Probability

Independent vs. Dependent events

Independent Events

- Two events A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.

- Examples- EX 1. Landing on heads from two different coins; EX 2. rolling a 4 on a die, then rolling a 3 on a second roll of the die.
- Probability of A and B occurring:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Experiment 1

- **A coin is tossed and a 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.**

Experiment 2

- **A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and an eight?**
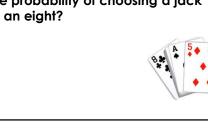
Experiment 3

- **A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and a yellow marble?**

Experiment 4

- **A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with replacement, what is the probability that all three students like pizza?**
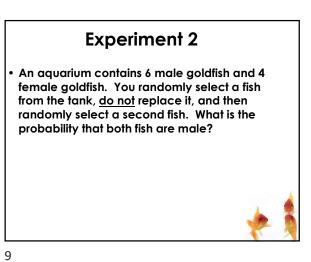
Dependent Events

- Two events A and B, are **dependent** if the fact that A occurs affects the probability of B occurring.
- Examples- Picking a blue marble and then picking another blue marble if I don't replace the first one.
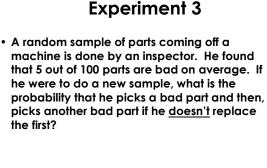- Probability of A and B occurring:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Experiment 1

- A jar contains three red, five green, two blue and six yellow marbles. A marble is chosen at random from the jar. A second marble is chosen <u>without</u> replacing the first one. What is the probability of choosing a green and a yellow marble?

Experiment 2

- An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, <u>do not</u> replace it, and then randomly select a second fish. What is the probability that both fish are male?

Experiment 3

- A random sample of parts coming off a machine is done by an inspector. He found that 5 out of 100 parts are bad on average. If he were to do a new sample, what is the probability that he picks a bad part and then, picks another bad part if he <u>doesn't</u> replace the first?

Independent vs. Dependent

Determining if 2 events are independent

Independent Events

- Two events are independent if the following are true:

$$P(A \mid B) = P(A)$$
$$P(B \mid A) = P(B)$$
$$P(A \text{ AND } B) = P(A) \cdot P(B)$$

- To show 2 events are independent, you must prove one of the above conditions.

Experiment 1

- Let event G = taking a math class. Let event H = taking a science class. Then, G AND H = taking a math class and a science class.
- Suppose P(G) = 0.6, P(H) = 0.5, and
  P(G AND H) = 0.3.
- Are G and H independent?

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Experiment 2

- In a particular college class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair.
- Let F be the event that the student is female. Let L be the event that the student has long hair.
- One student is picked randomly. Are the events of being female and having long hair independent?

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Approach #2

- If they are independent, P(L|F) should equal P(L).
- 0.75 ≠ 0.5

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