u> moves every point of a figure by the same distance in a given direction. We can "slide" a point or a figure left, right, up or down.

- Right: (x,y)→(x+a, y) This will shift the point "a" units **right**
- Left: (x,y)→(x-a, y) This will shift a point "a" units **left**.
- Up: (x,y)→(x, y+b) This will shift a point "b" units **up**
- Down: (x,y)→ (x, y-b) This will shift a point "b" units **down**.

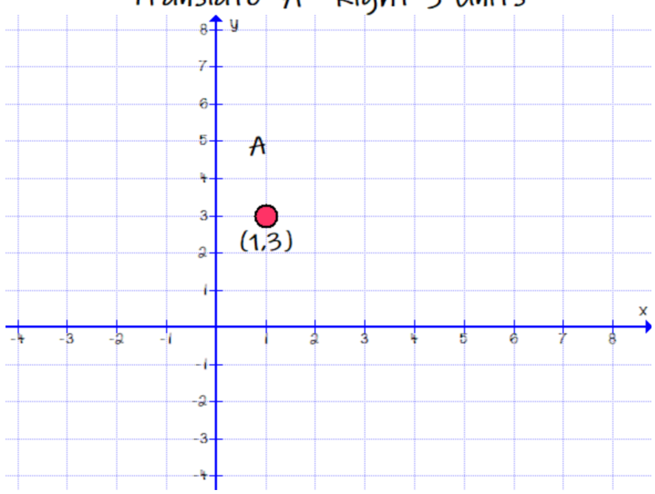
Define:

<u>Pre-Image-</u> ____.

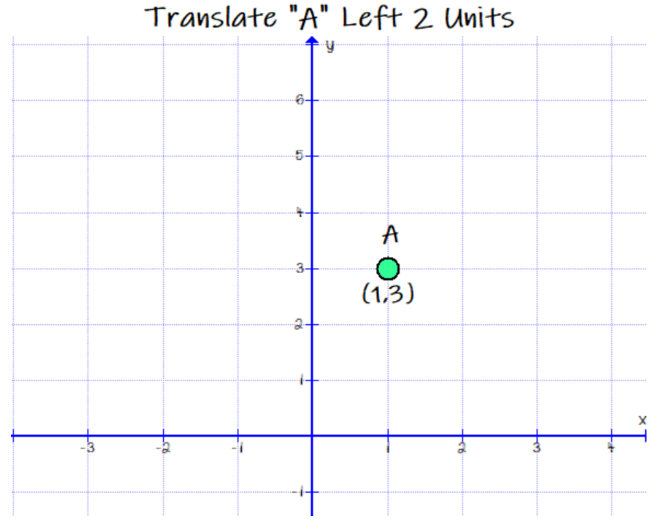
<u>Image-</u> ____.

Examples:

