Tangent Line Angles
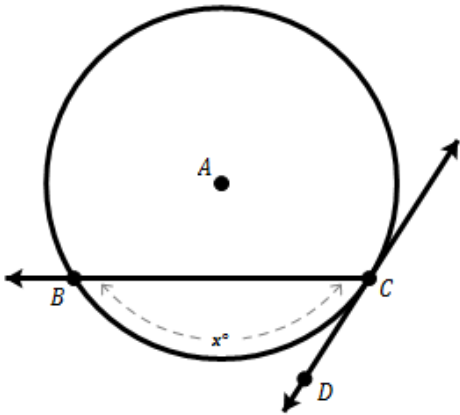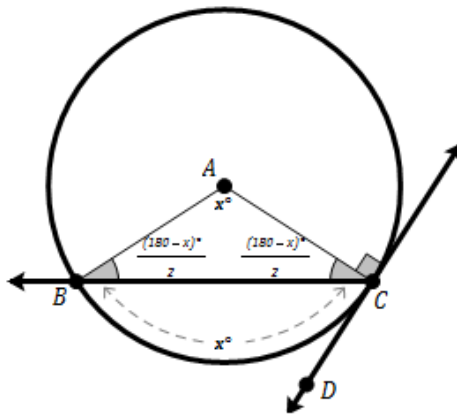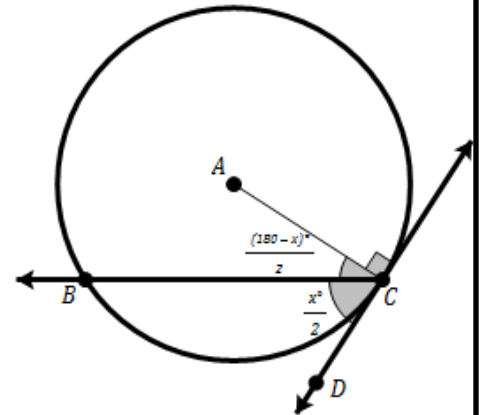
Consider the tangent line $\overleftrightarrow{DC}$ and the ray $\overrightarrow{CB}$ which intercepts arc $\overset{\frown}{BC}$ and has a measure of $x°$.

Draw an auxiliary segment $\overline{AB}$ and $\overline{AC}$ to create an isosceles triangles. We know that $m∡A = x°$ as a central angle and the interior angles of ΔABC sum to 180˚.
So, $m∡B + m∡C = 180° - x$
Also, $m∡B = m∡C$ because they are the base angles of an isosceles triangle. So, $m∡B + m∡B = 180° - x$ which simplifies:
$2 \cdot m∡B = 180° - x$ or

$$m∡B = \frac{180° - x}{2}$$

Finally, $\overline{AC}$ must be perpendicular to $\overleftrightarrow{DC}$ since it is tangent of the circle. The angle $∡DCB$ & $∡ACB$ must sum to 90˚.
So, we can find
$$m∡DCB = 90 - \frac{180° - x}{2}$$
which simplifies:
$$m∡DCB = \frac{x}{2}$$

"The measure of an angle formed by a tangent and a chord drawn to the point of tangency is exactly ½ the measure of the intercepted arc."

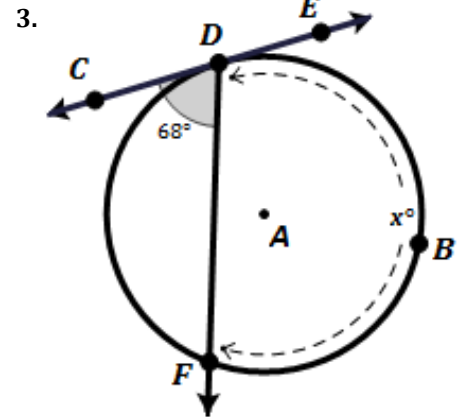**Find the most appropriate value for 'x' in each of the diagrams below. (Assume CE is tangent to the circle.)**
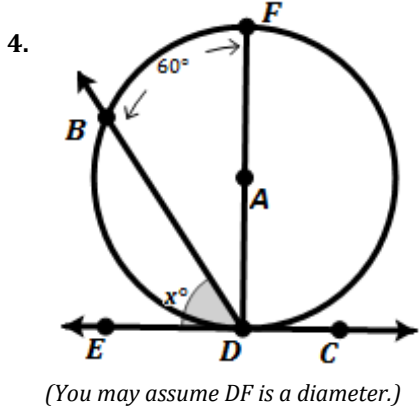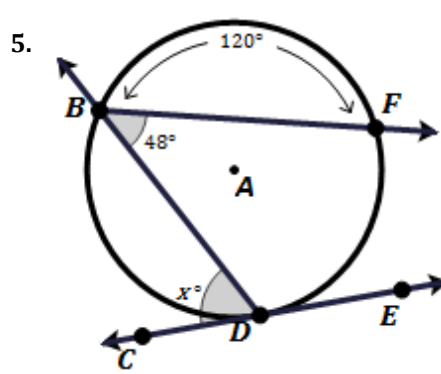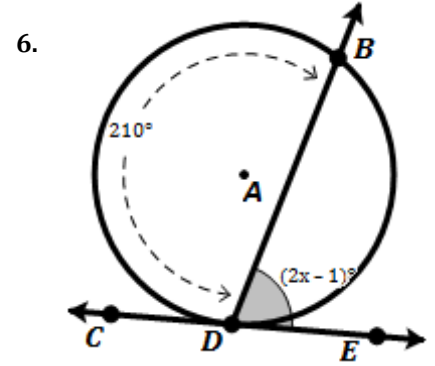
1.