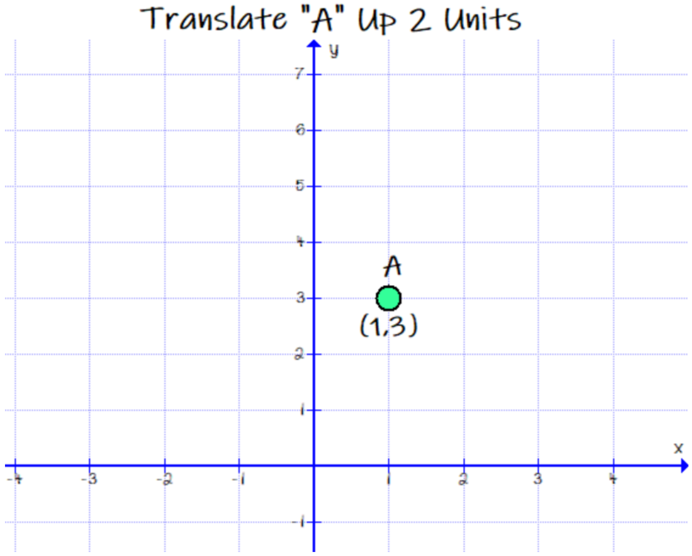
Translate "A" Right 3 Units



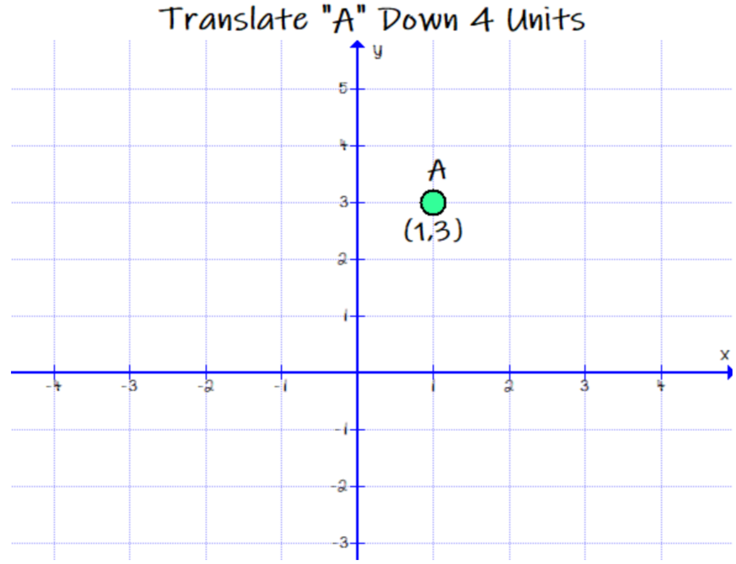
Translate "A" Left 2 Units



Translate "A" Up 2 Units

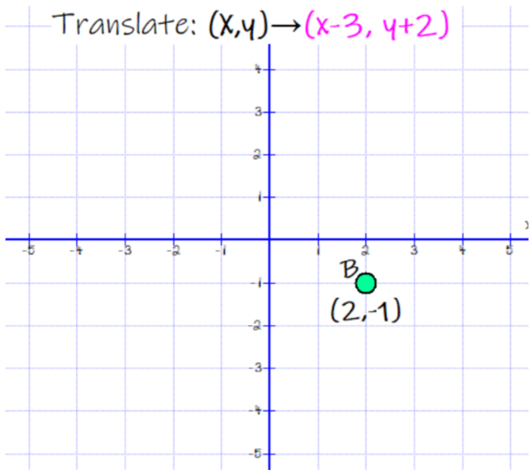


Translate "A" Down 4 Units

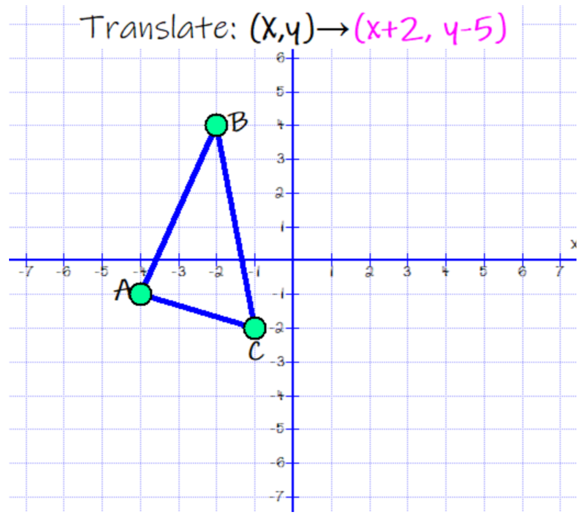


You Try!

1.



2.



3. **Working Backwards:** The coordinates shown were translated by the rule (x,y)→(x+5, y-2). What were the coordinates of the pre-image?

A (,) →A'(2,5)

B (,)→B' (4, 7)

C (,)→C' (5,-1)

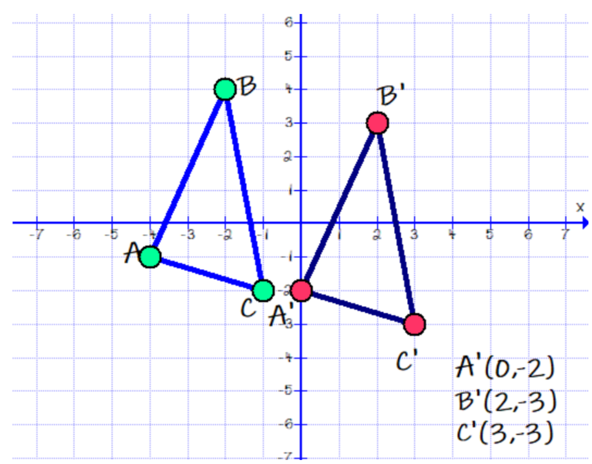
4. **Writing a rule**: Write a rule that would produce the translation shown below.

a. A (3,7) → A' (-5, 4) Rule: (x,y)→ ___________

b. B (4, 5) → B' (9, -2) Rule: (x,y)→ ___________

c. Using the figure, determine the rule for the translation that has occurred.

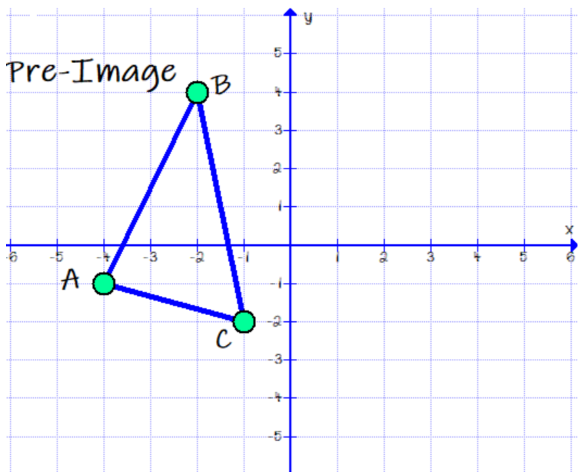
Rule: (x, y) → ___________

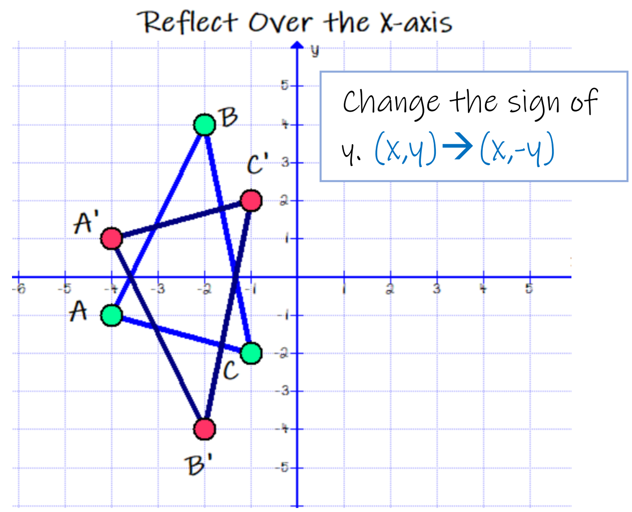


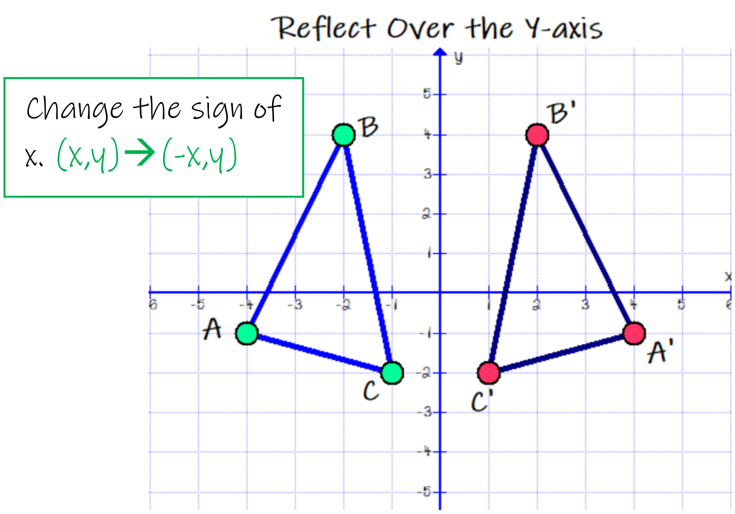
Reflections: A reflection "flips" a point or a figure over a given line. All the points of the image will be the same distance away from the line of reflection as the pre-image, just on the opposite side of the line.

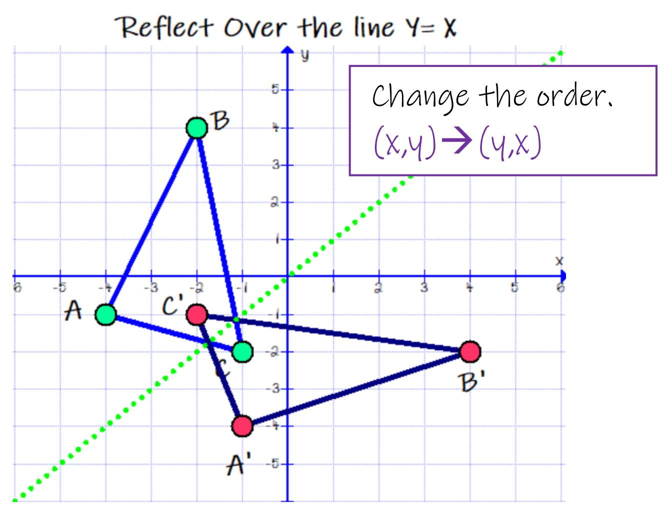
- Reflect over x-axis: Change the sign of y. (x,y)→(x,-y)
- Reflect over y-axis: Change the sign of x. (x,y)→(-x, y)
- Reflect over the line y = x: Change the order. (x,y)→(y,x)
- Reflect over the line y = -x: Change the order and the signs. (x,y)→(-y,-x)