214°

$x =$

2.

140°

$x°$

$x =$

3.

68°

$x°$

$x =$

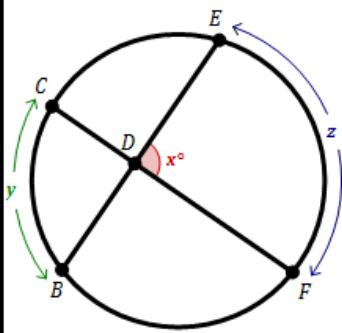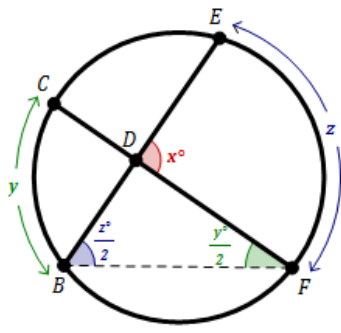**Find the most appropriate value for 'x' in each of the diagrams below. (Assume CE is tangent to the circle.)**

4.

(You may assume DF is a diameter.)

$x =$ ____

5.

$x =$ ____

6.
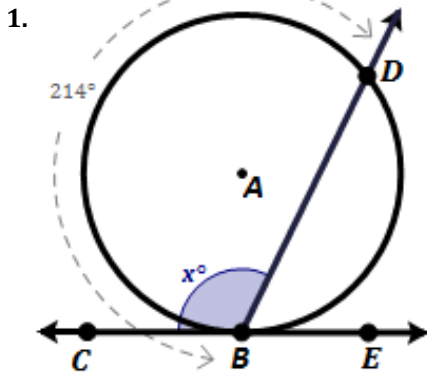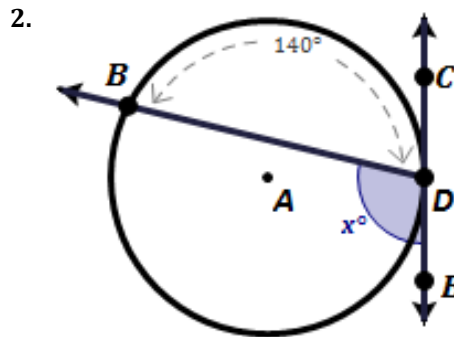
$x =$ ____

Intersecting Chords Interior Angles

|			
Consider the intersecting chords $\overline{BE}$ and $\overline{FC}$ that intercept the arcs $\overset{\frown}{CB}$ and $\overset{\frown}{FE}$.	Draw an auxiliary segment $\overline{BF}$ to create the inscribed angles that we know are half of the intercepted arc. So, $m\angle FBE = \frac{z°}{2}$ and $m\angle BFC = \frac{y°}{2}$	Since triangles interior angles sum to 180°. So we can subtract the 2 angles of triangle $\triangle DBF$ to find angle $m\angle BDF = 180° - \frac{y°}{2} - \frac{z°}{2}$	Finally, since $\angle FDE$ forms a linear pair with $\angle BDF$ we can subtract from 180° to find: $m\angle FDE = 180° - \left(180° - \frac{y°}{2} - \frac{z°}{2}\right)$ which simplifies to $m\angle FDE = \frac{y°+z°}{2}$
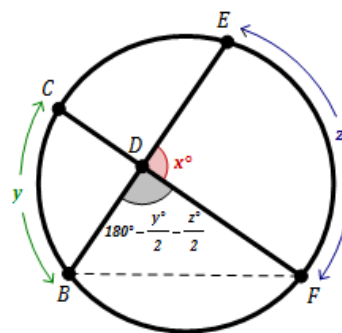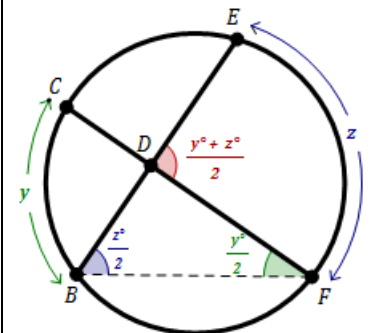
"The measure of an angle formed by two intersecting chords of the same circle is exactly ½ the measure of the sum of the two intercepted arcs."