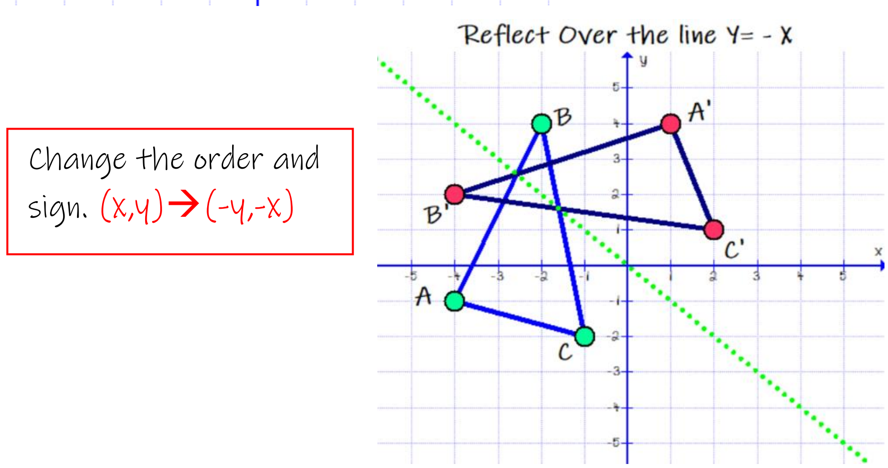
Examples:





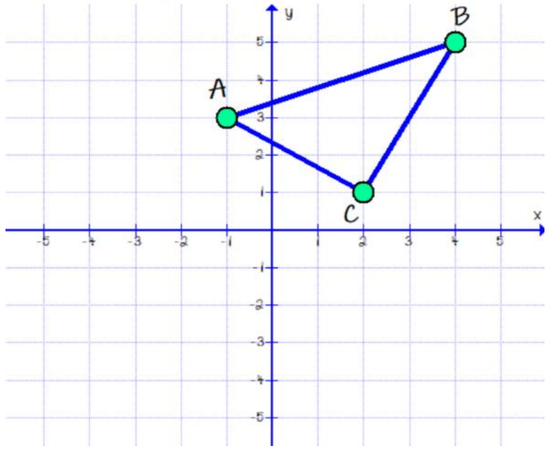




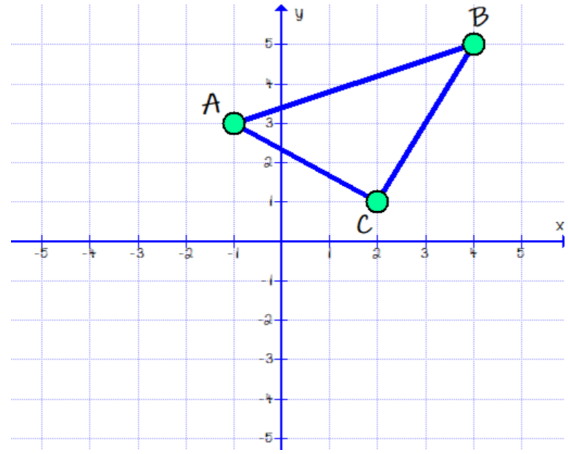


You Try!

1. Reflect Over the X-axis



2. Reflect Over the Y-axis



3. Apply the given reflection to the coordinates below.

a. Reflect over y = x

A(1,2) → A' ________

b. Reflect over y = -x

B(3,-4) → B' ________

c. Reflect over x-axis

C(-3,-2)→ C' ________

4. Determine the line of reflection:

a. Given the coordinate:

A(1,2)→A'(-2,-1)

____

b. Given the coordinate:

B(3,-4)→B'(-3,-4)

____

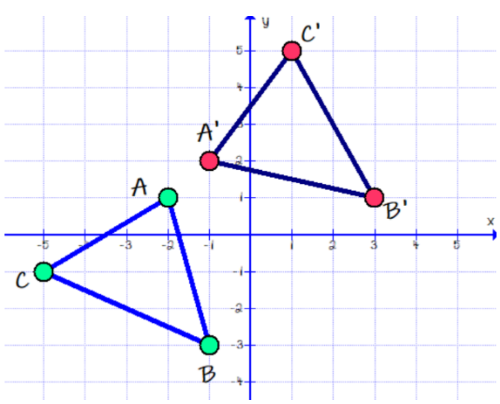
c. Given the coordinate:

C(-3,-2)→ C'(-2,-3)

____

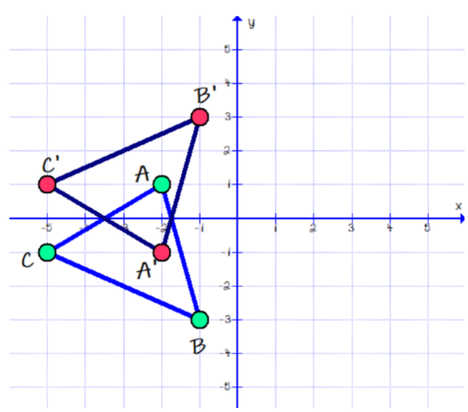
5. Determine the line of reflection from the figures:

a.



____

b.



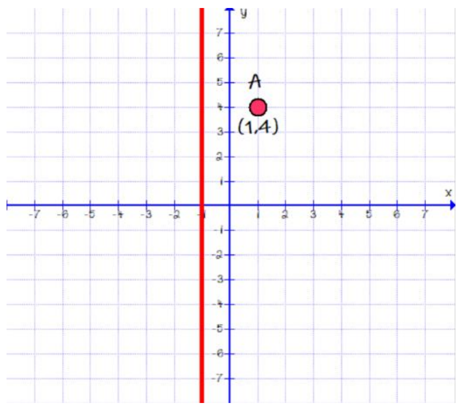
____

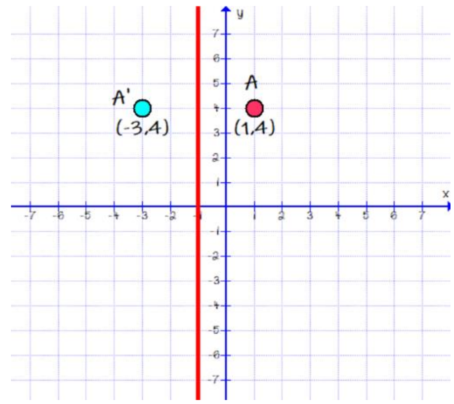
Reflecting over a given line: Mirror the points the same distance away on the other side

X = # is always a vertical line! Y = # is always a horizonal line!

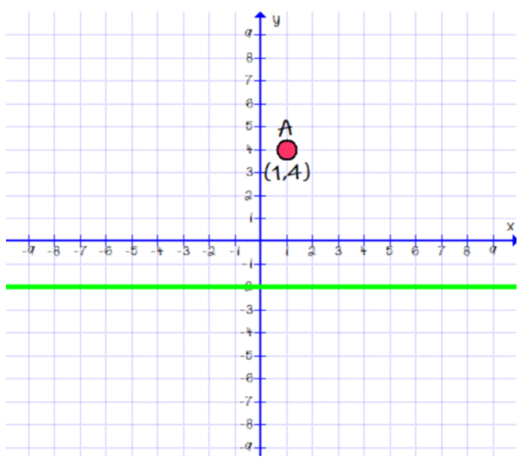
Examples:

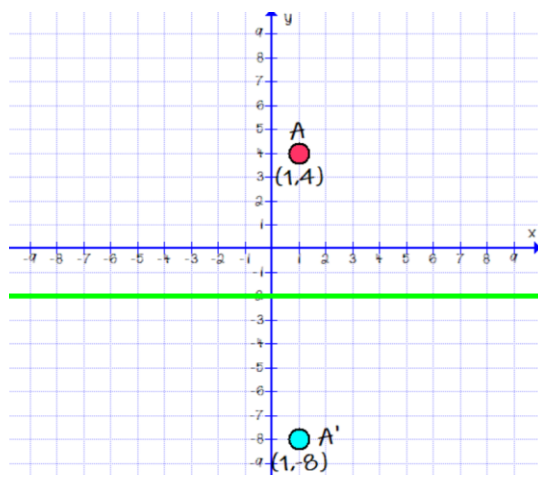
a. Reflect the point A over the line **x = -1**. "A" is two units away from the line **x = -1**, so we place A' two units away from **x = -1**, on the opposite side of the line.



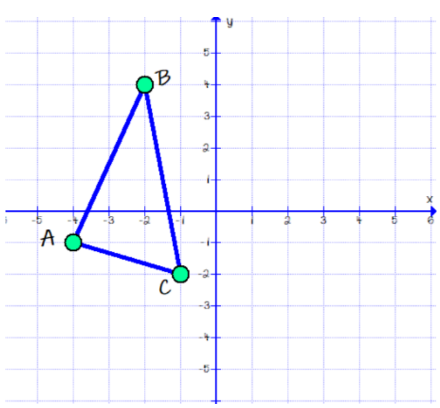


b. Reflect the point A over the line **y = -2**. The point A is six units from the line **y = -2**, so we place A' six units away from **y = -2** on the opposite side.