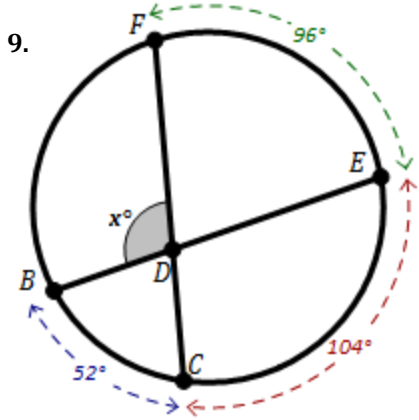**Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)**
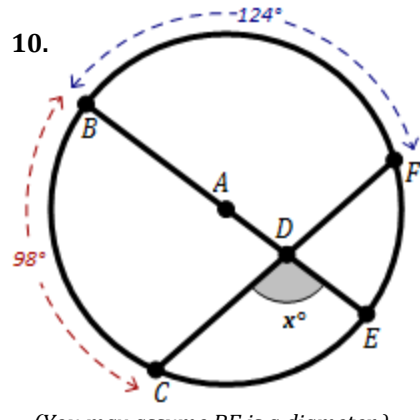
7.

$x =$ ____

8.
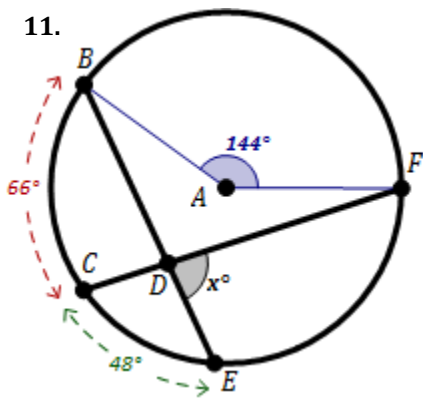

$x =$ ____

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Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)

9.

$x =$ ☐

10.
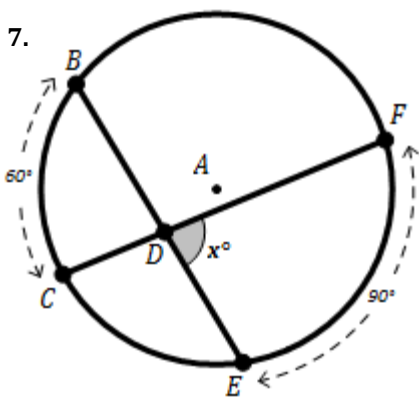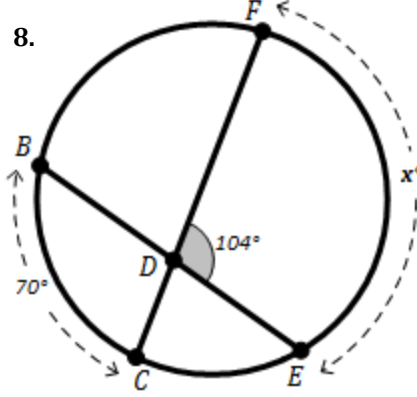
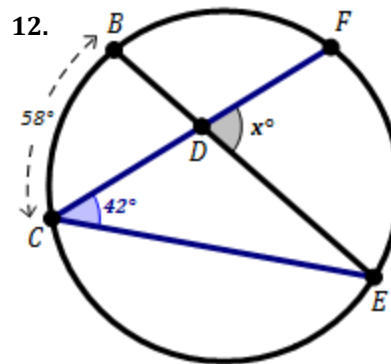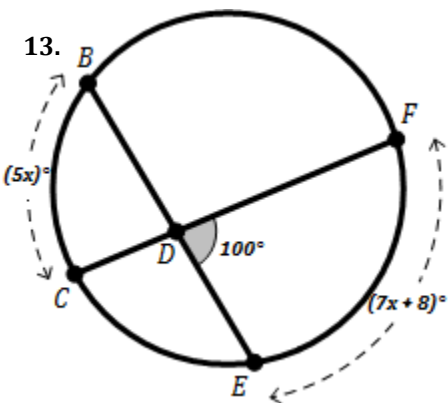*(You may assume BE is a diameter.)*

$x =$ ☐

11.