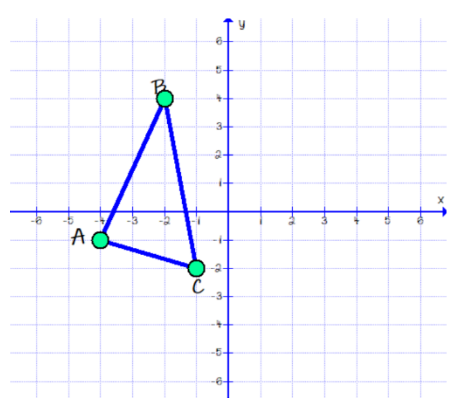




You Try! A. Reflect $\triangle ABC$ over the line y = 1.



B. Reflect $\triangle ABC$ over the line x = 1.



Rotations: When we rotate a point or figure, we are turning it about a fixed point called the **center of rotation.** We will assume that the center of rotation is the origin unless otherwise specified.

- When the center of rotation is the origin, we have a set of rules we can apply to our coordinate.
- The direction of rotation is understood to be **counter-clockwise** unless otherwise specified.

90 Degrees CCW is the same as 270 CW

- Use the rule $(x,y) \rightarrow (-y,x)$

270 Degrees CCW is the same as 90 CW

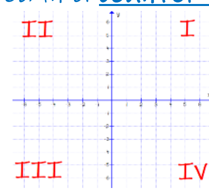
- Use the rule $(x,y) \rightarrow (y,-x)$

180 Degrees is the same in both directions

- Use the rule $(x,y) \rightarrow (-x,-y)$

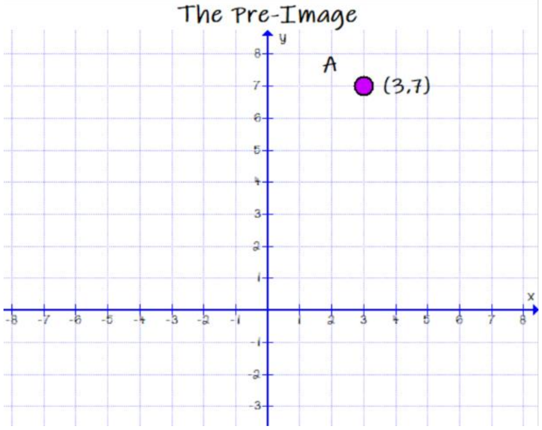
Why **Counter Clockwise??**

The quadrants of the coordinate plane are *numbered* in a counter clockwise direction.

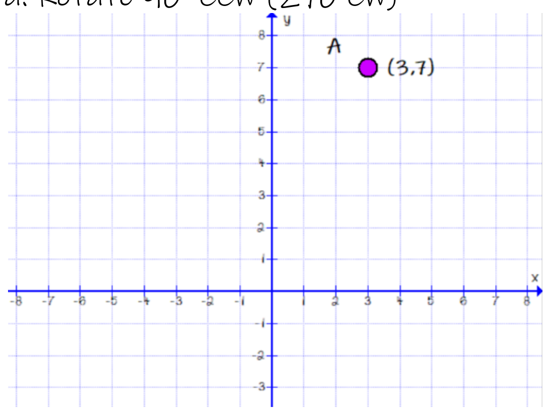


Examples with one point: A is the point (3,7). Let's look at what happens to it as we rotate.

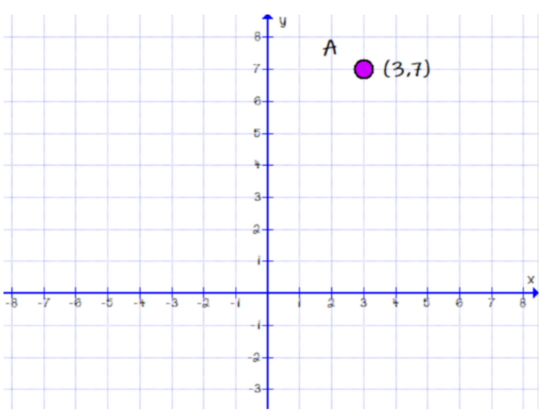
The Pre-Image



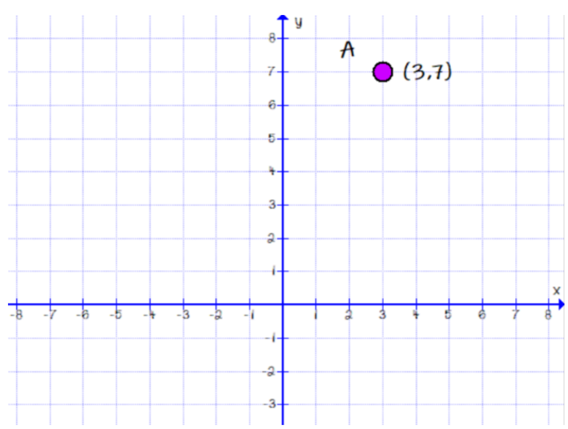
a. Rotate 90° CCW (270 CW)



b. Rotate 270° CCW (90CW)

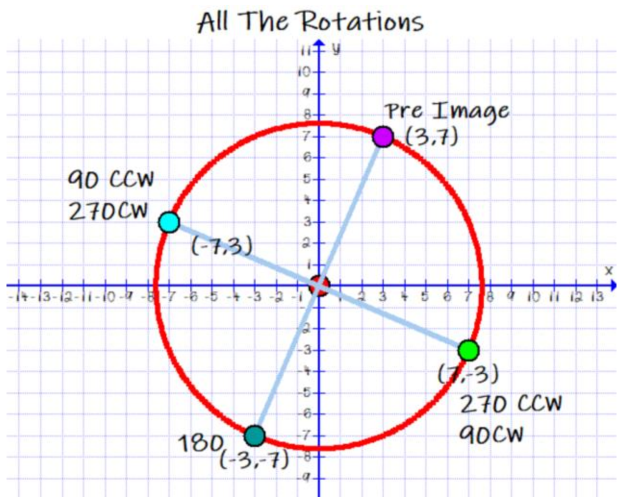


c. Rotate 180°



Look in Depth: What's really happening as we rotate?

- We are dragging our point along an imaginary circle! Each time we preform a rotation, the point will be the same distance away from the center of rotation, just a different spot!
- If we want to rotate about a point that is not the origin, the process is a little bit different….



When the center of rotation is NOT the origin, here's what we can do:

1. Subtract the center of rotation from your coordinate. This shifts the center of rotation back to the origin, allowing us to use our rules.
2. Apply the rule.
3. Add the center of rotation back to your coordinate. This shifts the center of rotation back to the right spot.

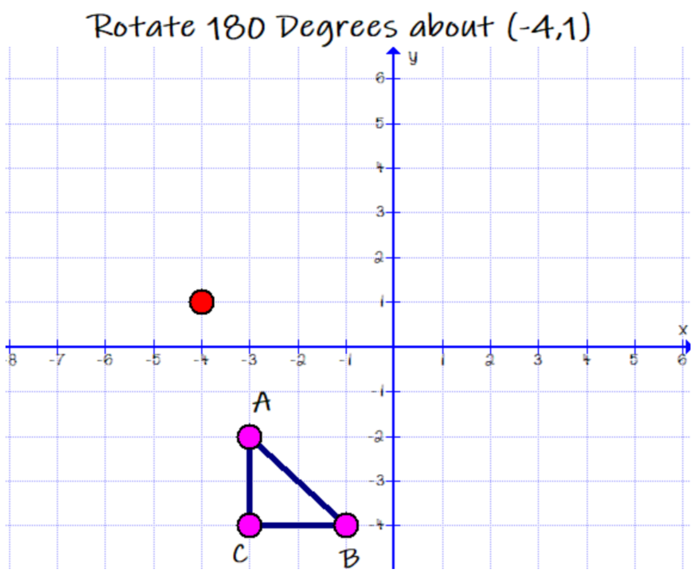
Take a Look: Rotate $\triangle ABC$ 180° about the point (-4,1)

1. Subtract the center of rotation from each coordinate:

A (-3,-2) becomes (-3 – -4 , -2 – 1) = A*(_______)

B(-1,-4) becomes (-1 – -4 , -4 – 1) = B*(________)

C (-3,-4) becomes (-3 – -4 , -4 – 1) = C*(_______)



2. Apply the Rule: 180 degrees (x,y)→(-x,-y)

A*(______) becomes A**(_________)

B*(______) becomes B**(_________)

C*(______) becomes C**(_________)

3. Add the Center of Rotation back in!

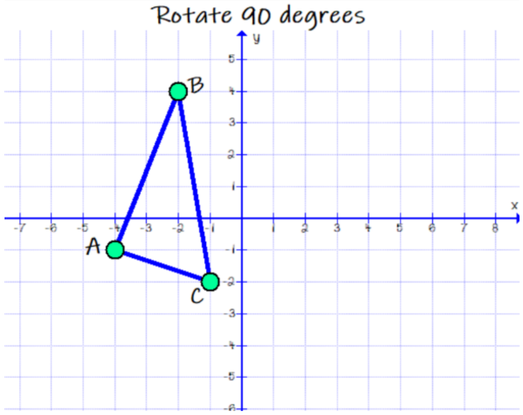
A**(________) becomes (____+ -4, ____+ 1)= A'(_______)

B**(________) becomes (____+ -4, ____+1)= B'(_______)

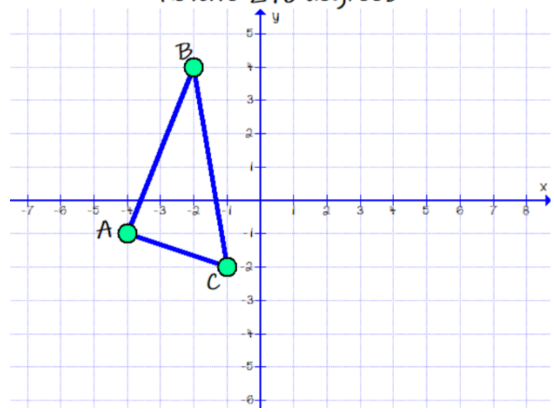
C** (________) becomes (____+-4, ____+1) = C'(_______)

You Try!