$x =$ ☐
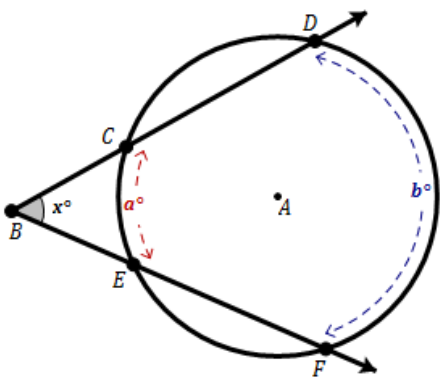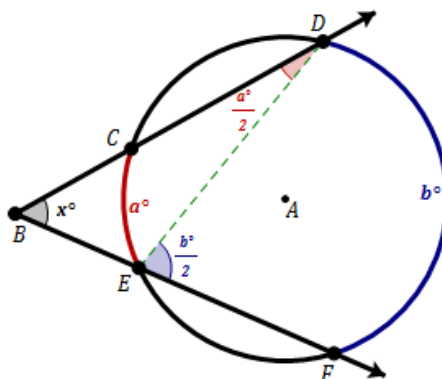
12.

$x =$ ☐

13.

$x =$ ☐

14.

$x =$ ☐
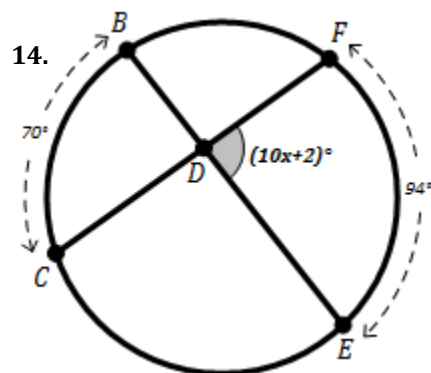
Consider the rays $\overrightarrow{BD}$ and $\overrightarrow{BF}$ which intercepts arc $\overarc{CE}$ and $\overarc{FD}$ which measure $a°$ and $b°$ respectively.

Draw an auxiliary segment $\overline{ED}$. We know that $m\angle DEF = \frac{b°}{2}$ and $m\angle CDE = \frac{a°}{2}$ because each is an inscribed angle.
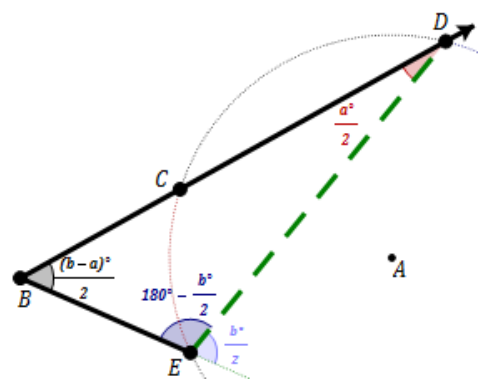
Finally, $m\angle DEB = 180° - \frac{b°}{2}$ since the two angles at point E form a linear pair. Furthermore, $m\angle B = 180° - \left(180° - \frac{b°}{2}\right) - \frac{a°}{2}$ since a triangle's interior angles sum to $180°$ and that would simplify to $m\angle B = \frac{(b-a)°}{2}$.

"The measure of an angle formed on the exterior of a circle by two intersecting secants of the same circle is exactly ½ the measure of the difference of the two intercepted arcs."