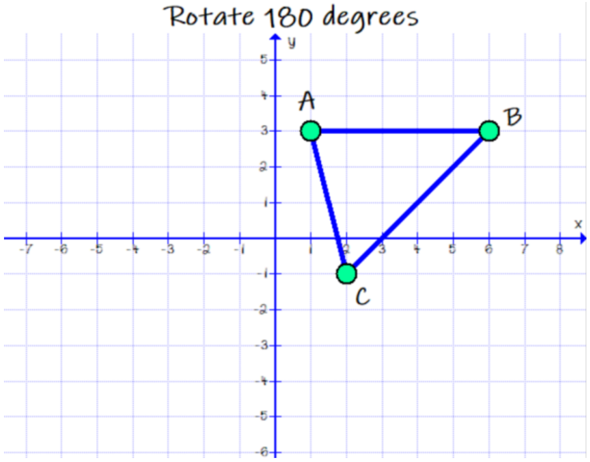
1. Rotate 90 degrees



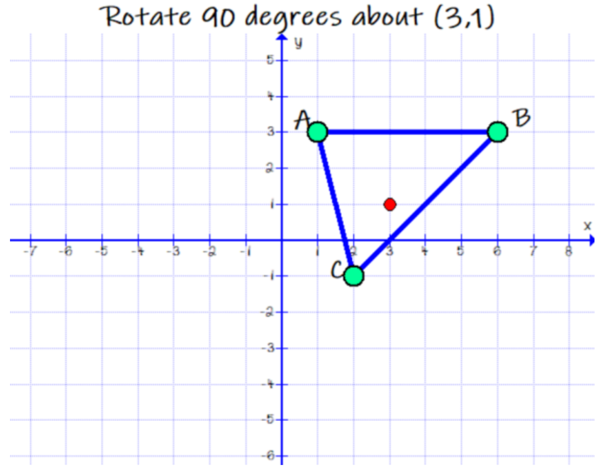
2. Rotate 270 degrees



3. Rotate 180 degrees



4. Rotate 90 degrees about (3,1)



5. Determine the transformation that has occurred from the coordinates:

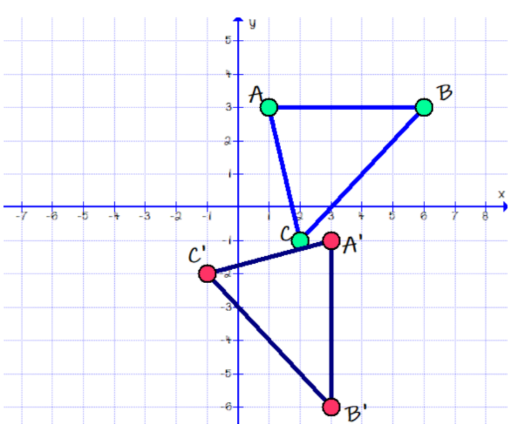
a. A(1,7) → A'(-7,1) ____________________

b. B'(-2,5)→ (5,2) ____________________

c. C (-2,-3)→ C'(2,3) ____________________

6. Determine the transformation that has occurred from the figures:

a.



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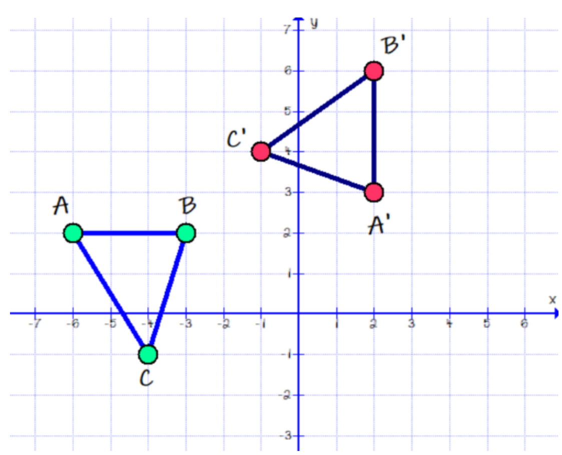
b.



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