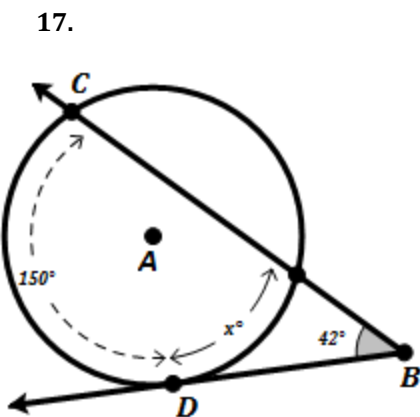**Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)**
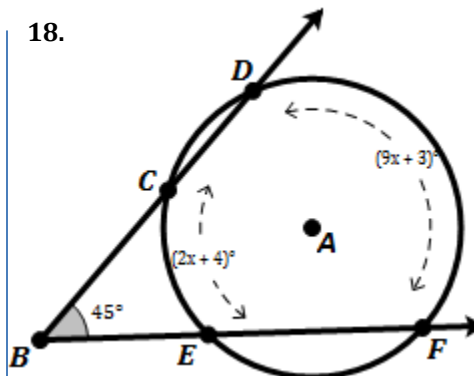
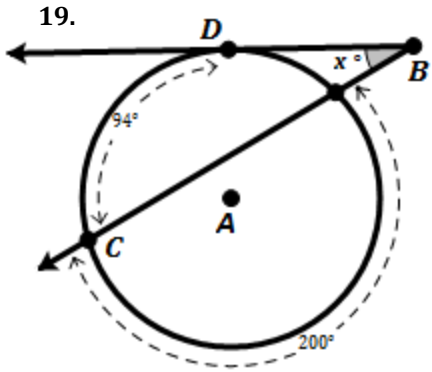**15.**

$x =$ ____

16.

$x =$ ____

17.

$x =$ ____

18.

$x =$ ____

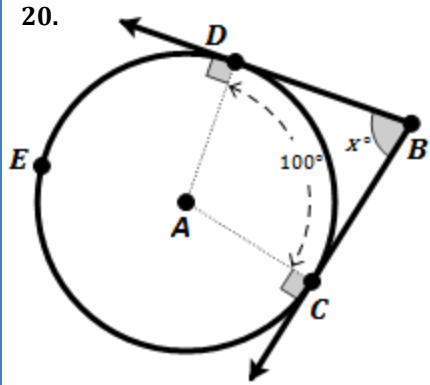**Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)**

19.
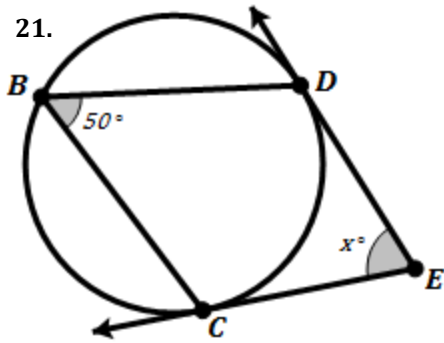

94°
200°
x°

You may assume DB is tangent to the circle.

x = _____

20.

100°
x°

x = _____

21.
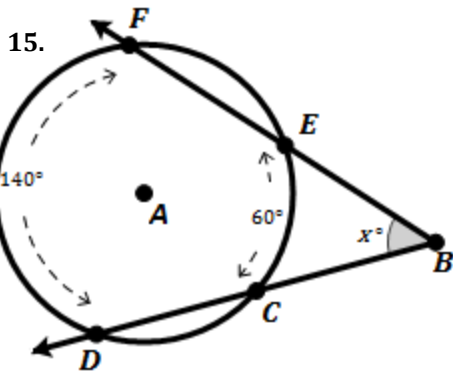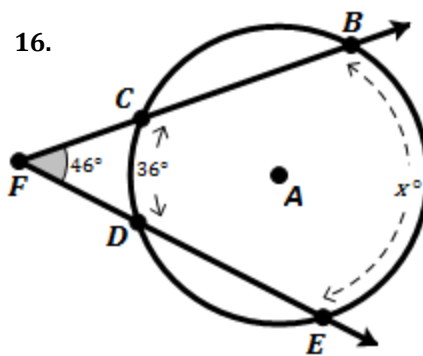
50°
x°

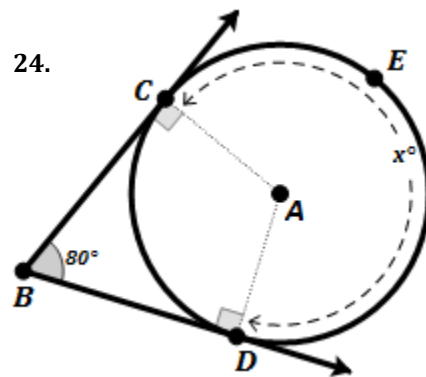*You may assume EC and DE are tangent to the circle.*

x = _____

22.

122°
156°
x°

You may assume DB is tangent to the circle.

x = _____

23.

x°
110°
140°

x = _____

24.

x°
80°

x = _____